# ”Short-sale constraints and real investments”

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# Short-sale constraints and real investments

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Stockholm, August 2019

# Research questions

**1. How do short-sale constraints influence the informational efficiency of market prices?**

- Short-sale constraints: costs of shorting or difficult shorting.

  Rebate rates (Jones-Lamont, 2002), regulatory or legal restrictions (Almazan et al, 2004), search frictions (Duffie-Garleanu-Pedersen, 2002).

- Informational efficiency: the ability of prices to aggregate / transmit information.

  Forecasting price efficiency (FPE) vs revelatory price efficiency (RPE) (Bond-Edmans-Goldstein, 2012).

**2. How do they affect the link of prices and economic activity?**

# Prevalent view about short-sale constraints

*"Short-selling improves liquidity and price informativeness in normal times*

*... but [it] reduces the ability of a firm to raise equity capital or to borrow money, and makes it harder for banks to attract deposits."*
*(SEC Press Release 2008-211, 19 September 2008)*

# This paper

- Informational effect of short-sale constraints:

  **They change the information content of security prices,**

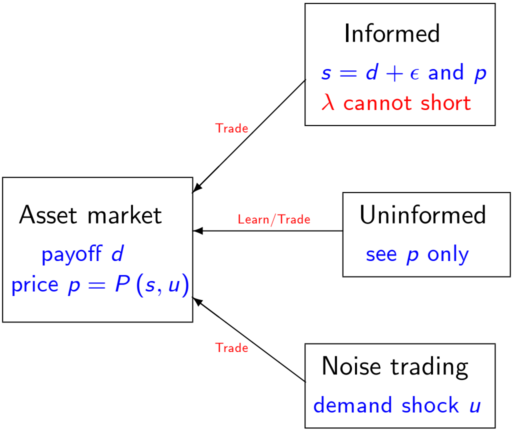
  - Prices contain less of the information of traders (FPE $\downarrow$), but...
  - ...provide more information to some agents with additional private information (RPE $\uparrow$).

  **hence can have real economic implications.**

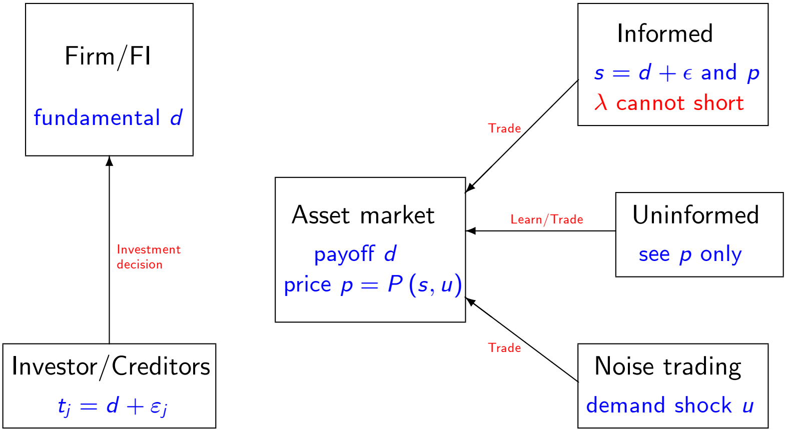
  - These agents are more willing to invest in good/profitable projects (Allocational E $\uparrow$).

- **Contribution**: analyze price informativeness under feedback and trading constraints, and to provide an informational argument in support of short-sale constraints.

# Structure


Informed
$s = d + \epsilon$ and $p$
$\lambda$ cannot short
Trade
Asset market
payoff $d$
price $p = P(s, u)$
Learn/Trade
Uninformed
see $p$ only
Trade
Noise trading
demand shock $u$

# Structure


Firm/FI
fundamental $d$
Investment decision
Investor/Creditors
$t_j = d + \varepsilon_j$
Asset market
payoff $d$
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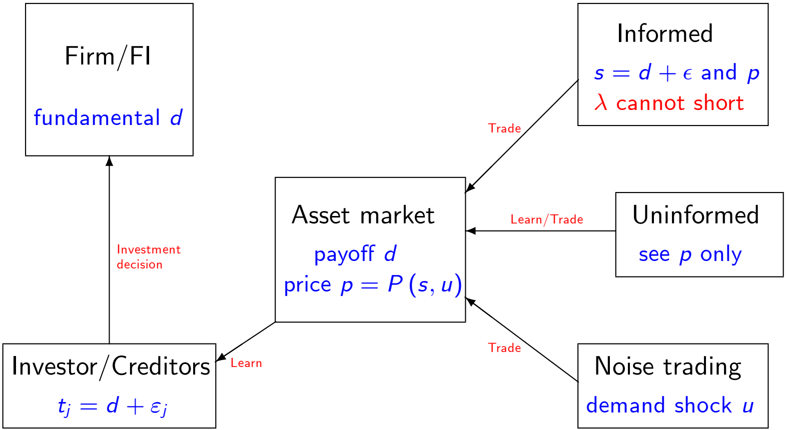
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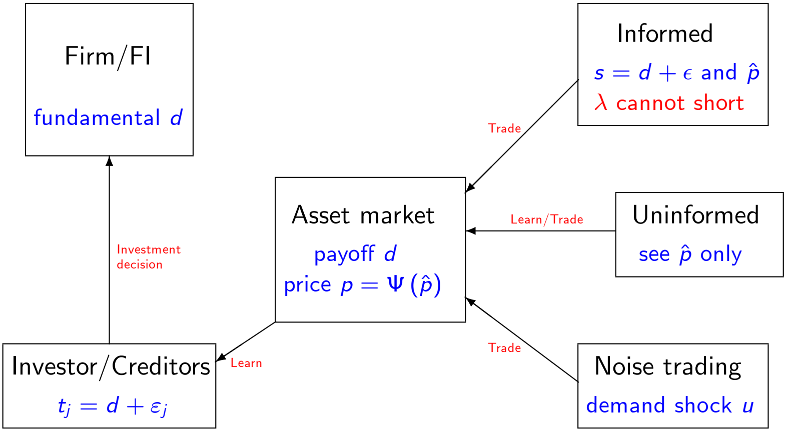
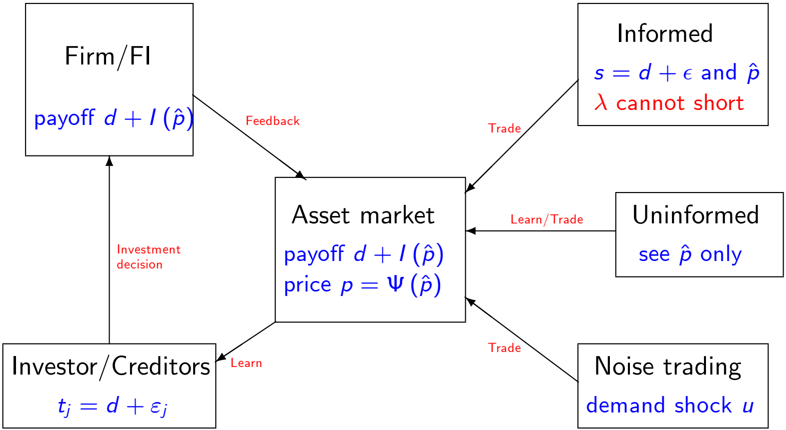
# Structure (cont'd)



# Structure extension (not today)



# Model: Outline

- Asset market:
  - Traded security and firm assets are correlated: other firm equity from the industry, or a derivative on the firm.
  - Noisy RE with asymmetric information (Grossman-Stiglitz).
  - Short-sale constraints on some informed traders.
- Firm with investors/short-term creditors:
  - Either invest (roll over short-term) or withdraw.
  - Face strategic complementarities.
  - Have private and public info, i.e., learn from a market price.

# 1. Asset market: Setup

- **Securities**:
  - Risky asset with payoff $d \sim N\left(0, \sigma_d^2\right)$, fixed supply $S$; price $p$. Noise traders demand $u \sim N\left(0, \sigma_u^2\right)$.
  - Bond with riskless rate 0, perfectly elastic supply.
- Rational **agents**: Maximize expected utility with CARA-coefficient $\alpha$:

$$E\left[-\exp\left(-\alpha W_i\right)|\mathcal{I}_i\right],$$

with $W_i$ final wealth, $\mathcal{I}_i$ information set of trader $i \in [0,1]$.

- Classes are different in **information**:
  - Informed traders: measure $\omega$, receive signal $s = d + \epsilon$, $\epsilon \sim N\left(0, \sigma_\epsilon^2\right)$.
  - Uninformed: measure $1 - \omega$, observe price only.
- $0 \leq \lambda < 1$ proportion of informed traders are subject to **short-sale constraints**: $x_i \geq 0$.

# Equilibrium concept

- **Noisy REE**: $\{P(s,u), x_I(s,p), x_{IC}(s,p), x_U(s,p)\}$ such that:
- Demands are optimal for informed traders:

$$\max_{x_I} E\left[-\exp\left(-\alpha\left[W_I^0 + x_I(d-p)\right]\right)|s, P=p\right],$$

$$\max_{x_{IC}} E\left[-\exp\left(-\alpha\left[W_{IC}^0 + x_{IC}(d-p)\right]\right)|s, P=p\right] \text{ s.t. } x_{IC} \geq 0.$$

- Demands are optimal for uninformed traders:

$$\max_{x_U} E\left[-\exp\left(-\alpha\left[W_U^0 + x_U(d-p)\right]\right)|P=p\right],$$

- Market clears:

$$\omega(1-\lambda)x_I(s, P(s,u)) + \omega\lambda x_{IC}(s, P(s,u)) + (1-\omega)x_U(P(s,u)) + u = S.$$

# Asset prices and short-sale constraints

- With SC ($\lambda > 0$), "conjecture and verify" **does not work**, but derive $\mathcal{F}_U$ from $MC$.

  Kreps (1977), Yuan (2005), Breon-Drish (2015), Pálvölgyi and Venter (2015).

- Plug same linear $I$ demand into $MC$:

$$\omega (1-\lambda) \frac{\beta_s s - p}{\alpha \sigma^2_{d|s}} + \omega \lambda \mathbf{1}_{s \geq \frac{1}{\beta_s} p} \frac{\beta_s s - p}{\alpha \sigma^2_{d|s}} + (1-\omega) x_U(p) + u = S,$$

and rearrange to obtain

$$\hat{p} = \begin{cases} \frac{1}{C}(s-E) + u & \text{if } s \geq E \\ \frac{1}{D}(s-E) + u & \text{if } s < E, \end{cases}$$

where in equilibrium we must have $\hat{p} = S - (1-\omega) x_U(p)$, $E = \frac{p}{\beta_s}$, and $D = \frac{1}{1-\lambda} C > C = \frac{\alpha\omega}{\sigma^2_\epsilon}$.

# Asset prices and short-sale constraints – Special case: uninformative prior

**Theorem**

*For $\lambda = 0$, there exists a linear equilibrium of the asset market with*

*$P_{GS}(s, u) = s + Cu$ and constant $C$.*

**Theorem**

*For $\lambda > 0$, stock price is given by the piecewise linear equation*

$$P_{SC}(s, u) = \begin{cases} s + C(u - E) & \text{if } u < E \\ s + D(u - E) & \text{if } u \geq E \end{cases}$$

*with $C = \alpha\sigma_{\varepsilon}^{2}/\omega$ and $D = C/(1 - \lambda) > C$ and $E$ constants.*

# Asset prices and short-sale constraints – General case

### Theorem

*For $\lambda = 0$, there exists a linear equilibrium of the asset market with*

$P_{GS}(s,u) = A + B\left(\frac{1}{C}s + u\right)$ *and constants* $A$, $B$, *and* $C$.

### Theorem

*For $\lambda > 0$, stock price is given by the implicit equation*

$P_{SC}(s,u) = \Psi\left(\hat{p}\left(P_{SC}(s,u)\right)\right)$,

*where* $\Psi(.)$ *is a strictly increasing function and*

$$\hat{p}(p) = \begin{cases} \frac{1}{C}\left(s - \frac{p}{\beta_s}\right) + u & \text{if } s \geq \frac{p}{\beta_s} \\ \frac{1}{D}\left(s - \frac{p}{\beta_s}\right) + u & \text{if } s < \frac{p}{\beta_s} \end{cases}$$

*with* $C = \alpha\sigma_\varepsilon^2/\omega$ *and* $D = C/(1-\lambda) > C$ *constants.*

# Price properties and empirical support

- Price informativeness **FPE decreases**:

  $Var\left[d|P_{SC}=p\right] > Var\left[d|P_{GS}=p\right]$

  ...but **asymmetrically**, as prices that are higher than the signal are more sensitive to the demand shock.

- The model predicts:
  1. Increase in volatility.
     - Ho (1996), Boehmer, Jones and Zhang (2013).
  2. Price discovery is slowed down, especially in down markets.
     - Saffi and Sigurdsson (2011), Beber and Pagano (2013).
  3. Announcement-day return ($d-p$; made between date 0 and 1) is more left-skewed, and larger in absolute terms.
     - Reed (2013).
  4. Market return ($p-E\left[d\right]$; made between date $-1$ and 0) is less negatively skewed.
     - Bris, Goetzmann and Zhu (2007).

# 2. Learning from prices with short-sale constraints

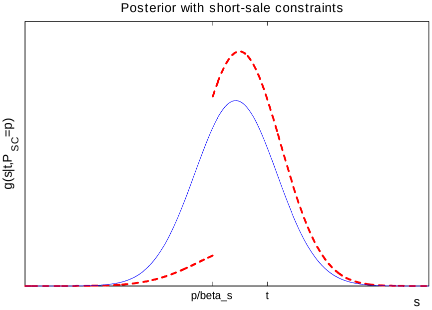
- Price signal:

$$\hat{p} = \begin{cases} \frac{1}{C}\left(s - \frac{p}{\beta_s}\right) + u & \text{if } s \geq \frac{p}{\beta_s} \\ \frac{1}{D}\left(s - \frac{p}{\beta_s}\right) + u & \text{if } s < \frac{p}{\beta_s} \end{cases}$$

- If informed traders are **buying** ($s \geq \frac{1}{\beta_s} p$), the price signal has the **same precision** as without short-sale constraints.
- If they are **shorting** ($s < \frac{1}{\beta_s} p$), demand shock is more prevalent.
  $\rightarrow$ Under short-sale constraints the same piece of public information $\hat{p}$ is a result of a lower $s$ signal.

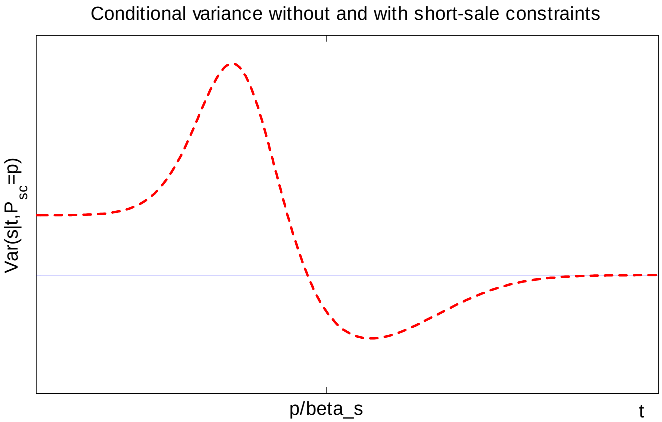
# Conditional distribution for high private signals

- Suppose **one more source** of info: $t = d + \eta$ with $\eta \sim N\left(0, \sigma_t^2\right)$.
- When $t$ is high, states with $s < \frac{1}{\beta_s} p$ are unlikely given private signal.
- For fixed $t$ and $p$, those states are even more unlikely under short-sale constraints as they correspond to lower $s$.

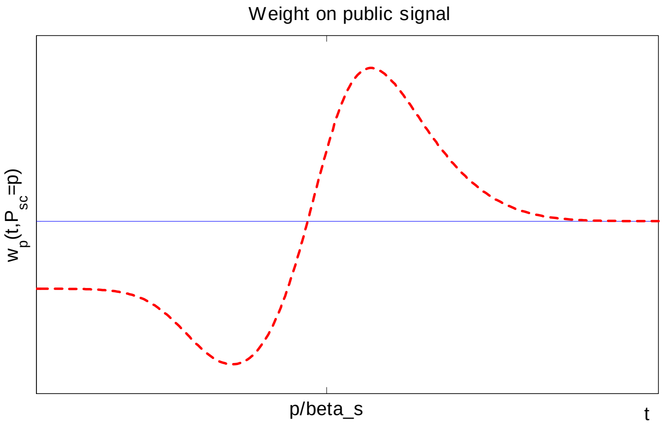



- For high $t$ agents, short-sale constraints can help to **rule out left tail events**. $\rightarrow$ **More precise** posterior, **RPE** $\uparrow$.

# Short-sale constraints and information percision



# Short-sale constraints and information percision (cont'd)



# 3. Application #1 - Single investor

- Single risk averse investor deciding the scale of investment; observes with private signal $t = d + \eta$, $\eta \sim N\left(0, \sigma_t^2\right)$, and $p$:

$$E\left[U|t,p\right] = \max_k E\left[d|t,p\right] k - \frac{c}{2} Var\left[d|t,p\right] k^2$$

- FOC implies

$$k = \frac{E\left[d|t,p\right]}{cVar\left[d|t,p\right]}$$

and we have

$$E\left[U|t,p\right] = \frac{E^2\left[d|t,p\right]}{2cVar\left[d|t,p\right]}$$

- Short-sale constraints **can increase the expected utility** of an investor with high $t$ via the effect on $Var\left[d|t,p\right]$,
  - and unconditional expected utility can be higher too (numerical).

# 4. Application #2 - Investor coordination: Setup

- Investors are risk neutral, receive net payoffs:

| | roll over ($i_j = 1$) | not ($i_j = 0$) |
|---|---|---|
| solvent ($d \geq 1 - I$) | $1 - c$ | 0 |
| fails ($d < 1 - I$) | $-c$ | 0 |

  where $c \in (0, 1)$, and proportion that rolls over: $I = \int i_j \mathrm{d}j$.

- Investor $j$ receives private signal $t_j = d + \eta_j$, $\eta_j \sim N\left(0, \sigma_t^2\right)$, and observes $p$.
- Optimal action is to invest iff $\Pr\left(\text{firm solvent} | t_j, p\right) \geq c$.
- Key question: How precisely can an agent predict what others know?

# Equilibrium

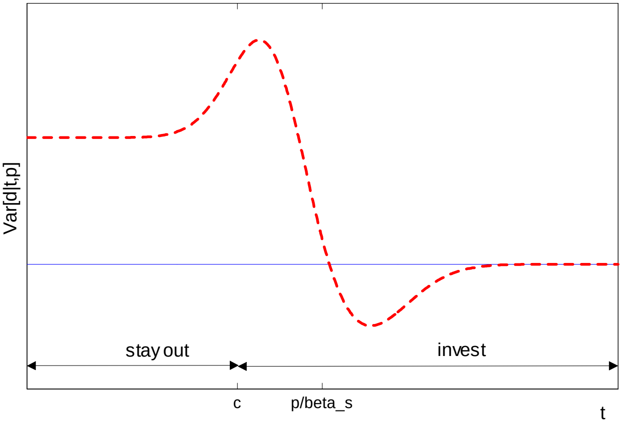
- Concept: Monotone PBE $(t^*(p), d^*(p))$ such that, for a given $p$
  - Investor $j$ invests if and only if $t_j \geq t^*(p)$.
  - Firm remains solvent if and only if $d \geq d^*(p)$.

## Theorem

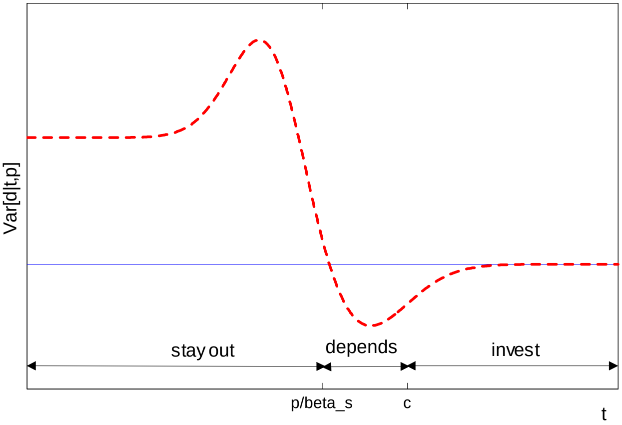
*In the economy without short-sale constraints, when $\sigma_t \to 0$, there exists a unique equilibrium with $t^* = d^* = c$.*

*In the economy with short-sale constraints, when $\sigma_t \to 0$, there exist either one or two equilibria. The equilibrium with $t^* = d^* = c$ always exists. Moreover, if $p < \beta_s c$, there exists an equilibrium with $t^* = d^* = p/\beta_s$.*

# No multiplicity for high p



# Multiple equilibria for low p



# Efficiency with short-sale constraints

- FPE $\downarrow$, RPE $\uparrow$ for a subset of investors.
- $p < \beta_s c$ implies more capital provision in the second equilibrium:
  - A firm with $d > 0$ has higher probability to remain solvent.
  - Allocational E $\uparrow$ in the real economy: more investment in good projects.
- Different from global games with multiplicity, because the second equilibrium is always better: SC provide "positive" public information.
  - In contrast to, e.g., Angeletos-Werning (2006) and Ozdenoren-Yuan (2008).
- (Not welfare.)

# Empirical/Policy implications

- When few investors (i.e., no coordination problem): financing is not affected by short-sale constraints.
- When multiple investors:
  - (Tighter) constraint in the market of the asset (higher $\lambda$) leads to smaller rollover/coordination risk, i.e., easier/cheaper ST financing.
  - The benefit of short-sale constraints on rollover is more pronounced for high proportion of ST debt...
  - ... and is an inverted U-shaped function of $c$.
- Regulation: if $c \uparrow$ (return for investors $\downarrow$), increase $\lambda$.
  - Tradeoff between worse security market conditions and fewer firm defaults.

# Related literature

- Information in asset prices under trading frictions and FPE.

  E.g. Miller (1977), Diamond-Verrecchia (1987), Yuan (2005, 2006), Bai et al (2006), Wang (2016).

  ⟶ Contribution: info effect for real investments (outside security market).

- Feedback and RPE.

  E.g. Hayek (1945), Leland (1992), Ozdenoren-Yuan (2008), Goldstein-Gümbel (2008), Goldstein et al (2013), Liu (2015); Bond et al (2010), Bond-Goldstein (2015).

  ⟶ Contribution: trading constraint in the feedback process.

- Bank runs and global games.

  E.g. Diamond-Dybvig (1983), Morris-Shin (1998, 2002, 2003, 2009).

  ⟶ Contribution: constraints introduce a broad class of multiple equilibria.

# Conclusion

- Due to short-sale constraints, price contains less information (FPE $\downarrow$)...
- ... but it provides more information to some agents with additional information (RPE $\uparrow$).
- Real effect: these agents are more willing to invest in good/profitable projects.
- In a coordination game it can lead to multiplicity, with the second equilibrium having higher allocative efficiency.
- Broader implications: Trading frictions change the ability of prices to incorporate and transmit information to decision makers.

# Appendix

# Appendix: Grossman-Stiglitz (1980) equilibrium

- Usual technique to solve the REE:
  - Conjecture price function, derive optimal demands given info, confirm that the price clears the market; see, e.g. Grossman and Stiglitz (1980), Brunnermeier (2001), Vives (2010), Veldkamp (2011).
- **Suppose** $\lambda = 0$; **assume a linear** form $P(s,u) = A + B\left(\frac{1}{C}s + u\right)$.
- **Joint normality** implies normal posteriors, so optimization program reduces to a mean-variance problem, and optimal demand is

$$x = \frac{E[d|\mathcal{I}] - p}{\alpha Var[d|\mathcal{I}]}.$$

- $I$ traders know $s$, price provides no additional information, so **optimal I demand** is

$$x_I(s,p) = \frac{\beta_s s - p}{\alpha \sigma^2_{d|s}}.$$

# Appendix: Grossman-Stiglitz (1980) equilibrium (cont'd)

- $U$ traders do not observe $s$, but they can partially infer it through the price signal:

$$P(s,u) = p = A + B\left(\frac{1}{C}s + u\right) \quad \Longrightarrow \quad \hat{p} \equiv \frac{p-A}{B} = \frac{1}{C}s + u.$$

- Combining with their priors, we compute $E[d|p] = E[d|\hat{p}]$ and $Var[d|p] = Var[d|\hat{p}]$, and get **uninformed demand**

$$x_U(\hat{p}) = \frac{\beta_{d|\hat{p}}\hat{p} - p}{\alpha\sigma^2_{d|\hat{p}}}.$$

## Theorem

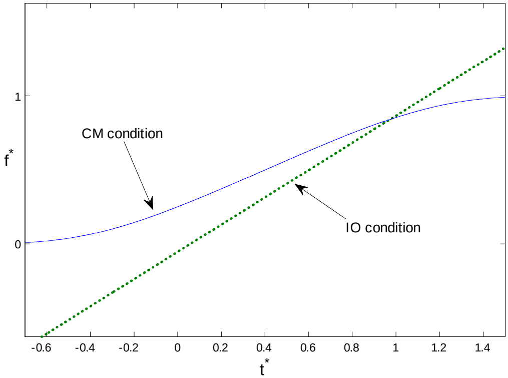
*There exists an equilibrium of the asset market with the price function given in the linear form $P_{GS}(s,u) = A + B\left(\frac{1}{C}s + u\right)$ with appropriate constants $A$, $B$, and $C$.*

# Appendix: Equilibrium

- Concept: Monotone PBE $(t^*(p), d^*(p))$ such that, for a given $p$
  - Investor $j$ invests if and only if $t_j \geq t^*(p)$.
  - Firm avoids bankruptcy if and only if $d \geq d^*(p)$.

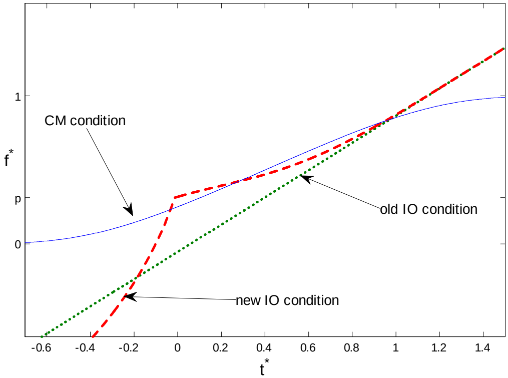
- Solution:
  - Critical Mass condition: if creditors with $t_j \geq t^*$ roll over, which is the marginal surviving firm $(d^*)$?
  - Individual Optimality condition: if a firm with $d \geq d^*$ stays solvent, what is the optimal $t^*$ strategy?

# Appendix: Equilibrium without short-sale constraints




- Unique equilibrium if $\sigma_t \to 0$, with $t^* = d^* = c$.

# Appendix: Equilibria with short-sale constraints




- Two equilibria even when $\sigma_t \longrightarrow 0$: (i) $t^* = d^* = c$; or

  (ii) $t^* = d^* = p/\beta_s$, only if $p < \beta_s c$.

# Towards welfare

- Calculate (numerically) the unconditional expected utilities for informed, uninformed and noise traders under short-sale constraints.
  - Latter: traders with CARA, who have to buy $u$ units of the risky asset (= constrained "supply-informed" agents).
  - Alternatively, simply calculate expected profits.
- Prices under short-sale constraints reveal less about the signal of informed agents, but uninformed can make more money on noise traders.

## Theorem (Proposition)

*Under short-sale constraints, the unconditional expected utilities of informed traders are higher/lower than in GS, those of uninformed agents and noise traders are lower than in GS. Overall, "welfare" (= weighted average of expected utilities) is lower under short-sale constraints.*