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Adverse Selection and Liquidity: From Theory to Practice

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Adverse Selection and Liquidity

Questions

- Is there a simple empirical measure of liquidity?
- How can theoretical liquidity parameters, like λ, be implemented empirically?
- How can empirical measures of liquidity be connected, through adverse selection, to properties of prices?

Answers: Use general principles

- Invariance implies a simple measure of liquidity: $L = \left(\frac{m^2 PV}{C\sigma^2}\right)^{1/3}$
- Liquidity L can be mapped to permanent price impact λ, temporary price impact κ, funding and trading liquidity.
- Theoretical dynamic models link L to resiliency of prices ρ and error variance of prices Σ.

Noise Trading and One-Period Models

One-period models are like block trading models.

- 1970s: Traditional ideology that financial markets are perfectly competitive. Grossman and Miller (1988): Customers demand immediacy. Perfect competition consistent with noise traders trading immediately.
- Kyle (1985): Breaks tradition: Large trader with monopolistic access to private information reduces price impact by reducing quantity traded.
  - "Market-order model" capture hybrid features of organized exchanges of 1980s: No limit orders but anonymity.
- Kyle (1989): Imperfectly competitive equilibrium in demand schedules approximates organized exchange.
  - Traders exercise monopoly power over private information by trading less aggressively than perfect competitors.
- Lee and Kyle (2018, 2017): In symmetric setup like Diamond and Verrecchia (1982) but with imperfect competition: prices are same as with competition, quantities are smaller, vanishing noise makes markets infinitely non-competitive, contradicting Grossman and Stiglitz (1980).

Dynamic Models

In dynamic models, traders have incentives to trade many times:

- Kyle (1985): Large trader trades gradually over time but noise traders demand immediacy.
  - Noise traders could reduce costs by trading gradually over time, as in model of Vayanos (1999).
- Kyle, Obizhaeva, Wang (2018): Well-specified stationary dynamic model of smooth trading on flow of private information with continuous limit order book. Trade motivated by overconfidence on private information.
  - Instantaneous liquidity vanishes; smooth trading as a flow over time, with permanent price impact λ and temporary price impact κ.
  - Prices reveal information immediately. Anomalies vanish when markets extremely illiquid. Liquidity leads to Keynesian beauty contest.
- Kyle and Lee (2018): Propose continuous scaled limit orders to end high frequency arms race; alternative to frequent batch auctions of Budish, Cramton, Shim (2015).

Invariance: From Theory to Practice

It is difficult to use theoretical microstructure models to make empirically operational predictions. Invariance makes it possible to

- Map parameters of theoretical models (order flow imbalances, quality of private information) into empirically observable quantities (dollar volume, return volatility).
- Infer difficult-to-observe market characteristics (market impact, average bet size, number of bets, pricing accuracy, resiliency) from easy-to-observe characteristics (volume and volatility).
- Derive precise empirical measures of trading and funding liquidity.
- Derive "universal price impact formula" which is a function of a few asset characteristics and applies to all assets simultaneously.
- Build a benchmark for assessing the role of industrial organization parameters (dealer markets, specialist system, Reg NMS or MiFiD), frictions (tick size, lot size), technology (high frequency trading).

Derivation of Invariance Formulas

There are many ways to derive invariance formulas:

- Meta-model: Reduced-form microstructure model.
- Dimensional analysis and leverage neutrality: Like physics.
- Kyle–Obizhaeva (ECMA, 2016): Ad hoc invariance hypotheses: Risk transfer of bets and transactions costs of bets the same for bets which transfer the same risk per unit of business time (arrival rate of bets).
- Dynamic theoretical model of Kyle–Obizhaeva (WP, 2018) in spirit of Kyle (1985) and Glosten–Milgrom (1985).

Market Microstructure Variables

Easy-to-observe quantities include price, volume, and volatility:

$$\text{Price} = P_{jt} = 40.00 \text{ dollars/share}$$

$$\text{Trading Volume} = V_{jt} = 1.00 \text{ million shares/day}$$

$$\text{Returns Volatility} = \sigma_{jt} = 0.02/\text{day}^{1/2}$$

Hard-to-measure quantities that vary greatly across assets and time include bet size, number of bets, and the price impact coefficient:

$$\text{Size of Bet} = Q_{jt} = 10\,000 \text{ shares}$$

$$\text{Number of Bets} = \gamma_{jt} = 100/\text{day}$$

$$\text{Execution Horizon} = H_{jt} = 1 \text{ day}$$

$$\text{Price Change per Bet} = \Delta P_{jt} = 0.04 = \text{ dollars/share}$$

$$\text{Price Impact Coefficient} = \lambda_{jt} = 5 \times 10^{-5} \text{dollars/share}^2$$

$$\text{Price Error} = \Sigma^{1/2} = \text{ var}^{1/2}\left[\log\left(\frac{F}{P}\right)\right] = \log(2) \text{ (dimensionless)}$$

$$\text{Price Resiliency} = \rho = 0.0040/\text{day (dimensionless)}$$

Market Microstructure Variables (continued)

Invariant Parameters: Hard-to-measure variables which (we hypothesize do) not vary much, if at all:

$$\text{Avg Dollar Cost per Bet} = C = 2000 \text{ dollars},$$

$$\text{Moment ratio} = m = \frac{\mathrm{E}\left[|Q|\right] \cdot \sqrt{\mathrm{E}\left[|Q|^{2\beta}\right]}}{\mathrm{E}\left[|Q|^{\beta+1}\right]} = \sqrt{2/\pi} \text{ (dimensionless)}$$

$$\text{Moment ratio} = m_\beta = \frac{\left(\mathrm{E}\left[|Q|\right]\right)^{\beta+1}}{\mathrm{E}\left[|Q|^{\beta+1}\right]} = \sqrt{2/\pi} \text{ (dimensionless)}$$

$$\text{Probability a bet is informed} = \theta = 1/2 \text{ (dimensionless)},$$

$$\text{Information content of signal} = \tau = 0.0080^2 \text{ (dimensionless)}.$$

$$\text{Information content of a bet} = \theta^2\tau = 0.0040^2 \text{ (dimensionless)},$$

$$\text{Cost of an Informative Signal} = \bar{c}_I = 2000 \text{ dollars}.$$

Notation for Linear Price Impact

Finance theorists like linear models of price impact:

$$\text{Price Impact} = \Delta P = \lambda \cdot Q$$

Price impact cost in dollars:

$$\text{Dollar Price Impact Cost} = \lambda \cdot Q^2$$

Price impact cost G as fraction of value traded:

$$G = \frac{\Delta P}{P} = \frac{\lambda \cdot Q^2}{P \cdot Q} = \frac{\lambda \cdot Q}{P}$$

Note: λ has units of dollars per share-squared, Q has units of shares, P has units of dollars per share, so G is dimensionless.

Approach I: Meta-Model

Suppose power function price impact for a bet Q: $\Delta P = \lambda \cdot Q^{\beta}$.
Define γ = number of bets per day.
Now assume three-equation "meta-model":

$$V = \gamma \cdot \mathrm{E}\left[|Q|\right] \qquad \text{(Definition of volume)}$$

$$\sigma^2 = \gamma \cdot \mathrm{E}\left[\left(\frac{\Delta P}{P}\right)^2\right] \qquad \text{(Bets generate all volatility),}$$

$$\mathrm{E}\left[(\Delta P)^2\right] = \lambda^2 \cdot \mathrm{E}\left[|Q|^{2\beta}\right] \qquad \text{(Volatility from one bet).}$$

Three easy-to-measure quantities: V, σ, P.
Five hard-to-measure quantities:

$$\gamma, \quad \mathrm{E}\{|Q|\}, \quad \mathrm{E}\{|Q|^{2\beta}\}, \quad \mathrm{E}\left[(\Delta P)^2\right], \quad \alpha.$$

Since three log-linear constraints, need to know e.g. γ or $\mathrm{E}\{|Q|\}$ to implement solution empirically.

Empirical Motivation for Invariance

- Emprical Problem: Parameters like bet arrival rate γ and bet size $\mathrm{E}\{|Q|\}$ are hard to measure or estimate.
- Can they be replaced with a parameter that is either easier to estimate or does not vary much across assets?
- Empirical strategy: Introduce a parameter C (dollars), which does not vary (much) across assets and time.
- Use "invariant" parameter C to replace hard-to-measure and varying across assets and time parameters γ or $\mathrm{E}\{|Q|\}$.
- Assume "transactions cost invariance": Ex ante expected dollar cost of a bet is constant (almost?)

$$C = \mathrm{E}\{|Q| \cdot \Delta P\} = \lambda \, \mathrm{E}\left[|Q|^{1+\beta}\right].$$

Augmented Four Equation Meta-Model

Add transactions cost invariance to obtain four equations:

$$V = \gamma \cdot \; \mathrm{E}\{|Q|\} \qquad \text{(Definition of volume)}$$

$$\sigma^2 = \gamma \cdot \; \mathrm{E}\left[\left(\frac{\Delta P}{P}\right)^2\right] \quad \text{(Bets generate all volatility),}$$

$$\mathrm{E}\left[(\Delta P)^2\right] = \lambda^2 \cdot \; \mathrm{E}\left[|Q|^{2\beta}\right] \qquad \text{(Price Impact of one bet),}$$

$$C = \lambda \; \mathrm{E}\left[|Q|^{1+\beta}\right] \quad \text{(Dollar impact cost of a bet).}$$

Need two invariant moment ratios:

$$m := \frac{\mathrm{E}\{|Q|\} \cdot \sqrt{\mathrm{E}\{|Q|^{2\beta}\}}}{\mathrm{E}\{|Q|^{\beta+1}\}}, \qquad m_\beta := \frac{(\;\mathrm{E}\{|Q|\})^{\beta+1}}{\mathrm{E}\{|Q|^{\beta+1}\}}.$$

Assume six parameters are easy to measure or almost constant:

$$P, \quad V, \quad \sigma, \quad C, \quad m, \quad m_\beta.$$

Solve six equations and six hard-to-measure parameters:

$$\gamma, \quad \alpha, \quad \mathrm{E}\left[\Delta P^2\right], \quad \mathrm{E}\{|Q|\}, \quad \mathrm{E}\{|Q|^{1+\beta}\}, \quad \mathrm{E}\{|Q|^{2\beta}\}.$$

Solution with Invariance

Define observable "illiquidity" $1/L$ as volume-weighted expected cost:

$$\frac{1}{L} := \frac{C}{\mathrm{E}\{|PQ|\}} = \left(\frac{\sigma^2 C}{m^2 PV}\right)^{1/3}.$$

Then expected bet size and number of bets are given by

$$\mathrm{E}\{|PQ|\} = C \cdot L, \qquad \gamma = \frac{1}{m^2} \cdot \sigma^2 L^2.$$

Price impact is

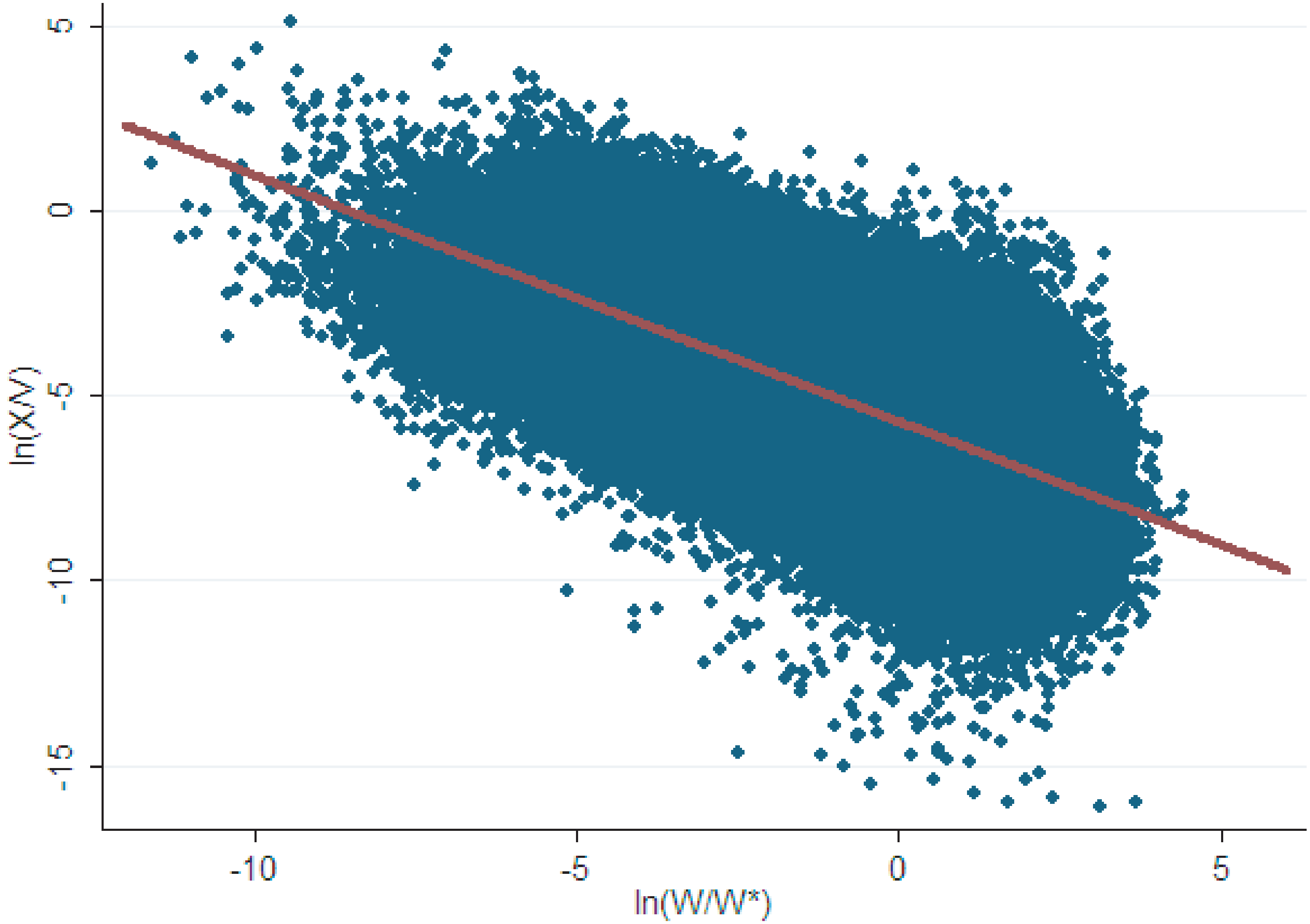
$$\frac{\Delta P}{P} = \frac{1}{L} \cdot m_\beta \cdot |Z|^\beta, \qquad \text{where} \qquad Z := \frac{Q}{\mathrm{E}\{|Q|\}} = \frac{PQ}{CL},$$

Result: If C, m, and m_β are invariant across assets, then we have a universal market impact formula and universal formula for size and number of bets, which requires estimation of only these three parameters and β!

(Preliminary) Calibration: $C = \$2,000$; if $\beta = 1$, then $m \approx 0.25$ and $m_\beta = m^2$. If $\beta = 1/2$, then $m_\beta = m \approx 0.40$.

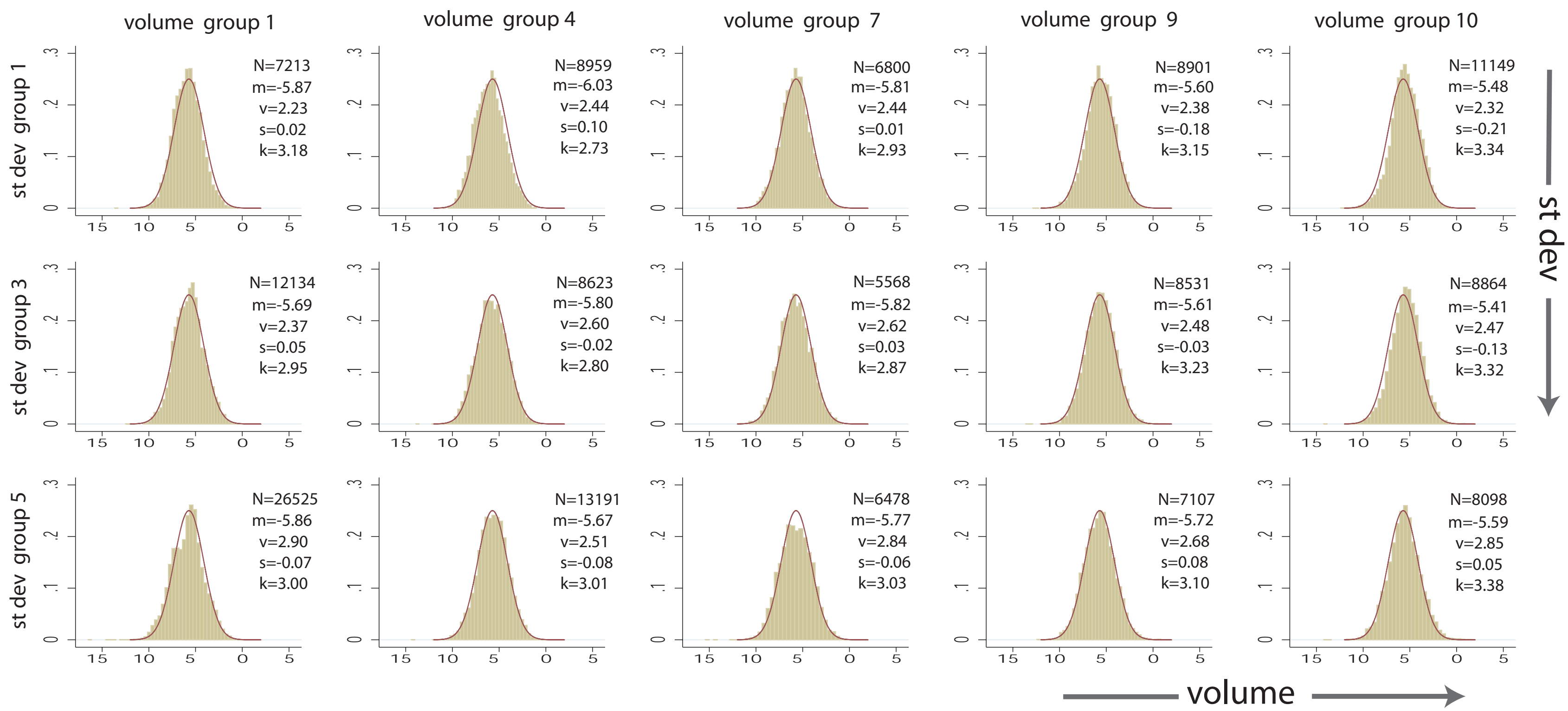
Portfolio Transition Order Size

Figure from Kyle-Obizhaeva-ECMA-2016



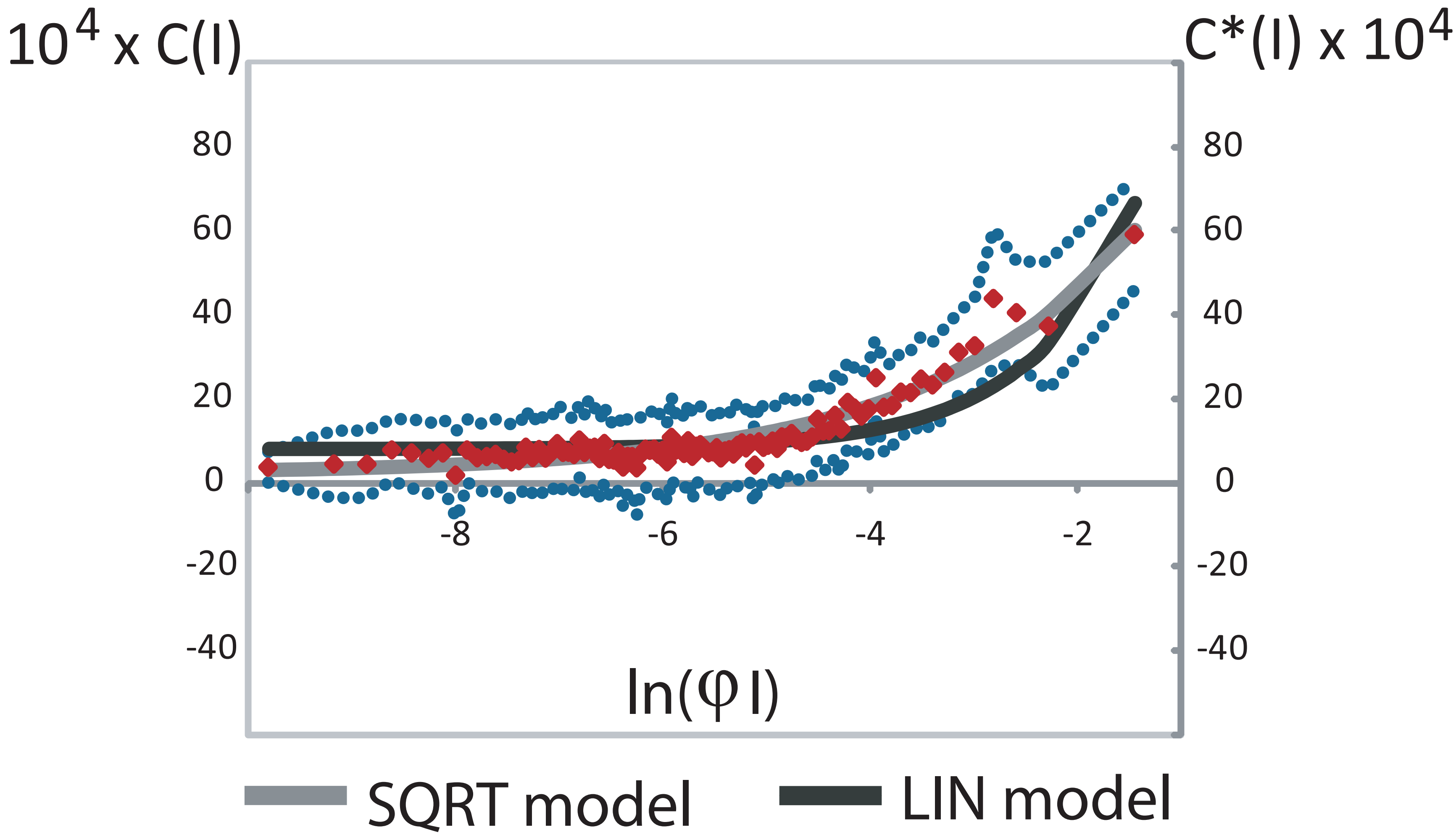
Distribution of Portfolio Transitions Orders

Figure from Kyle-Obizhaeva-ECMA-2016



Square Root Model in Portfolio Transition Orders

Figure from Kyle-Obizhaeva-ECMA-2016



Approach II: Dimensional Analysis

Physics researchers obtain powerful results by using dimensional analysis to reduce the dimensionality of problems (the size and number of molecules in a mole of gas, the size of the explosive energy, turbulence).

- Physics: Fundamental units of mass, distance, and time & conservation laws based on laws of physics.
- Finance: Fundamental units of time, currency, and shares & conservation laws based on no-arbitrage restrictions.

Dimensional Analysis Approach (continued)

Basic idea: Use dimensional analysis like a physicist (units consistency, Buckingham π Theorem):

- "Guess" correct functional form, i.e, correct list of explanatory variables.
  - Warning: Incorrect guess may lead to nonsense.
- Reduce dimensionality of problem by factoring out units, making remaining parameters dimensionless.
- Add restriction of "leverage neutrality" (Modigliani–Miller Theorem) to reduce dimensionality further. The cost of exchanging cash is zero. Dollar market impact cost of exchanging a risky bundle of assets is the same for any positive or negative amount of cash-equivalent assets included with the bundle.
- Make empirically motivated invariance assumption.

Implication: Results similar to meta-model

Where is Adverse Selection?

- Meta-model derives an empirical formula for liquidity L without an underlying model of adverse selection.
  - This is consistent with mechanical aspects of trading experienced in markets, where information is invisible.
- We need theories based in economics to link meta-model to adverse selection.
- This enables further link to pricing accuracy, probability of informed trading, and precision of signals.

We next show that the meta-model and dimensional approach are consistent with theoretical models of both block trading and smooth trading.

Model of Treynor (1971)

How to determine bid price B, ask price A, bid-ask spread S, and mid-point P?

- Informed traders and liquidity traders buy or sell fixed quantity $\pm \bar{Q}$ with competitive risk neutral market maker.
- Probability θ: Informed trader observes information about fundamental value $P \pm \Delta F$ with equal probabilities.
- Probability $1 - \theta$: Noise trader trades randomly.

Result: Bid-ask spread satisfies

$$S = A - B = 2\theta \Delta F.$$

How to map to data? Since model maps to meta-model (two states consistent with linearity), bid-ask spread satisfies reasonable, empirically testable hypothesis

$$\frac{1}{2} \cdot \frac{S}{P} = \left(\frac{\sigma^2 C}{PV}\right)^{1/3} = \frac{1}{L} \qquad \text{with} \qquad m = 1.$$

One-period Model of Kyle (1985): Setup

Assumptions:

- Informed trader observes fundamental value:

$$F = P_0 + \sigma_F Z_F, \qquad \text{with} \qquad Z_F \sim \mathrm{N}(0, 1).$$

- Noise traders trade $Q_U = \sigma_U Z_U$, with $Z_U \sim \mathrm{NID}(0, 1)$.

Definition of Equilibrium (following nice approach of Bagnoli, Holden, Viswanathan (2001)):

- Informed trader exercises market power, trades Q_I given by

$$Q_I(F) = \underset{x}{\operatorname{argmax}} \; \mathrm{E}\left[(F - p(x, Q_U)) \cdot x_I \mid F\right],$$

- Market makers set price as symmetric function

$$p(Q_I, Q_U) = \mathrm{E}\left[F \mid \{Q_I, Q_U\}\right].$$

One-Period Model of Kyle (1985): Equilibrium

Simple derivation shows equilibrium given by

$$Q_I = \beta\,(F - P_0), \qquad p(Q_I, Q_U) = P_0 + \lambda(Q_I + Q_U),$$

with coefficients for trading intensity β and market impact λ given by

$$\beta = \frac{\sigma_U}{\sigma_F} \text{ and } \lambda = \frac{1}{2}\frac{\sigma_F}{\sigma_U}.$$

Equilibrium maps into meta-model with two bets, Q_I and Q_U:

$$V = \gamma \cdot \ \mathrm{E}\left[|Q|\right] \qquad \text{(Definition of volume)}$$

$$\sigma^2 = \gamma \cdot \ \mathrm{E}\left[\left(\frac{\Delta P}{P}\right)^2\right] \qquad \text{(Bets generate all volatility),}$$

$$\mathrm{E}\left[\Delta P^2\right] = \lambda^2 \cdot \ \mathrm{E}\left[Q^2\right] \qquad \text{(Market impact of one bet),}$$

$$C = \lambda \cdot \ \mathrm{E}\left[Q^2\right] \qquad \text{(Dollar impact cost of a bet.)}$$

One-Period Model of Kyle (1985): Invariance

Therefore, theoretical parameters map empirically to L as

$$\sigma_F := \frac{2mP}{L} = \frac{2P\sigma}{\sqrt{\gamma}}, \qquad \text{and} \qquad \sigma_U := \frac{CL}{mP} = \frac{\mathrm{E}\left[|Q|\right]}{m}.$$

Solution to model implies

$$\text{Trading Profit} = C = \frac{\sigma_F\ \sigma_U}{2} = \text{Dollar Market Impact Cost}$$

Market microstructure invariance is consistent with assumption that informed trader pays C for a private signal, in which case

$$C = \frac{\sigma_F\ \sigma_U}{2} = \text{Cost of Private Signal.}$$

Model describes what happens during time period when two bets arrive, both of which are executed as block trades.

- Model operates over different horizons for securities with different liquidity (in this case measured by $\gamma = \sigma^2 L^2/m^2$).

Smooth Trading Model: Setup

Simplified version of Kyle, Obizhaeva, Wang (2018):

- With probability θ, informed trader receives one signal at random time, trades gradually on signal over time. Risk neutrality consistent with $\theta = 1/2$.
- With probability $1 - \theta$, noise trader trades randomly with fake signal.

Trading has temporary and permanent price impact:

$$P(t) = P_{-*}(t) + \lambda S(t) + \kappa \cdot x(t), \qquad \text{where} \qquad x(t) = \frac{\mathrm{d}S(t)}{\mathrm{d}t}.$$

Equilibrium definition is analogous to Kyle (1985):

- Trader maximizes profit by incorporating fraction θ of private information into prices.
- Market makers set efficient price based on past order flow.

Smooth Trading Model: Equilibrium

Prices reveal information immediately:

$$\mathrm{E}\left[P(t+h) \,\middle|\, \text{Bet arrives}\right] = P(t) + \theta \cdot \Delta F.$$

Trader trades total quantity $Q(t)$ smoothly over time:

$$Q(t) = \int_{h=t}^{\infty} \Delta x(t) \cdot \, \mathrm{e}^{-\rho h} \cdot \, \mathrm{d}h = \frac{\Delta x(t)}{\rho},$$

The trade rate Δx is determined by some additional assumption about intensity of noise trading.

Smooth Trading Model: Invariance

This equilibrium satisfies meta-model-like equations

$$V = \gamma \cdot \frac{\mathrm{E}\left[|\Delta x|\right]}{\rho} \qquad \text{(Definition of volume)}$$

$$\sigma^2 = \gamma \cdot \mathrm{E}\left[\left(\frac{\Delta P}{P}\right)^2\right] \qquad \text{(Bets generate all volatility),}$$

$$\mathrm{E}\left[(\Delta P)^2\right] = \kappa^2 \cdot (\Delta x)^2 \qquad \text{(Market impact of one bet),}$$

$$C = \kappa \cdot \frac{(\Delta x)^2}{\rho} \qquad \text{(Dollar impact cost of a bet.)}$$

Maps into meta-model with invariance under assumptions:

$$Q \mapsto \frac{|\Delta x|}{\rho} \qquad \text{(Bet Size)}$$

$$\lambda \mapsto \kappa \; \rho \qquad \text{(Market Impact)}$$

Temporary price impact κ defines liquidity as a flow concept.

The “bet” size Q is total contribution of flow trading to volume over time.

Smooth Trading Model: Information and Bet Size

Implications of meta-model mapping:

$$\mathrm{E}\left[\frac{|\Delta x|}{V}\right] = \theta^2 \tau,$$

$$\frac{\kappa \ \mathrm{E}\left[|\Delta x|\right]}{P} = \frac{m^2}{L}.$$

Interpret Δx as trader's fraction of volume when new bet starts.

Results map information content of bet into expected flow-bet-size.

Flow price impact maps into observable liquidity.

Pricing Accuracy

Liquidity L can be linked through information flow to pricing accuracy and resiliency.

Define error variance of prices like Black (1986):

$$\Sigma = \text{var}\left[\log\left(\frac{F}{P}\right)\right] \approx \text{var}\left[\frac{F-P}{P}\right].$$

Private signals have invariant precision τ, which, from perspective of informed trader, reduces error variance by fraction τ from Σ to $(1-\tau)\Sigma$.

Then each bet reduces price variance only by fraction $\theta^2\tau$ because market cannot distinguish informed bets from noise.

$$\frac{\sigma^2}{\gamma} = \text{E}\left[\left(\frac{\Delta P}{P}\right)^2\right] = \frac{\lambda^2 \cdot \text{E}[Q^2]}{P^2} = \theta^2\tau \cdot \Sigma.$$

Pricing accuracy can be inferred from market liquidity L:

$$\Sigma = \frac{\sigma^2}{\gamma(\theta^2\tau)} = \frac{m^2}{(\theta^2\tau)L^2}.$$

Market Resiliency

Define "market resiliency" ρ as rate at which prices converge to changing fundamental value; also equal to rate at which noise shocks die out from prices:

$$\rho = (\theta^2 \cdot \tau) \cdot \gamma.$$

Resiliency ρ is function of invariance parameters $\frac{\theta^2 \cdot \tau}{m^2}$ and observable volatility σ and liquidity L:

$$\rho = \frac{\sigma^2}{\Sigma} = \frac{\theta^2 \cdot \tau}{m^2} \cdot \sigma^2 L^2.$$

Trading Liquidity, Funding Liquidity, and Time

Liquidity L is related to the rate at which bets arrive γ (business time):

$$L = \left(\frac{m^2 P V}{C \sigma^2}\right)^{1/3} = \frac{\gamma^{1/2} m}{\sigma}.$$

Riskiness of large bets in risky assets depends on horizon over which liquidation takes place:

- Margins and Repo Haircuts: Liquidation of defaulted collateral takes time.
- Bank Capital: Bank assets extremely illiquid. Selling bank assets is impractical. Raising new capital takes time.
- Bank Equity Issuance: Issuing equity takes time.
- Government Securities: Can be sold quickly.
- Stock Market Crashes: Can be caused by large sales over short periods of time. See Kyle and Obizhaeva (2016) paper on stock market crashes.

This implies that margins, repo haircuts, and bank capital should be proportional to $1/L$, taking into account time dimension of liquidity of underlying assets.

Conclusion

Theoretical models generate empirically realistic predictions when augmented with invariance principles.

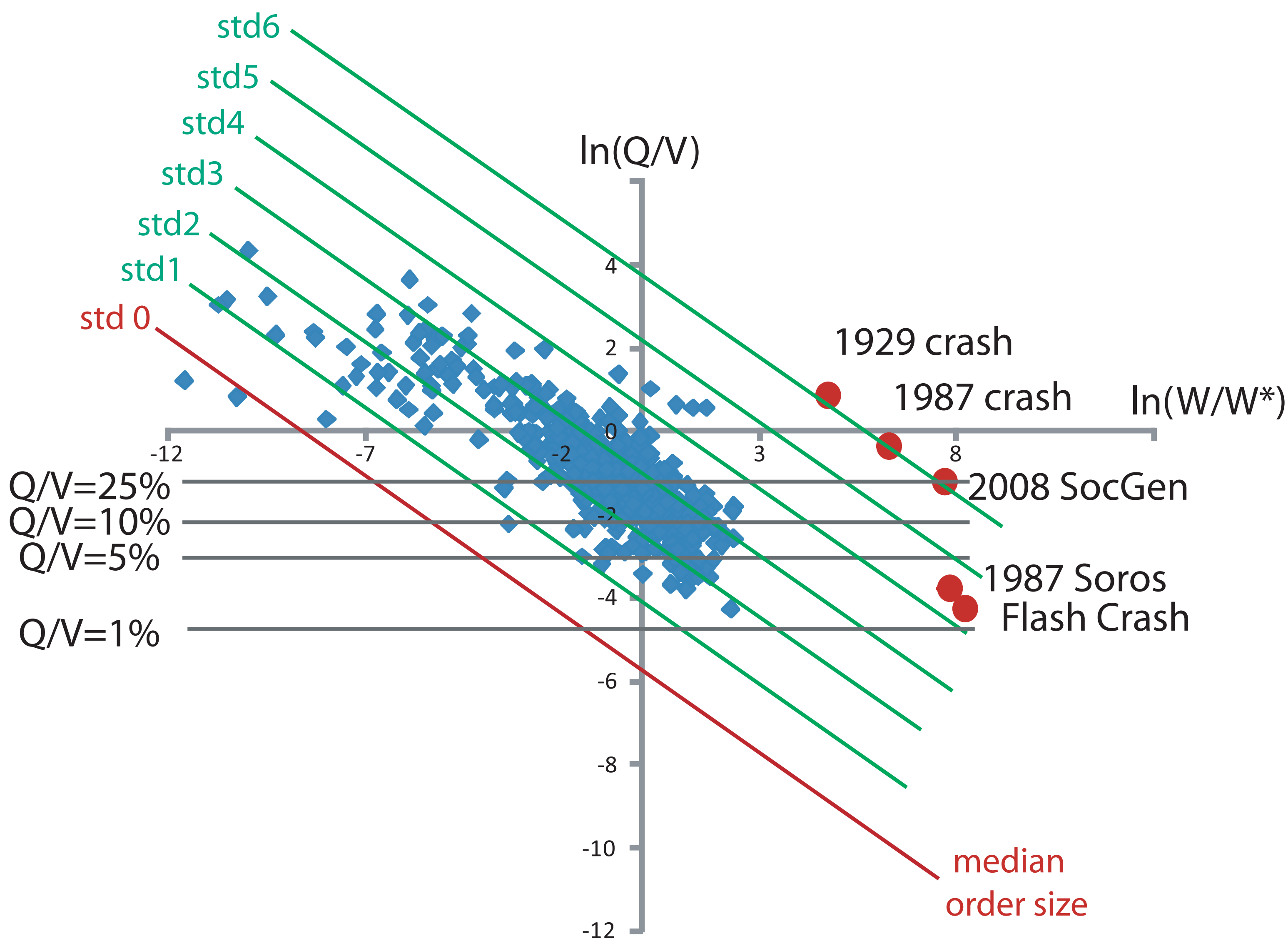
- Meta-model implies liquidity measure L.
- This measure is consistent with both block trading and smooth trading models.
- Generates many empirically useful implications relating bet size to liquidity.
- By connecting liquidity to time, provides approach for setting bank capita.

More Empirical Evidence

Working papers can be found on SSRN.

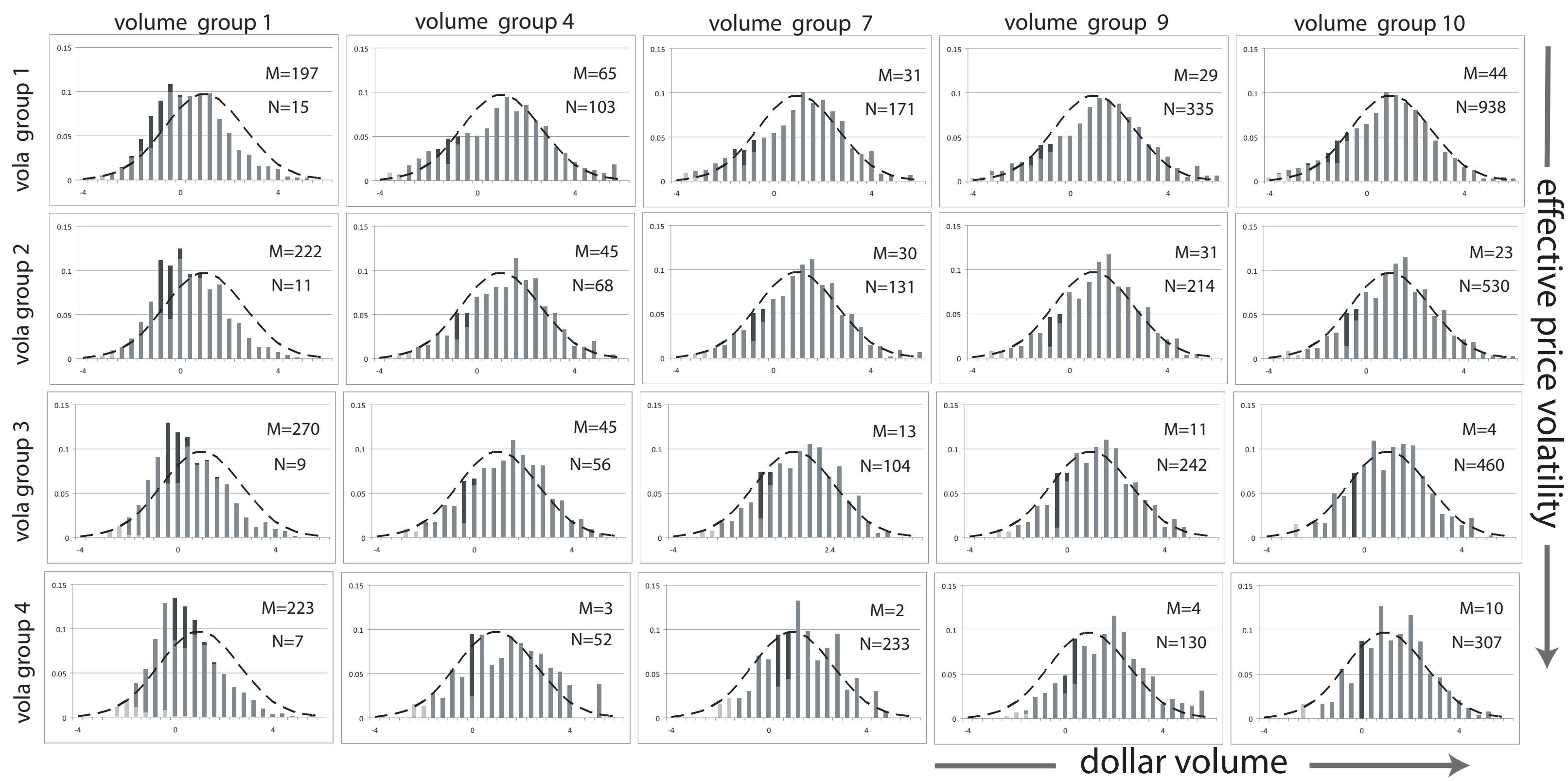
Large Bets and Stock Market Crashes

Figure from "Large Bets and Stock Market Crashes"



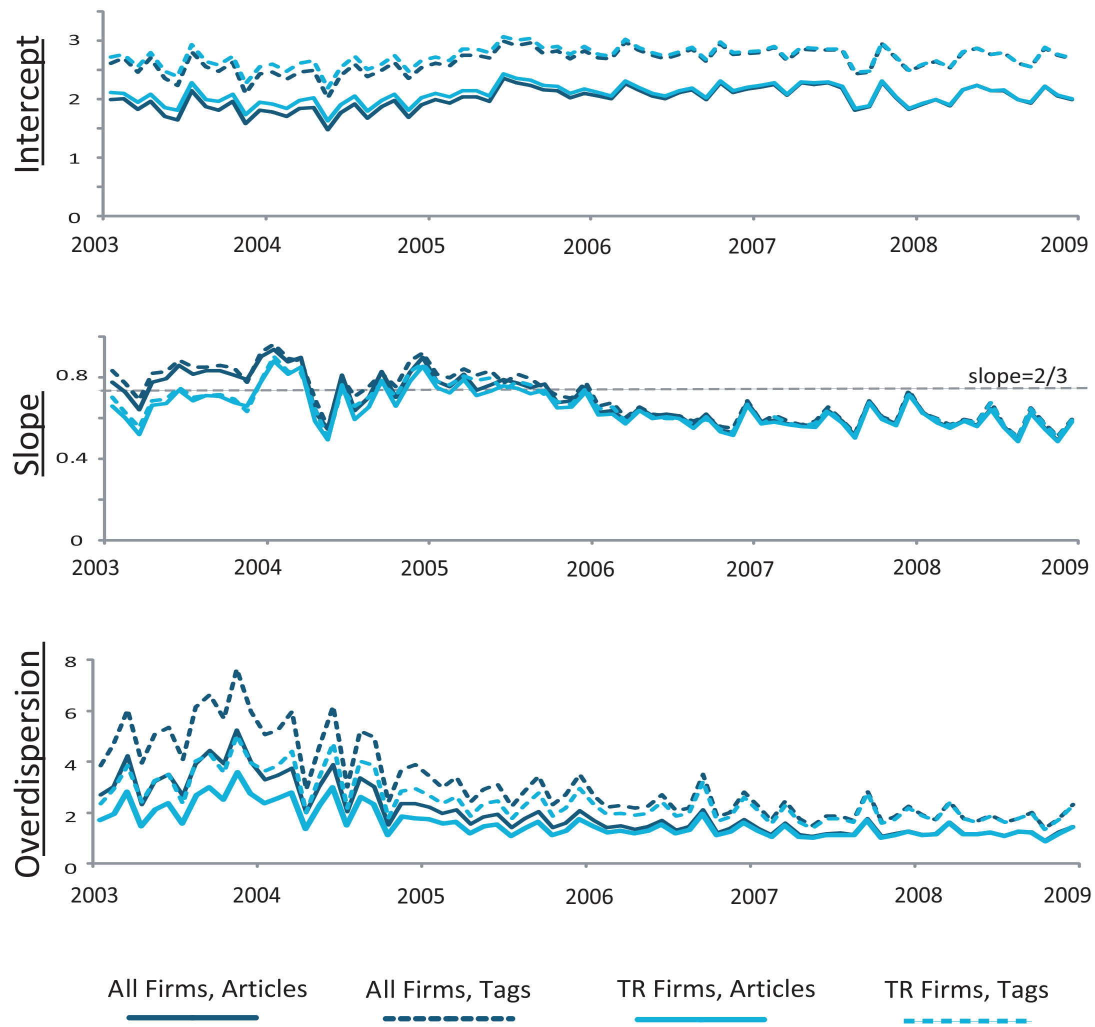
Volume-Weighted Distribution of NASDAQ Trades (1993)

Figure from Kyle-Obizhaeva-Tuzun-2018



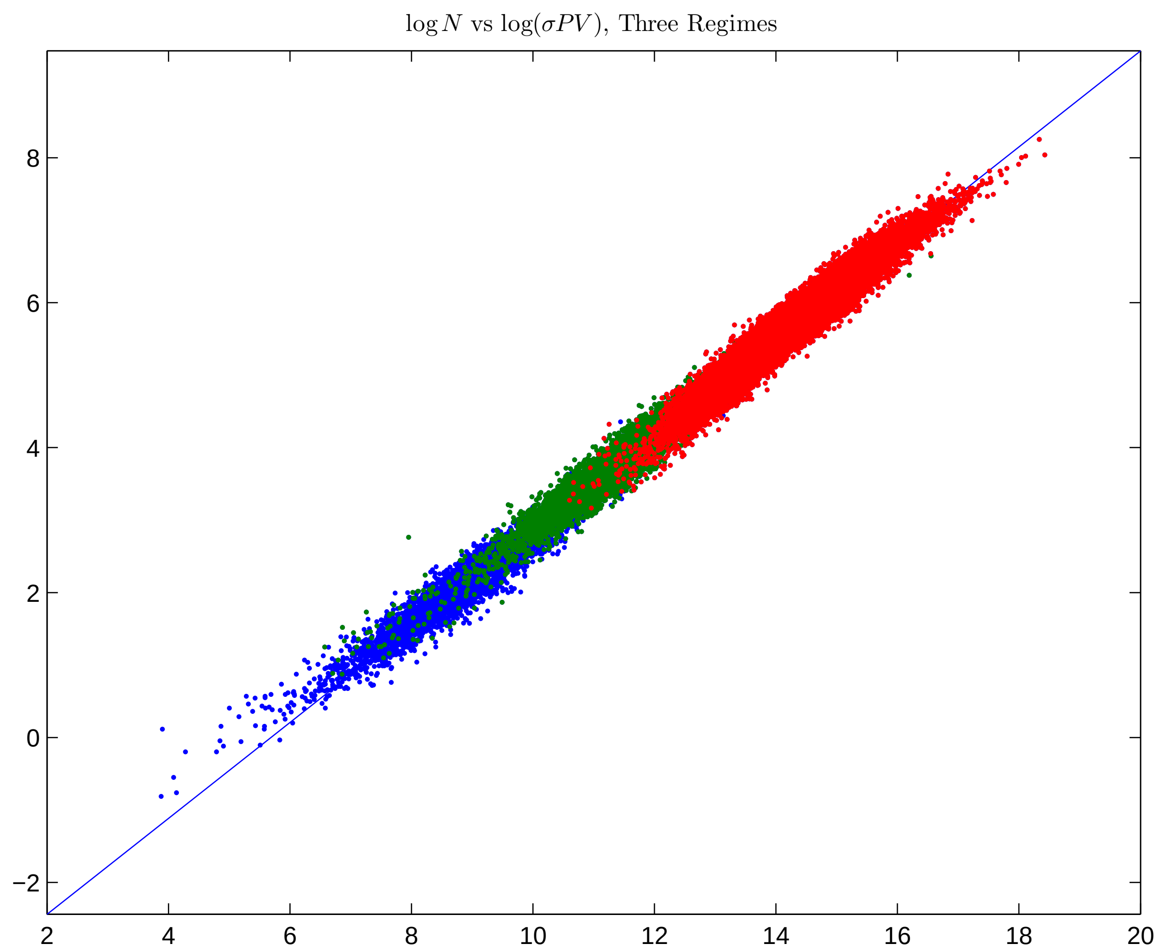
Reuters News Articles

Figure from Kyle-Obizhaeva-Sinha-Tuzun-2010



Trade Size in S&P 500 E-mini Futures Contracts

Figure from Andersen-Bondarenko-Kyle-Obizhaeva-2017



Switching Points on the Korea Exchange

Figure from Bae-Kyle-Lee-Obizhaeva-2016