

# Tomas Björk: The pedagog

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University of Copenhagen  
Conference in memory of Tomas Björk  
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Unlike the two previous speakers, I didn't know Tomas Björk personally

But I have been a fly on the wall:

- Tomas Björk and Bent Jesper Christensen starting consistent term structure modelling; live at Hindsgavl '95
- Regime-switching with varying elegance; PhD-defense in Aarhus '98
- *Pointing with a Coke bottle*; Cph.'98
- "Damir Filipovic checked 56 Lie bracket cases manually"; PhD-course, Aarhus '01



# Memento mori

Fischer Black is no longer alone

Every time I see a certain type of photo posted on social media, I think "Oh, no ..."

But we live forever through our writings

Ragnar Norberg



Tomas Björk



Ole Barndorff-Nielsen



Stephen Ross



Peter Carr



Marc Yor



Mark Joshi



Mark Rubinstein



Mark Davis



Marco Avellaneda



...

# Magnum opus

## Arbitrage Theory in Continuous Time

1<sup>st</sup> edition '98, 4<sup>th</sup> edition '20

0<sup>th</sup> edition: Many claim midwifery,

- KTH
- KU-MATH
- ETH



**Rolf Poulsen**

### Tomas Björk in Memoriam

The late Swedish financial mathematician influenced the author's views on quantitative finance enormously.

**O**n August 17, 2022, the Swedish financial mathematician Tomas Björk (1962-2022) died from a brain tumor. He had been diagnosed with the disease in 2019, and he passed away on August 17, 2022, at the age of 59.

The first edition of *Arbitrage Theory in Continuous Time* was published in 1998, and the fourth edition (which is the basis for my reference in the following) came out in 2020. The book has been translated into Chinese, Japanese, and Korean, and it is also available in a Chinese edition.

Björk's research has been widely cited in the financial mathematics community, and his work has been a major source of inspiration for many researchers in the field.

The book is a pedagogical masterpiece; it is written in a lucid style that makes continuous-time mathematical finance easy to understand.

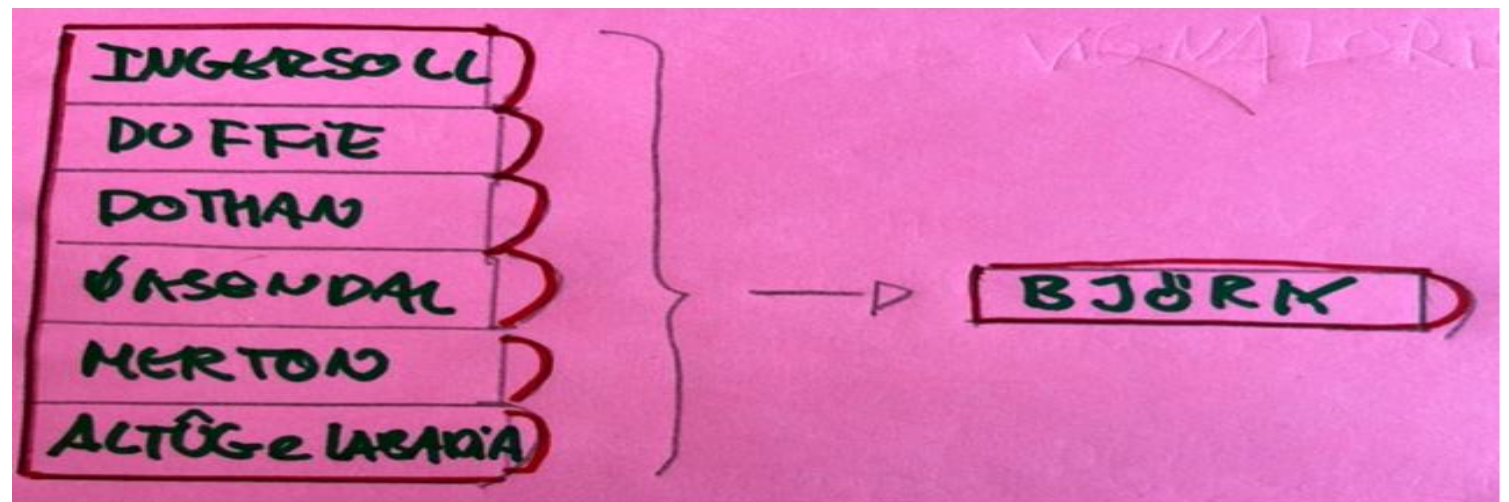
**The book is a pedagogical masterpiece; it is written in a lucid style that makes continuous-time mathematical finance easy to understand.**

**easy**

# E pluribus unum

When I was a student ('91-'99) we didn't have Björk

Since '00 I/we have been teaching 2-3 courses a year based on it at KU-MATH



# Cur ita bonum est?

Arrogant causality: We teach directly from it at KU-MATH – and we are very picky! (Case: [Lando & Poulsen \(2022\), "Finance 1 and Beyond"](#))

Generic praise: Lucid exposition; explains why, not just how; gives the reader intuition – *yada-yada-yada*

The brilliant thing: Continuous-time finance made easy – but not too easy



Some books, papers, online sources are deceptively simple; irreproducible argumentation, “This is trivial if you don’t think about it too carefully”, what is a function of what?



# Highlights from The Good Book

Chapter 6 on portfolio dynamics. It is *not* as obvious as it seems that  $dV^\phi = \phi^\top dS$  is the self-financing condition in continuous time. Causes confusion to this day

Chapters 7 and 8 that address and fix a potential circularity when deriving the Black-Scholes pricing PDE



# Björk's lemma

Helps us prove, construct, and analyse for instance

- HJM drift condition
- Musiela parametrization
- Cheyette Markovian representation
- Expectations hypotheses

## Proposition 22.5

1. If  $p(t, T)$  satisfies (22.2), then for the forward rate dynamics we have

$$df(t, T) = \alpha(t, T)dt + \sigma(t, T)dW(t),$$

where  $\alpha$  and  $\sigma$  are given by

$$\begin{cases} \alpha(t, T) = v_T(t, T) \cdot v(t, T) - m_T(t, T), \\ \sigma(t, T) = -v_T(t, T). \end{cases} \quad (22.4)$$

2. If  $f(t, T)$  satisfies (22.3) then the short rate satisfies

$$dr(t) = a(t)dt + b(t)dW(t),$$

where

$$\begin{cases} a(t) = f_T(t, t) + \alpha(t, t), \\ b(t) = \sigma(t, t). \end{cases} \quad (22.5)$$

3. If  $f(t, T)$  satisfies (22.3) then  $p(t, T)$  satisfies

$$dp(t, T) = p(t, T) \left\{ r(t) + A(t, T) + \frac{1}{2} \|S(t, T)\|^2 \right\} dt + p(t, T)S(t, T)dW(t),$$

where  $\|\cdot\|$  denotes the Euclidean norm, and

$$\begin{cases} A(t, T) = - \int_t^T \alpha(t, s)ds, \\ S(t, T) = - \int_t^T \sigma(t, s)ds. \end{cases} \quad (22.6)$$

# I also really like (plus shameless self-promotion)

Risk and Stochastics, pp. 43-67 (2019)

No Access

## Chapter 3: The Pedestrian's Guide to Local Time

Tomas Björk

[https://doi.org/10.1142/9781786341952\\_0005](https://doi.org/10.1142/9781786341952_0005) | Cited by: 0

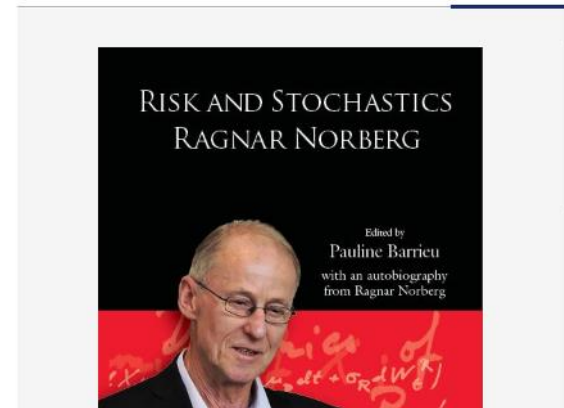
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## How I learned to stop worrying and love the Dirac- $\delta$ -function

September 14, 2022

### My [Wilmott](#) Collection

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Published

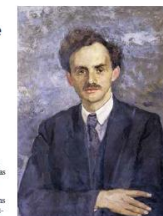
[The Fed Isn't Federal – And Other Odd Things in Finance](#), issue 88 (March 2017), pp. 34-45.

[American  \$\pi\$ : Piece of Cake?](#), issue 91 (September 2017), pp. 12-13.

[Binary Backwards](#), issue 94 (March 2018), pp. 20-21.

### Things I Learned This Semester the Fourth

The fundamental theorem of derivative trading extends to non-simple claims! Stop worrying and love the Dirac- $\delta$ -function! Selection bias lurks!



**T**hat the fundamental theorem of derivative trading (as discussed at length in my September 2018 column) extends to more complex claims in the way that we could most optimally hope for. I will just refer the reader to Example 1 in Section 4 of Dupire (2019), an article where functional calculus is developed.

**To stop worrying and love the Dirac- $\delta$ -function Volatility formulas.** The following arguments – heavily inspired by pages 20–42 from Sarno (2016) – allow us to derive the Gyöngy–Dupire–Derman–Kani volatility formula without referring to the Fokker–Planck–Kolmogorov equation and without the usual multiple integrations by parts. Consider a zero interest rate, arbitrage-free model in which some stock price follows an Itô process under a risk-neutral measure  $\mathbb{Q}$ :

$$dS_t = \sigma_t S_t dW_t^{\mathbb{Q}}$$

where  $W$  is some stochastic process. For a positive real number  $K$ , look at the function given by  $f(S) = (S - K)^+$ , and define the stochastic process  $Y$  via  $Y_t = f(S_t)$ . From the extended Itô formula (Theorem 4.3 in Björk (2019)) we have that

$$(S_T - K)^+ - Y_0 = \int_0^T \int_{\mathbb{R}} \delta(S_t - K) \sigma_t^2 dt + \frac{1}{2} \int_0^T \delta(S_t - K) V_t dt$$

where  $\delta$  denotes the Dirac- $\delta$ -function, which is the second derivative of  $f$  in distributional sense – see Proposition 2.3 in Björk (2019). Now take risk-neutral expectation on (\*) i.e. apply  $\mathbb{Q}^T$ . The left-most side is the time-0 price of a strike  $K$ , expiry  $T$  call option. Let's give that the name  $\text{Call}(K, T)$ . The term in the middle on the right-most side disappears, it is 0 because  $\delta$  is  $\mathbb{Q}^T$ -martingale. So, by using changing expectation and integration we have

$$\text{Call}(K, T) = (S_0 - K)^+ + \frac{1}{2} \int_0^T \mathbb{E}^{\mathbb{Q}^T}[\delta(S_t - K) V_t] dt \quad (*)$$

Now let  $\delta_{t+1}(s, y)$  and  $\delta_t(s)$  denote, respectively, the (joint) density of  $(S_t, V_t)$  and the (marginal) density of  $S_t$  – both seen from time  $t$ . The expectation inside the integral on the right-hand side of (\*), say  $(*)'$ , can then be written as

$$(*)' = \int_0^T \int_{\mathbb{R}} \delta(S_t - K) V_t \delta_{t+1}(s, y) dt ds$$

By the fundamental integration property of the Dirac- $\delta$ -function,  $\int_{\mathbb{R}} \delta(s - y) g(s) ds = g(y)$ , the innermost integral in  $(*)'$  is  $\delta_{t+1}(K, y)$ . Now remember that the density of  $V_t$  conditional on  $S_t = K$  (by definition) is  $\delta_{t+1}(K, y) / \delta_t(K)$ . Thus,

$$(*)' = \int_0^T \int_{\mathbb{R}} \delta_t(K) \delta_{t+1}(K, y) V_t dt dy = \int_0^T \int_{\mathbb{R}} \delta_t(K) \delta_{t+1}(K, y) V_t dt dy$$

# New editions: Sacrilegious or the ultimate tribute?

Could/would we in this room do it?  
Possibly: Bourbaki-style; profits going  
to support of young finance scholars.

What would the family say? I guess  
*OUP* wouldn't mind.

