# Risk Revealed

## Cautionary Tales, Understanding and Communication

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RiskLab and ETH Risk Center

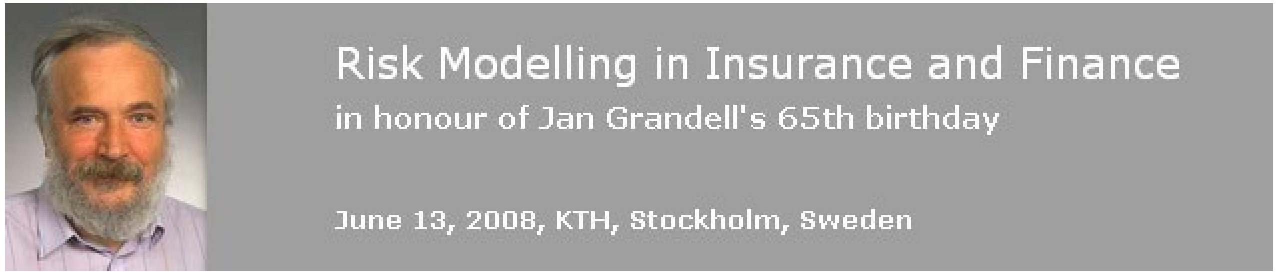
**Conference in memory of Tomas Björk (1947 – 2021)**
Stockholm, October 10 - 11, 2022

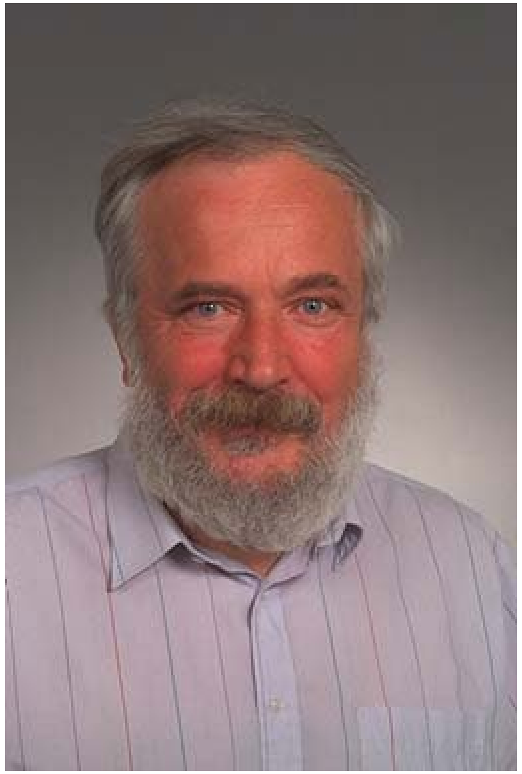
# Tomas Björk (1947 – 2021)

# My first academic contact with Tomas Björk:

Exponential inequalities for ruin probabilities in the Cox case
**Tomas Björk & Jan Grandell**
Scandinavian Actuarial Journal, Volume **1988**, 1988-Issue 1-3, 77 – 111

**Abstract**: We consider an insurance model where the underlying point process is a Cox process. Using a martingale approach we obtain extensions of the classical Lundberg inequality.

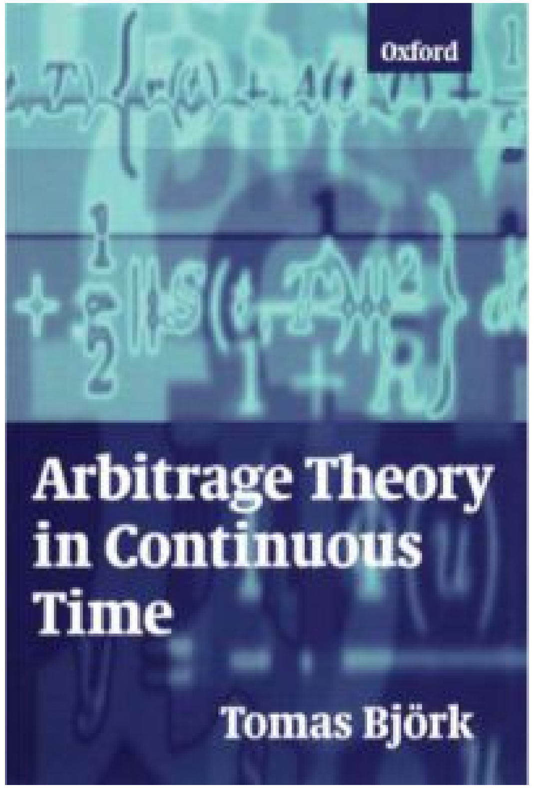


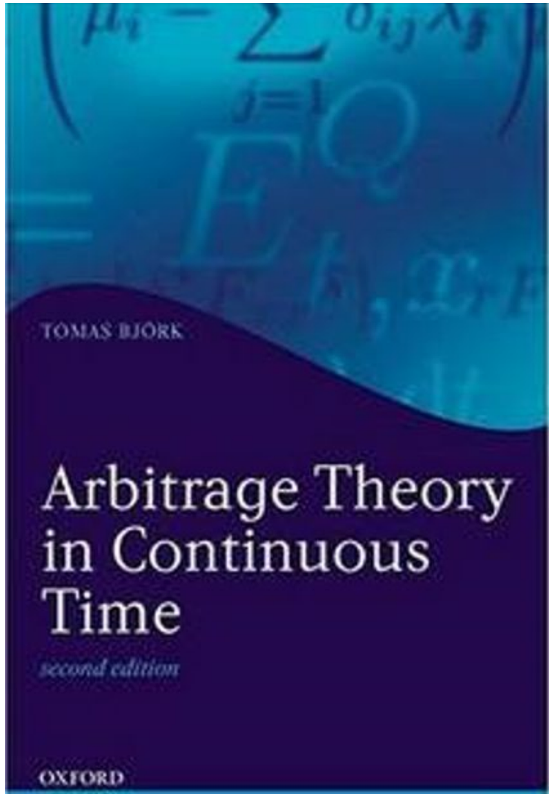



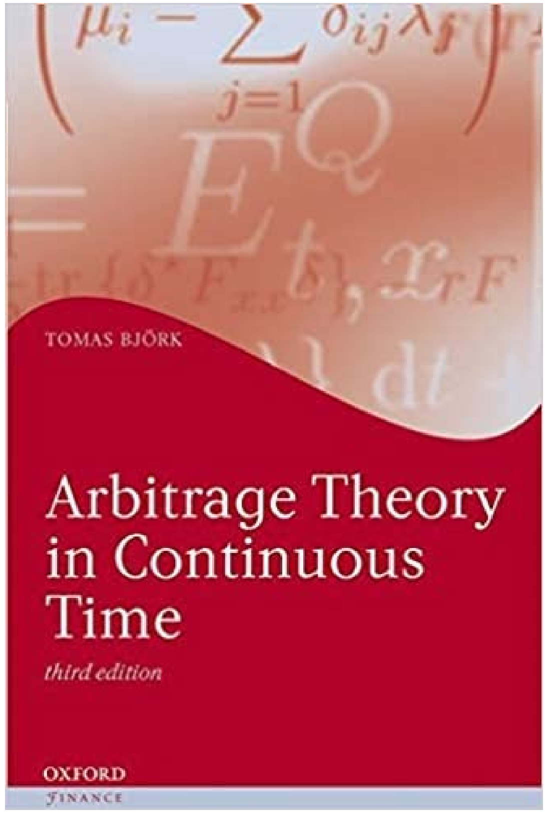
Paul Embrechts:
Statistics and Quantitative Risk Mnagement

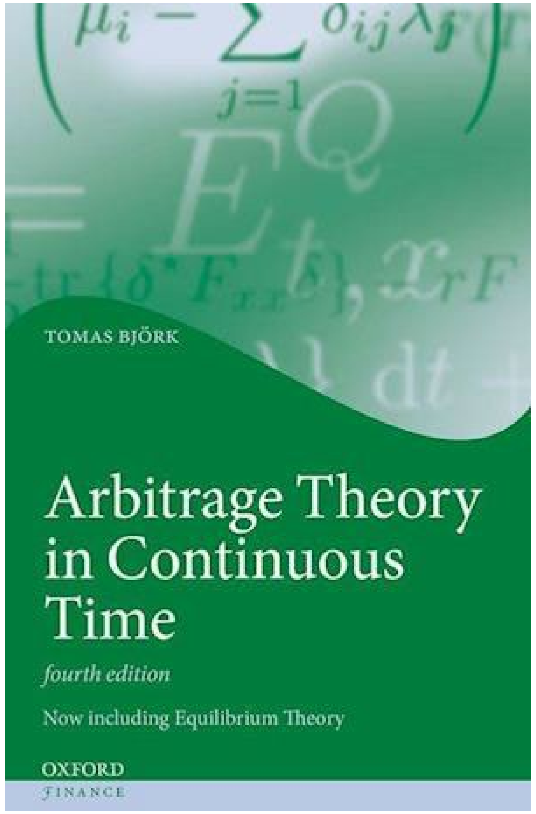
**Tomas Björk:**
**Time Inconsistent Stochastic Control**

# A personal story related to his famous book:

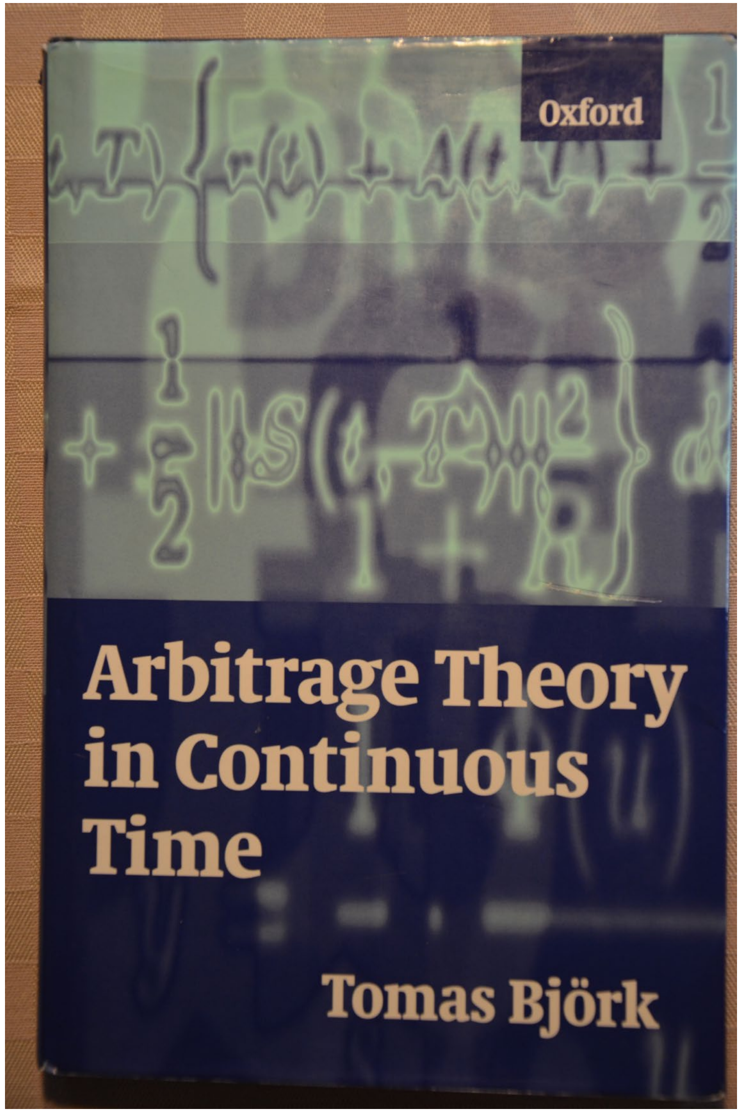


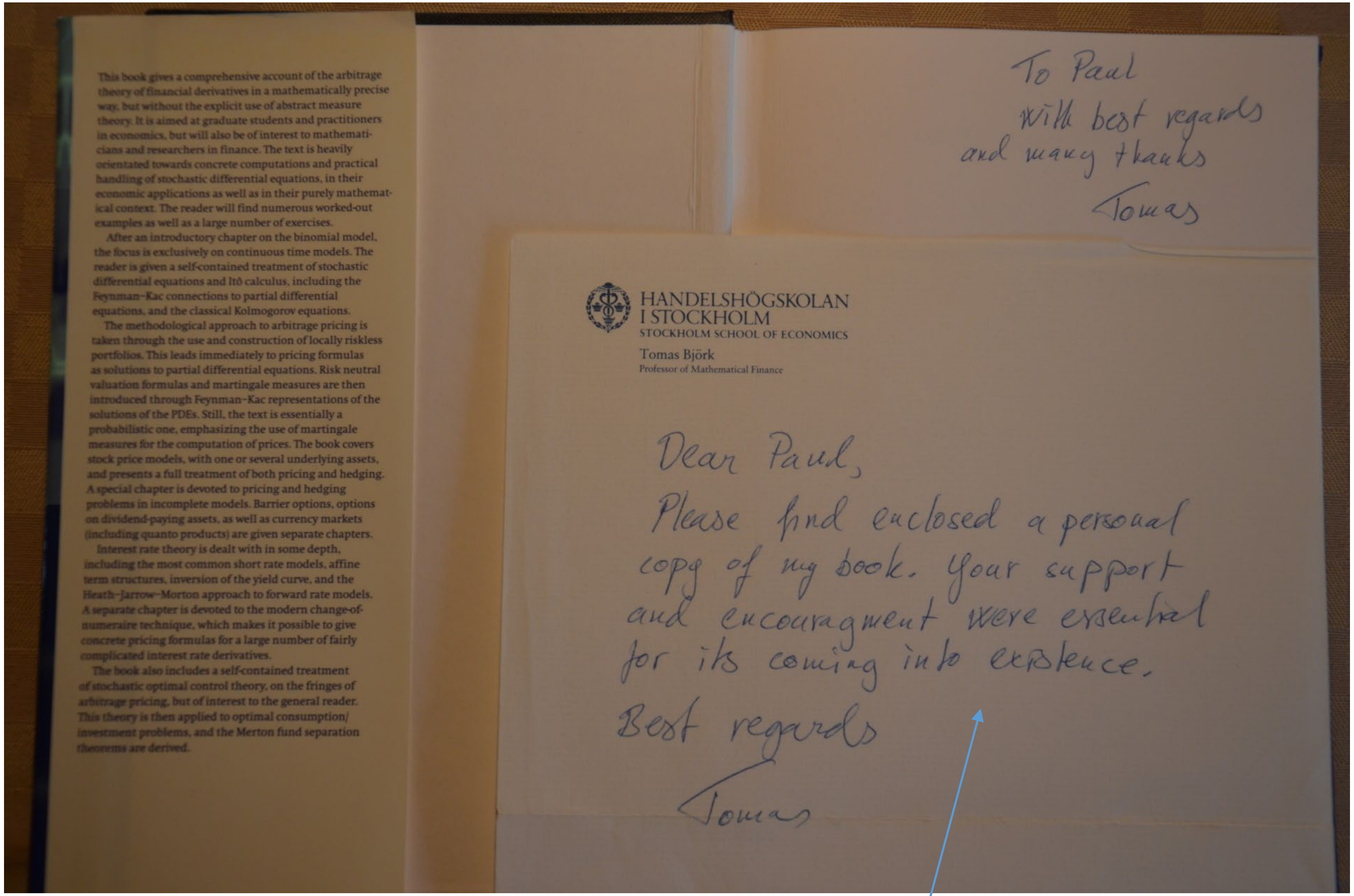




**1998** . . . **2019**



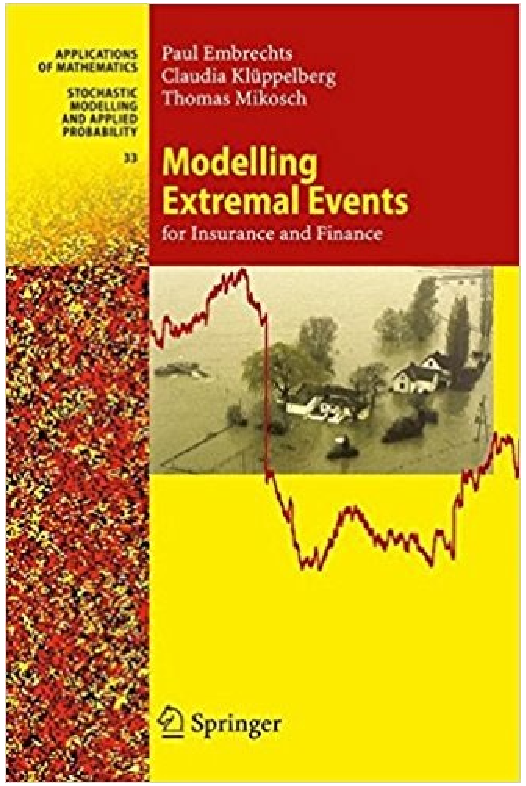



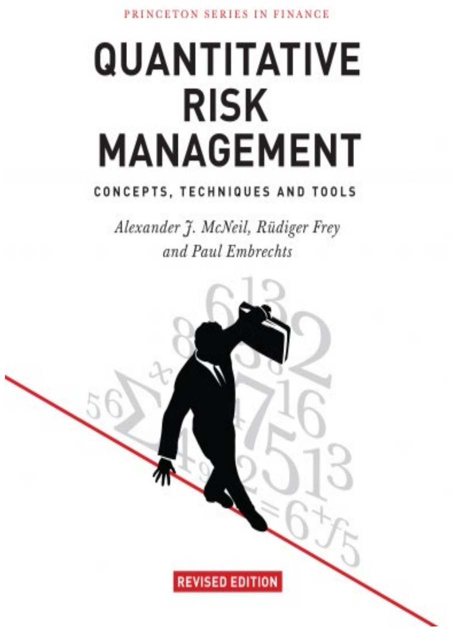

The personal story …

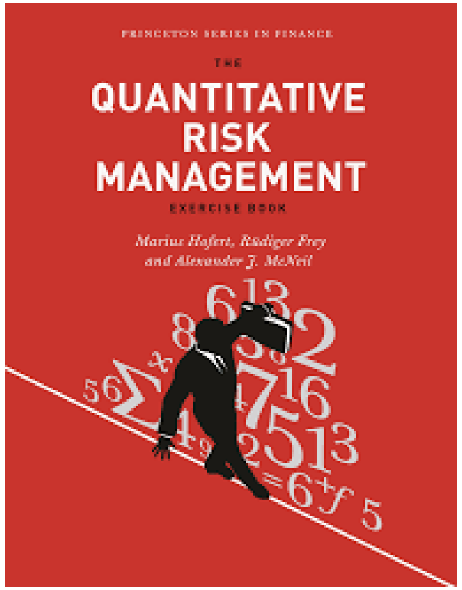
# From its Introduction:

"Hans Bühlmann, Paul Embrechts and Hans Gerber gave me the opportunity to give a series of lectures for a summer school at Monte Verita in Ascona **1995**. This summerschool was for me an extremely happy and fruitful time, as well as the start of a partially new career.

The set of lecture notes produced for that occasion is the basis for the present book."



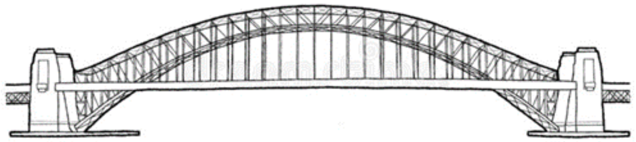

www.qrmtutorial.org (!!!)

Valérie Chavez-Demoulin

PE

Marius Hofert

A new book **"Risk Revealed: Cautionary Tales, Understanding and Communication"** (*)



More

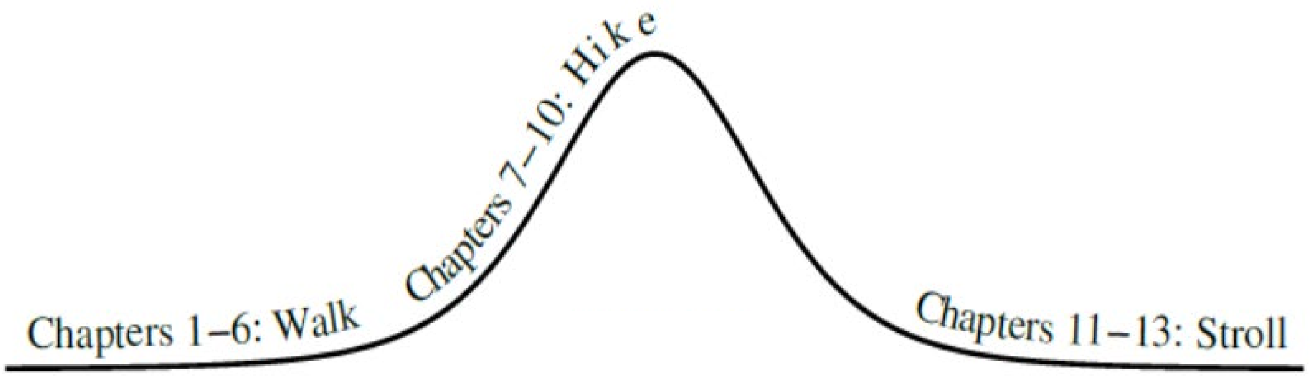
**General Public**

(*) Cambridge University Press, 2022, to appear

# Table of Contents of the book

**The wider realm of risk: a stroll**





An alternative title for the book would be

From *If* to ***What If***

and

**EVT** (**E**xtreme **V**alue **T**heory)

**We start with two motivating examples**

# A comparison between Onagawa and Fukushima Dai-ichi NPPs after the March 11, 2011 Tōhoku earthquake and tsunami


Earth Observatory Singapore

Tsunami arrival and **max height**

40 – 45’

**13 – 14 m**

*“what if”* **(14 m)**

*“if”* **(5.8 m)**

**Protective seawalls**

Yanosuke Hirai (1902-1986)

**Interconnection of risks**

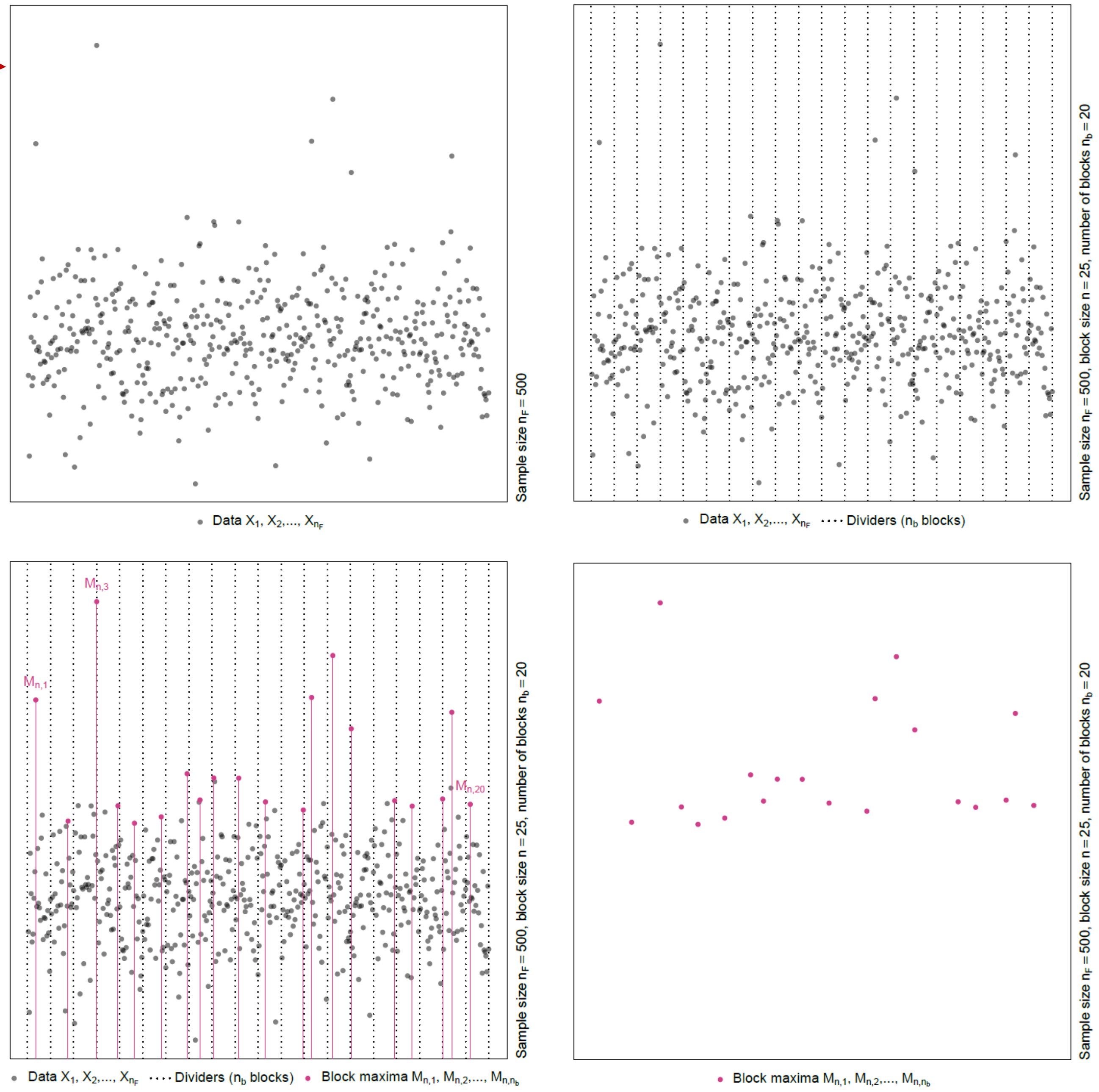
# The basic results of EVT

- **The Fisher-Tippett-Gnedenko Theorem**
- **The Pickands-Balkema-de Haan Theorem**

and their statistical interpretations

# EVT: the basics of "*What If*" thinking

## The Block Maxima Method (BMM)




**Data:** $X_1, \ldots, X_n$ iid with df F ($n = n_F$)
**Maximum**: $M_n = \max(X_1, \ldots, X_n)$

## The Fisher-Tippett-Gnedenko Theorem

If $\exists\ c_n > 0$ and $d_n$ so that for $n \to \infty$

$$(M_n - d_n)/c_n \Rightarrow H \quad \text{(n.d.) (F} \in \text{MDA}(H))$$

then $H$ is of the following type:

$$H_{\xi,\mu,\sigma}(x) = \begin{cases} \exp(-(1+\xi(x-\mu)/\sigma)^{-1/\xi}), & \xi \neq 0, \\ \exp(-e^{-(x-\mu)/\sigma}), & \xi = 0, \end{cases}$$

The **G**eneralized **E**xtreme **V**alue distribution (**GEV**)
Notation: $H_{\xi,0,1} = H_\xi$

# The Peaks Over Threshold Method (POTM)

The **conditional excess distribution**

*What If*

$$F_u(x) = P(X - u \leq x \mid X > u)$$

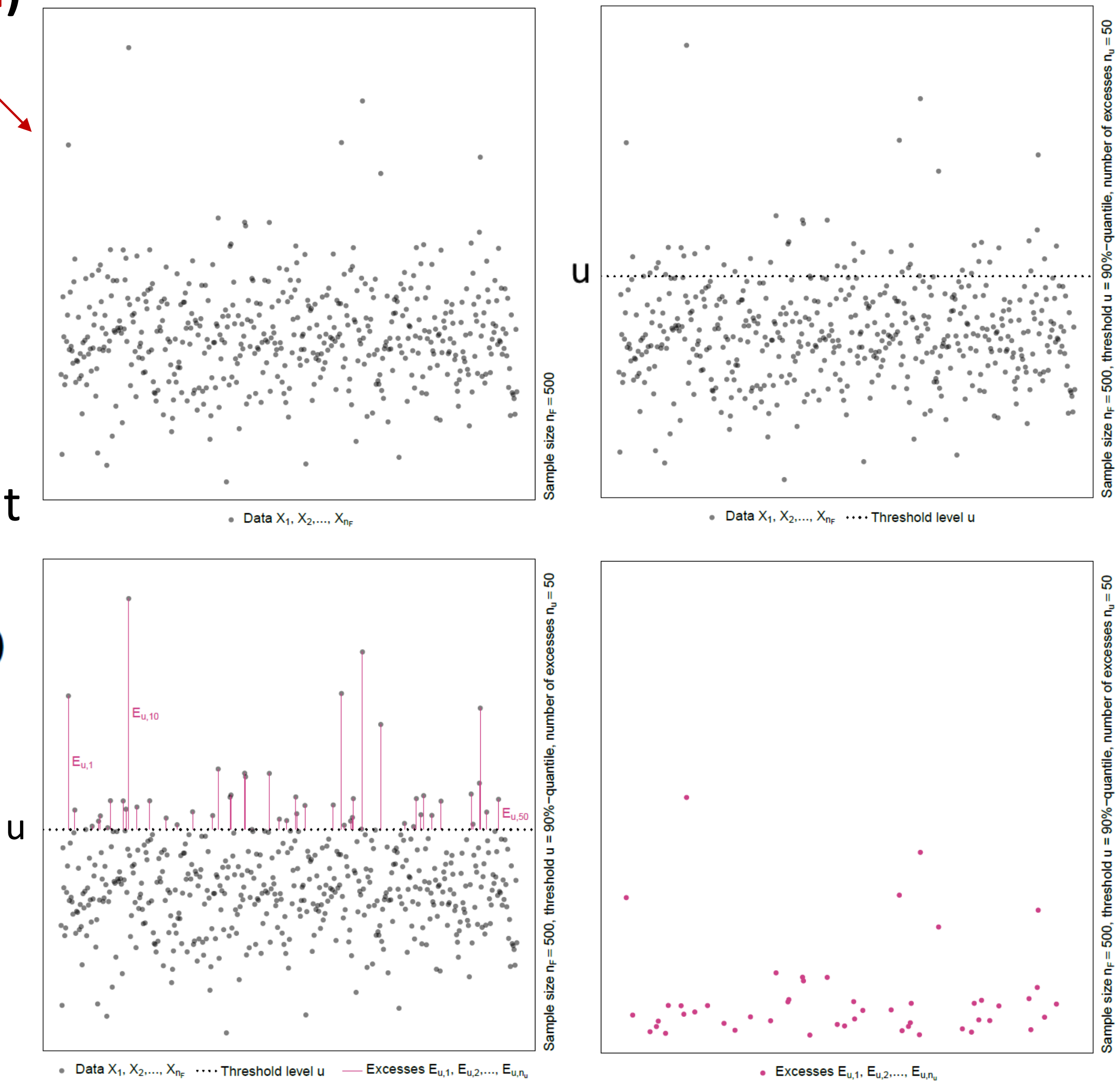
## The Pickands-Balkema-de Haan Theorem

F ϵ MDA($H_\xi$) iff ∃ β: (0,∞) → (0,∞) so that

$$\lim_{u \to x_F} \sup_{0 \leq x < x_F - u} \left| F_u(x) - G_{\xi,\beta(u)}(x) \right| = 0$$

$$G_{\xi,\beta}(x) = \begin{cases} 1 - (1 + \xi x/\beta)^{-1/\xi}, & \xi \neq 0, \\ 1 - \exp(-x/\beta), & \xi = 0, \end{cases}$$

The Generalized Pareto Distribution (GPD)



# A distributional-tail estimator

# The POT/GPD tail estimator

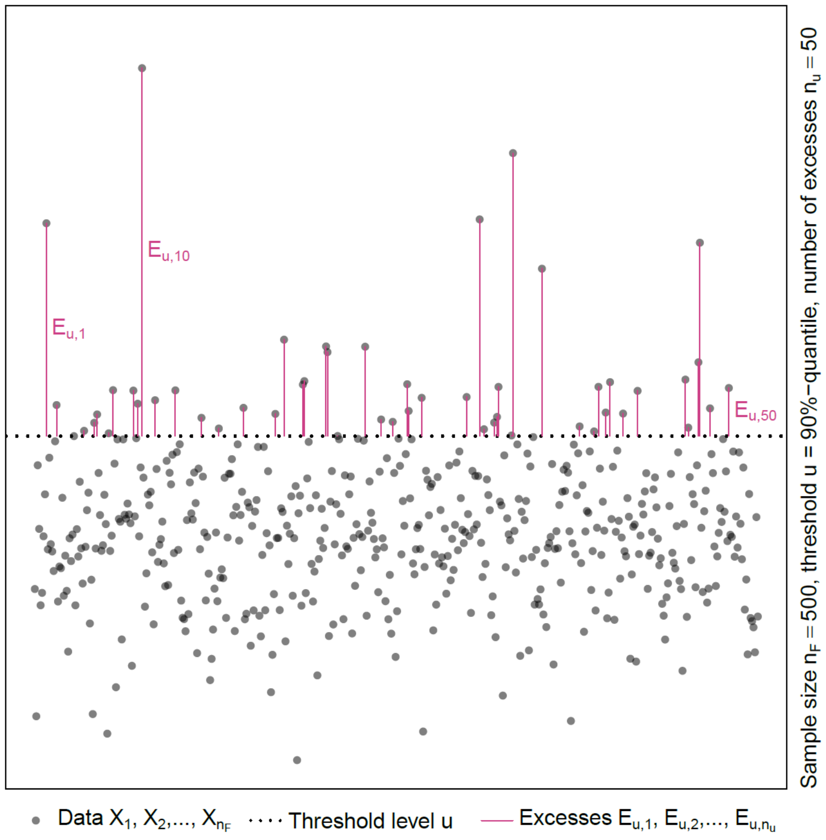
$$P(X > x) = \bar{F}(x) = 1 - F(x) = (1 - F(u))\frac{1 - F(x)}{1 - F(u)} = \mathbb{P}(X > u)\mathbb{P}(X > x \mid X > u)$$

$$= \mathbb{P}(X > u)\mathbb{P}(X - u > x - u \mid X > u) = \bar{F}(u)(1 - F_u(x - u)).$$

EVT-POT estimator

$$\hat{\bar{F}}(x) = \frac{n_u}{n_F}\left(1 + \hat{\xi}_{n_u}\frac{x - u}{\hat{\beta}_{n_u}}\right)^{-1/\hat{\xi}_{n_u}}, \quad x \geq u,$$

(MLE regular for $\xi > -1/2$)




## The EVT-based VaR estimator

$$\widehat{\mathrm{VaR}}_\alpha = \hat{F}^{-1}(\alpha) = u + \frac{\hat{\beta}_{n_u}}{\hat{\xi}_{n_u}}\left(\left(\frac{1 - \alpha}{n_u/n_F}\right)^{-\hat{\xi}_{n_u}} - 1\right), \quad \alpha \in [1 - n_u/n_F, 1).$$

## The normal-based VaR estimator

$$\widehat{\mathrm{VaR}}_\alpha = \hat{\mu} + \hat{\sigma}\Phi^{-1}(\alpha), \quad \alpha \in (0, 1).$$

## How to estimate high return levels/exceedance probabilities?

As in the BMM, once we have a distribution for the largest values, we can determine the return level as a quantile of said distribution. In the POTM, these largest values are the exceedances over $u$ and thus the *return level* $r_{u,p}$ is

$$r_{u,p} = F^{-1}(1-p), \quad p \in (0, \bar{F}(u)].$$

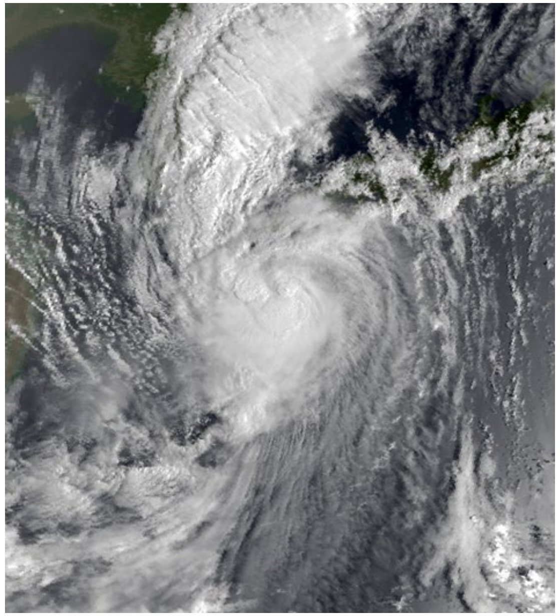
The estimated return level can thus be expressed in terms of the POT quantile estimator

$$\hat{r}_{u,p} = u + \frac{\hat{\beta}_{n_u}}{\hat{\xi}_{n_u}}\left(\left(\frac{p}{n_u/n_F}\right)^{-\hat{\xi}_{n_u}} - 1\right), \quad p \in (0, n_u/n_F].$$

An insightful application,
perhaps **the** EVT-”showcase” example

# The sinking of the MV Derbyshire

- Massive (294x44m, 160’000 tonnes) ore-bulk-oil carrier built 1976
- The Derbyshire started her final voyage on July 11, 1980
- Off the coast of Japan she ran into Typhoon Orchid

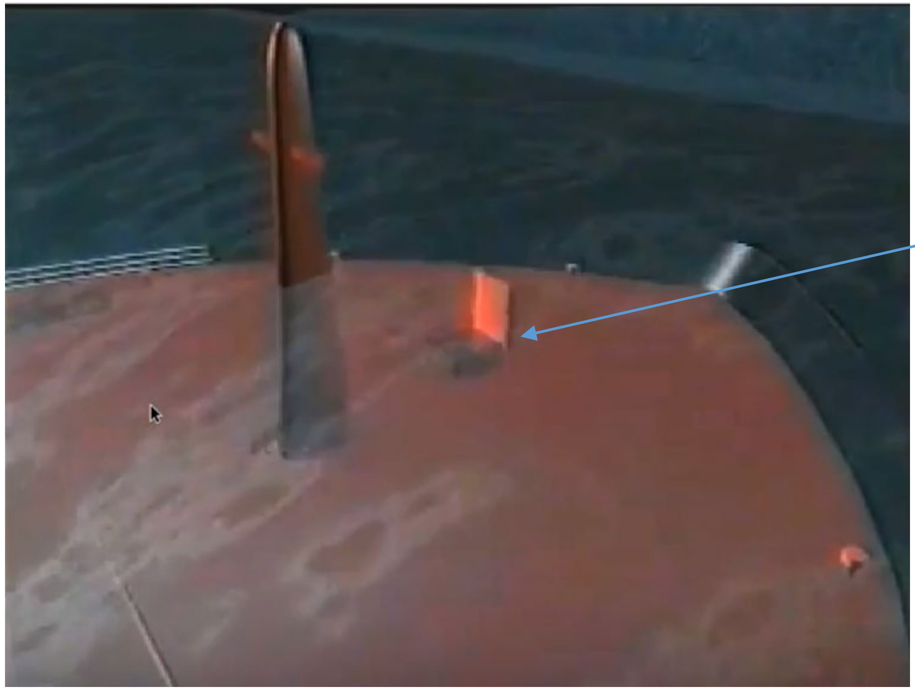


# The sinking of the MV Derbyshire

- On 9 Sep 1980, she sinks in the Pacific Ocean, close to Japan, during Typhoon Orchid
- All 44 people on board died
- Sunk suddenly: no mayday calls, no lifeboats
- Largest UK ship to have been lost at sea
- Basic references on the statistical EVT analysis: "**J.E. Heffernan & J.A. Tawn** (2003), JRSS(C) **52(3)**, 337–354 (University of Lancaster) and J.E. Heffernan and J.A. Tawn (2001), Extremes **4**, 359 -378."
- What happened?

# The data

- In June 1994, the wreck of Derbyshire was found at a depth of 4 km, spread over 1.3 km. An additional expedition spent over 40 days photographing and examining the debris field looking for evidence of what sank the ship ([200 hours of video, 137 000 photographs]).
- Laboratory measurements were based on a 1:65 scale replica of the Derbyshire in a wave tank provided by MARIN, the Maritime Research Institute located in Wageningen in the Netherlands. The statistical design for the latter was a so-called 2x4x3 factorial design accounting for an intact versus a damaged ship (hence 2), 4 wave conditions and 3 speeds.
- Satellite images of the storm and sea conditions.



Ventilation shafts

Maximum sustainable load on hatch covers is 42 kPa

# The question (in reduced form)

P(wave impact > 42 kPa on hatch cover 1| various ship & sea conditions) (*)

or using EVT language, estimate

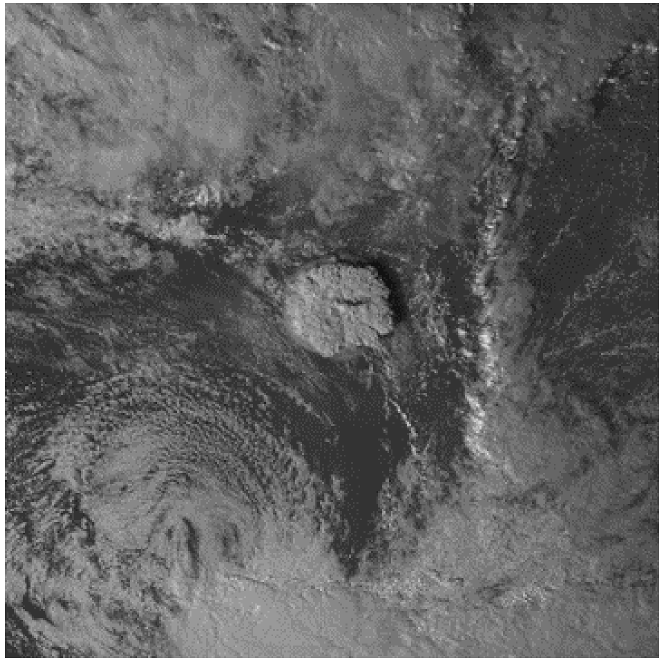
$F_u$(x;**z**) = P(impact ≤ x | impact > u, **z**)

for some high threshold u (10 kPa, say) and a vector **z** of relevant covariates and then estimate (*), further check for model uncertainty and goodness of fit.

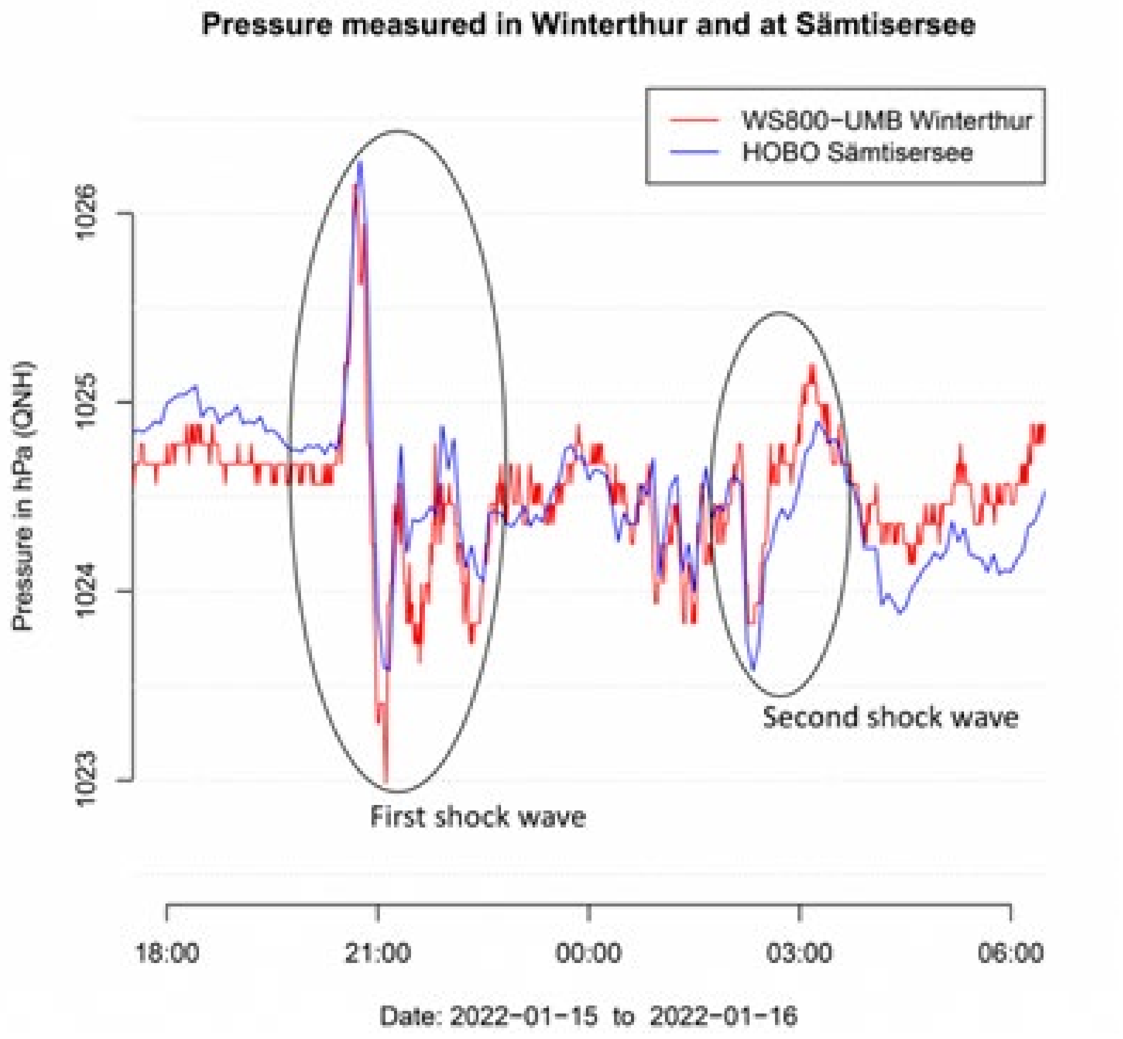
The **relevance of EVT** through the work of Jonathan Tawn and Jane Heffernan becomes clear from the following conclusion of the presiding **Judge Colman**: “The contribution that the extremes values group at Lancaster made to identify the cause of the sinking was of absolutely fundamental importance to the outcome of this Investigation.”

As a result of a further, more involved EVT-based statistical analysis, Lloyds Registry of Shipping eventually proposed a 35% strengthening of the hatch covers.

# A picture is worth a thousand words



The Hunga Tonga – Hunga Ha'apai volcanic eruption of 15/1/22




Source: ZHAW

# Conclusion

- Changing our perception of risk from ***If*** to ***What If*** is absolutely crucial
- **EVT** yields one important methodological tool to do just that
- Going forward, **interconnectivity** within the risk landscape will increase together with a need for truly **global models**
- **Climate change** will be a major risk driver across all fields of application leading to non-stationarity/change(tipping) points
- **Environmental** and **social resilience** have to go hand in hand
- **Risk communication** increasingly plays a fundamental role

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**Thank you!**