### **Risk Revealed**

**Cautionary Tales, Understanding and Communication** 

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Conference in memory of Tomas Björk (1947 – 2021) Stockholm, October 10 - 11, 2022

# Tomas Björk (1947 – 2021)



### My first academic contact with Tomas Björk:

Exponential inequalities for ruin probabilities in the Cox case **Tomas Björk & Jan Grandell** Scandinavian Actuarial Journal, Volume **1988**, 1988-Issue 1-3, 77 – 111

**Abstract**: We consider an insurance model where the underlying point process is a Cox process. Using a martingale approach we obtain extensions of the classical Lundberg inequality.



Risk Modelling in Insurance and Finance in honour of Jan Grandell's 65th birthday

June 13, 2008, KTH, Stockholm, Sweden

Paul Embrechts: Statistics and Quantitative Risk Mnagement

Tomas Björk: Time Inconsistent Stochastic Control



### A personal story related to his famous book:



. . .

1998

2019



### Arbitrage Theory in Continuous Time

Tomas Björk

This book gives a comprehensive account of the arbitrage theory of financial derivatives in a mathematically precise way, but without the explicit use of abstract measure theory. It is aimed at graduate students and practitioners in economics, but will also be of interest to mathematicians and researchers in finance. The text is heavily orientated towards concrete computations and practical handling of stochastic differential equations, in their economic applications as well as in their purely mathematical context. The reader will find numerous worked-out examples as well as a large number of exercises.

After an introductory chapter on the binomial model, the focus is exclusively on continuous time models. The reader is given a self-contained treatment of stochastic differential equations and H0 calculus, including the Feynman-Kac connections to partial differential equations, and the classical Kolmogorov equations.

The methodological approach to arbitrage pricing is taken through the use and construction of locally riskless portfolios. This leads immediately to pricing formulas as solutions to partial differential equations. Risk neutral valuation formulas and martingale measures are then introduced through Feynman-Kac representations of the solutions of the PDEs. Still, the text is essentially a probabilistic one, emphasizing the use of martingale measures for the computation of prices. The book covers stock price models, with one or several underlying assets, and presents a full treatment of both pricing and hedging. A special chapter is devoted to pricing and hedging problems in incomplete models. Barrier options, options on dividend-paying assets, as well as currency markets (including quanto products) are given separate chapters. Interest rate theory is dealt with in some depth.

including the most common short rate models, affine term structures, inversion of the yield curve, and the Heath-Jarrow-Morton approach to forward rate models. A separate chapter is devoted to the modern change-ofnumeraire technique, which makes it possible to give concrete pricing formulas for a large number of fairly complicated interest rate derivatives.

The book also includes a self-contained treatment of stochastic optimal control theory, on the fringes of arbitrage pricing, but of interest to the general reader. This theory is then applied to optimal consumption/ investment problems, and the Merton fund separation theorems are derived. HANDELSHÖGSKOLAN I STOCKHOLM STOCKHOLM SCHOOL OF ECONOMICS Tomas Björk

Tomas

Dear Paul, Please find enclosed a personal copy of my book. your support and encouragment were essential for its coming into explance. Best regards 1

To Paul

and many thanks

Jouas

The personal story ...

### From its Introduction:

"Hans Bühlmann, Paul Embrechts and Hans Gerber gave me the opportunity to give a series of lectures for a summer school at Monte Verita in Ascona 1995. This summerschool was for me an extremely happy and fruitful time, as well as the start of a partially new career.

The set of lecture notes produced for that occasion is the basis for the present book."





#### Table of Contents of the book

#### Six cautionary tales: a walk

- 1. The 1953 Great Flood
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#### Methodology: a hike

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#### References

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### An alternative title for the book would be

From *If* to *What If* 

and

**EVT** 

(Extreme Value Theory)

We start with two motivating examples

# A comparison between Onagawa and Fukushima Dai-ichi NPPs after the March 11, 2011 Tōhoku earthquake and tsunami



Earth Observatory Singapore

Interconnection of risks

### The basic results of EVT

- The Fisher-Tippett-Gnedenko Theorem
- The Pickands-Balkema-de Haan Theorem

and their statistical interpretations

#### EVT: the basics of "What If" thinking

The Block Maxima Method (BMM)—

**Data:**  $X_1, ..., X_n$  iid with df F  $(n = n_F)$ **Maximum:**  $M_n = \max(X_1, ..., X_n)$ 

#### **The Fisher-Tippett-Gnedenko Theorem**

If  $\exists c_n > 0$  and  $d_n$  so that for  $n \to \infty$ 

$$(M_n - d_n)/c_n \Rightarrow H \text{ (n.d.) (F } \in \text{MDA}(H))$$

then *H* is of the following type:

$$H_{\xi,\mu,\sigma}(x) = \begin{cases} \exp(-(1+\xi(x-\mu)/\sigma)^{-1/\xi}), & \xi \neq 0, \\ \exp(-e^{-(x-\mu)/\sigma}), & \xi = 0, \end{cases}$$

The Generalized Extreme Value distribution (GEV) Notation:  $H_{\xi,0,1} = H_{\xi}$ 



Block maxima M<sub>n,1</sub>, M<sub>n,2</sub>,..., M<sub>n,n</sub>

···· Dividers (n<sub>b</sub> blocks) • Block maxima M<sub>n.1</sub>, M<sub>n.2</sub>,...

The Peaks Over Threshold Method (POTM)

The conditional excess distribution *What If*  $F_{\mu}(x) = P(X - u \le x | X > u)$ 

The Pickands-Balkema-de Haan Theorem

 $F \in MDA(H_{\xi})$  iff  $\exists \beta: (0,\infty) \rightarrow (0,\infty)$  so that

 $\lim_{u \to x_F} \sup_{0 \le x < x_F - u} |F_u(x) - G_{\xi,\beta(u)}(x)| = 0$ 

$$G_{\xi,\beta}(x) = \begin{cases} 1 - (1 + \xi x/\beta)^{-1/\xi}, & \xi \neq 0, \\ 1 - \exp(-x/\beta), & \xi = 0, \end{cases}$$

The Generalized Pareto Distribution (GPD)



Data X<sub>1</sub>, X<sub>2</sub>,..., X<sub>n<sub>F</sub></sub> ···· Threshold level u — Excesses E<sub>u,1</sub>, E<sub>u,2</sub>,..., E<sub>u,n<sub>i</sub></sub>

### A distributional-tail estimator

#### The POT/GPD tail estimator

$$P(X > x) = \bar{F}(x) = 1 - F(x) = (1 - F(u))\frac{1 - F(x)}{1 - F(u)} = \mathbb{P}(X > u)\mathbb{P}(X > x \mid X > u)$$
$$= \mathbb{P}(X > u)\mathbb{P}(X - u > x - u \mid X > u) = \bar{F}(u)(1 - F_u(x - u)).$$

estimator

$$\hat{\bar{F}}(x) = \frac{n_u}{n_F} \left( 1 + \hat{\xi}_{n_u} \frac{x - u}{\hat{\beta}_{n_u}} \right)^{-1/\hat{\xi}_{n_u}}, \quad x \ge u,$$

(MLE regular for  $\xi > -1/2$ )

The EVT-based VaR estimator

$$\widehat{\operatorname{VaR}}_{\alpha} = \widehat{F}^{-1}(\alpha) = u + \frac{\widehat{\beta}_{n_u}}{\widehat{\xi}_{n_u}} \Big( \Big(\frac{1-\alpha}{n_u/n_F}\Big)^{-\widehat{\xi}_{n_u}} - 1 \Big), \quad \alpha \in [1-n_u/n_F, 1).$$

The normal-based VaR estimator

$$\widehat{\operatorname{VaR}}_{\alpha} = \hat{\mu} + \hat{\sigma} \Phi^{-1}(\alpha), \quad \alpha \in (0, 1).$$



• Data X<sub>1</sub>, X<sub>2</sub>,..., X<sub>n<sub>F</sub></sub> ···· Threshold level u — Excesses E<sub>u,1</sub>, E<sub>u,2</sub>,..., E<sub>u,n<sub>u</sub></sub>

#### How to estimate high return levels/exceedance probabilities?

As in the BMM, once we have a distribution for the largest values, we can determine the return level as a quantile of said distribution. In the POTM, these largest values are the exceedances over u and thus the *return level*  $r_{u,p}$  is

$$r_{u,p} = F^{-1}(1-p), \quad p \in (0, \bar{F}(u)].$$

The estimated return level can thus be expressed in terms of the POT quantile estimator

$$\hat{r}_{u,p} = u + \frac{\hat{\beta}_{n_u}}{\hat{\xi}_{n_u}} \left( \left( \frac{p}{n_u/n_F} \right)^{-\hat{\xi}_{n_u}} - 1 \right), \quad p \in (0, n_u/n_F].$$

An insightful application, perhaps the EVT-"showcase" example

# The sinking of the MV Derbyshire

- Massive (294x44m, 160'000 tonnes) ore-bulk-oil carrier built 1976
- The Derbyshire started her final voyage on July 11, 1980
- Off the coast of Japan she ran into Typhoon Orchid





## The sinking of the MV Derbyshire

- On 9 Sep 1980, she sinks in the Pacific Ocean, close to Japan, during Typhoon Orchid
- All 44 people on board died
- Sunk suddenly: no mayday calls, no lifeboats
- Largest UK ship to have been lost at sea
- Basic references on the statistical EVT analysis: "J.E. Heffernan & J.A. Tawn (2003), JRSS(C) 52(3), 337–354 (University of Lancaster) and J.E. Heffernan and J.A. Tawn (2001), Extremes 4, 359 -378."
- What happened?

# The data

- In June 1994, the wreck of Derbyshire was found at a depth of 4 km, spread over 1.3 km. An additional expedition spent over 40 days photographing and examining the debris field looking for evidence of what sank the ship ([200 hours of video, 137 000 photographs]).
- Laboratory measurements were based on a 1:65 scale replica of the Derbyshire in a wave tank provided by MARIN, the Maritime Research Institute located in Wageningen in the Netherlands. The statistical design for the latter was a so-called 2x4x3 factorial design accounting for an intact versus a damaged ship (hence 2), 4 wave conditions and 3 speeds.
- Satellite images of the storm and sea conditions.



#### Ventilation shafts



Maximum sustainable load on hatch covers is 42 kPa

# The question (in reduced form)

P(wave impact > 42 kPa on hatch cover 1| various ship & sea conditions) (\*)

or using EVT language, estimate

 $F_u(\mathbf{x};\mathbf{z}) = P(\text{impact} \le \mathbf{x} \mid \text{impact} > \mathbf{u}, \mathbf{z})$ 

for some high threshold u (10 kPa, say) and a vector **z** of relevant covariates and then estimate (\*), further check for model uncertainty and goodness of fit.

The **relevance of EVT** through the work of Jonathan Tawn and Jane Heffernan becomes clear from the following conclusion of the presiding **Judge Colman**: "The contribution that the extremes values group at Lancaster made to identify the cause of the sinking was of absolutely fundamental importance to the outcome of this Investigation."

As a result of a further, more involved EVT-based statistical analysis, Lloyds Registry of Shipping eventually proposed a 35% strengthening of the hatch covers.

### A picture is worth a thousand words



The Hunga Tonga – Hunga Ha'apai volcanic eruption of 15/1/22



Pressure measured in Winterthur and at Sämtisersee

Source: ZHAW

### Conclusion

- Changing our perception of risk from *If* to *What If* is absolutely crucial
- EVT yields one important methodological tool to do just that
- Going forward, **interconnectivity** within the risk landscape will increase together with a need for truly **global models**
- Climate change will be a major risk driver across all fields of application leading to non-stationarity/change(tipping) points
- Environmental and social resilience have to go hand in hand
- Risk communication increasingly plays a fundamental role



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# Thank you!