

In memory of Professor Tomas Björk

Voting with Decentralized Policy Contingent Promises

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Tomas Björk and time inconsistency

- All started with Strotz (1955): beautiful continuous time, deterministic model with non constant discount rate.
- Tomas introduced very the idea of time inconsistency in stochastic settings in a standard model in math finance:
 - ▶ Mean variance preferences
- The problem of time inconsistency is much broader because it appears endogenously (i.e not from the preferences) in
 - ▶ Dynamic games: game between central banks and private agents
 - ▶ Collective decision making: Voting

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- This paper: **Vote trading through promises contingent on the collective decision of a committee ruled by a qualified majority rule in the absence of any stigma/constraint.**

This paper

- A committee $\mathbb{I} = \{1, \dots, I\}$ of I members vote to adopt/reject a reform with a super majority rule κ (If $I = 3$, $\kappa = 2$).
 - ① Intensity of preferences for the reform are known $u_1 \leq u_2 \leq \dots \leq 0 \leq \dots \leq u_I$.
 - ② The reform is socially optimal $\sum_i u_i > 0$.

- **Timing of the model:**

- ① The promises $\mathbf{r} = (r_1, \dots, r_I)$ (resp. $\mathbf{s} = (s_1, \dots, s_I)$) contingent on adopting (resp. rejecting) the reform are done within the committee:

$$(\mathbf{r}, \mathbf{s}) \in \mathcal{P}^2 \Leftrightarrow \sum_i r_i = \sum_i s_i = 0$$

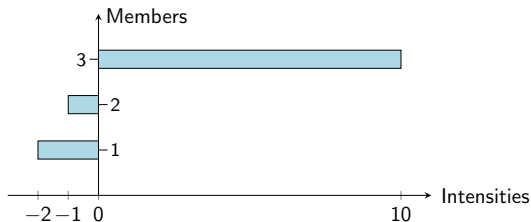
- ② Committee member i vote for or against the reform to maximize the *ex post* intensity

$$v_i^{\mathbf{r}, \mathbf{s}} := \begin{cases} u_i + r_i & \text{if the reform is adopted;} \\ s_i, & \text{otherwise} \end{cases}$$

- ③ The promises are enforced.
- In this framework, we define the “*political equilibrium*” and provide insights on the structure of promises that need to be done to implement the political equilibrium.

An example: Committee with 3 members ruled by majority ($\kappa = 2$)

Ex ante utilities: $\mathbf{u} = (-2, -1, 10)$



- Reform is defeated without promises: $\mathbf{v}^0 = (0, 0, 0)$
- Promises generate gains from trades

$$\begin{aligned} \mathbf{r} = (+3, +2, -5) \quad \mathbf{s} = \mathbf{0} \quad \mathbf{v}^{(\mathbf{r}, \mathbf{0})} &= (1, 1, 5); \\ \mathbf{r} = (+2, +2, -4) \quad \mathbf{s} = \mathbf{0} \quad \mathbf{v}^{(\mathbf{r}, \mathbf{0})} &= (1, 2, 6). \end{aligned}$$

- Reform is adopted after the promises are made!
- Too many degrees of freedom: **Stability at the cheapest cost**

Main results

Equilibrium Definition

1) No **blocking coalition** exists, 2) The total promises are minimized

- All equilibria implement the socially optimal reform
 - ▶ **Intuition:** If the reform is defeated, blocking coalitions emerge to “grow the total size of the pie” and get a better payoff.
- Multiple equilibria
 - ▶ **Intuition:** indeterminacy in cost sharing and distribution of enticements.
- Equilibrium promises feature some general properties:
 - ▶ Voluntary participation for those who make the promises
 - ▶ Push toward equality: Top-down flow of promises.
 - ▶ When the reform is defeated under no trade: Frustrated minority coalition compensates a majority coalition to sway their vote in favour of the reform.
 - ▶ When the reform is adopted under no trade: Promises may be needed to preempt the emergence of frustrated minorities

Notations

- Decisive coalitions:

$$\mathcal{D}^R = \{\mathcal{C} \subseteq \mathbb{I} : |\mathcal{C}| \geq \kappa\}, \quad \mathcal{D}^S = \{\mathcal{C} \subseteq \mathbb{I} : |\mathcal{C}| \geq I - \kappa + 1\}$$

- Supportive coalitions: $\mathcal{C}^R = \{i : u_i \geq 0\}$, $\mathcal{C}^S = \{i : u_i < 0\}$
- Aggregate intensity of preferences: $U_R = \sum_{i \in \mathcal{C}^R} |u_i|$, $U_S = \sum_{i \in \mathcal{C}^S} |u_i|$.
- For any given transfer promises (\mathbf{r}, \mathbf{s}) from the set $\mathcal{P} = \{x \mid \sum_{i \in \mathbb{I}} x_i = 0\}$:
 - ▶ The committee decision is $D(\mathbf{r}, \mathbf{s}) \in \{R, S\}$

$$D(\mathbf{r}, \mathbf{s}) := \begin{cases} R, & \text{if } |\{i : u_i + r_i \geq s_i\}| \geq \kappa; \\ S, & \text{otherwise} \end{cases}$$

- ▶ The voting outcome: $v_i^{r,s}$ is given by

$$v_i^{r,s} := \begin{cases} u_i + r_i, & \text{if } D(\mathbf{r}, \mathbf{s}) = R; \\ s_i, & \text{otherwise} \end{cases}$$

The political equilibrium

A coalition \mathcal{C} of at least two members blocks the promises $(\mathbf{r}, \mathbf{s}) \in \mathcal{P}^2$ iff

- ① **When $D(\mathbf{r}, \mathbf{s}) = R$:** There exists a promise $\tilde{\mathbf{s}} \in \mathcal{P}$ such that:
 - ▶ $\tilde{\mathbf{s}}_i \neq 0$ if and only if $i \in \mathcal{C}$; $D(\mathbf{r}, \mathbf{s} + \tilde{\mathbf{s}}) = S$ and,
 - ▶ $v_i^{\mathbf{r}, \mathbf{s} + \tilde{\mathbf{s}}} > v_i^{\mathbf{r}, \mathbf{s}}$ for all $i \in \mathcal{C}$.
- ② **When $D(\mathbf{r}, \mathbf{s}) = S$:** There exists a promise $\tilde{\mathbf{r}} \in \mathcal{P}$ such that:
 - ▶ $\tilde{\mathbf{r}}_i \neq 0$ if and only if $i \in \mathcal{C}$; $D(\mathbf{r} + \tilde{\mathbf{r}}, \mathbf{s}) = R$ and,
 - ▶ $v_i^{\mathbf{r} + \tilde{\mathbf{r}}, \mathbf{s}} > v_i^{\mathbf{r}, \mathbf{s}}$ for all $i \in \mathcal{C}$.

$(\mathbf{r}, \mathbf{s}) \in \mathcal{P}^2$ is an equilibrium (\mathcal{E}) iff

- ① No blocking coalition exists: (\mathbf{r}, \mathbf{s}) is stable (\mathcal{S}_0),
- ② **Cheapest cost of enticement:** The total promise $\mathcal{T}_{\mathbf{r}, \mathbf{s}} = \frac{1}{2} \sum_{\mathbb{I}} |r_i| + \frac{1}{2} \sum_{\mathbb{I}} |s_i|$ is minimized

Equilibrium analysis

Observation

Equilibria with minimal total promises have the form $(\mathbf{r}, \mathbf{0})$ or simply \mathbf{r} .

Intuition: if (\mathbf{r}, \mathbf{s}) is stable, then $(\mathbf{r} - \mathbf{s}, \mathbf{0})$ is also stable and $\mathcal{T}_{\mathbf{r}-\mathbf{s}, \mathbf{0}} \leq \mathcal{T}_{\mathbf{r}, \mathbf{s}}$.

Proposition 1: Characterization of the stable promises

A promise \mathbf{r} is stable iff

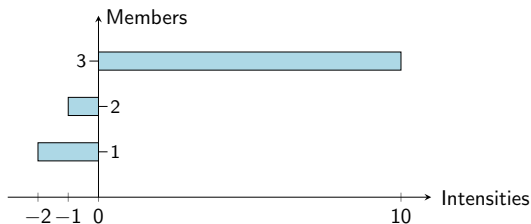
$$\sum_{\mathcal{C}} (u_i + r_i) \geq 0 \text{ for all coalitions } \mathcal{C} \in \mathcal{D}^S.$$

Proposition 2: Existence, indeterminacy and efficiency

Stable promises \mathbf{r} are indeterminate they all implement the reform: $D(\mathbf{r}) = R$.

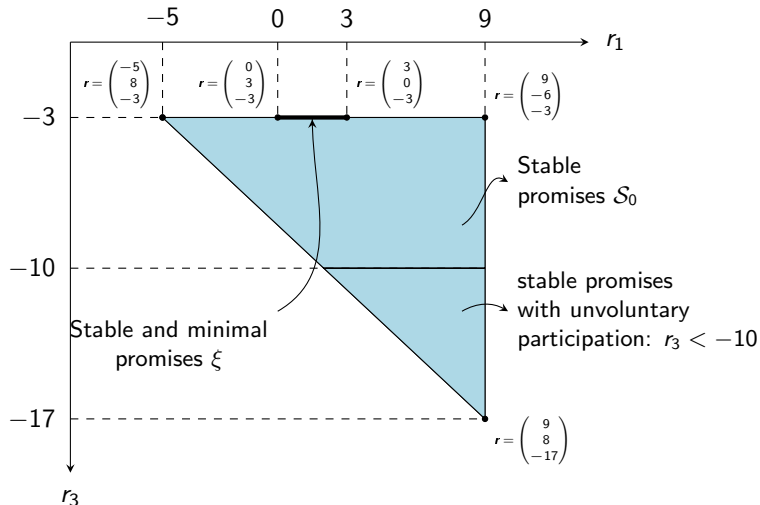
The equilibrium promises are also indeterminate: the multiplicity is not removed by minimizing the total payment $\mathcal{T}_{\mathbf{r}}$.

Example continued: Committee with 3 members ruled by majority



- $\mathbf{u} = (u_1, u_2, u_3) = (-2, -1, 10)$.
- No trading is not an equilibrium: $u_1 + u_2 = -3 < 0$
- The equilibrium payment promises satisfy $r_1 + r_2 \geq 3$, $r_1 + r_3 \geq -8$, $r_2 + r_3 \geq -9$ and, $r_1 + r_2 + r_3 = 0$.
- Member 3 need to pay 3 to the coalition $\{1, 2\}$.
- The set of equilibrium payment promises satisfies $\mathcal{T}_r = 3$

Visualization of the example: $u = (-2, -1, 10)$



- Minimality i) reduces multiplicity but does not eliminate it and, ii) imply voluntary participation: no one promises more than ex ante utility

General properties of the equilibrium promises

Proposition 3: Voluntary participation

Stable promises can violate voluntary participation for those who make the promises. Equilibrium promises are consistent with voluntary participation for those who make the promises.

Proposition 4: Push toward equality

For any equilibrium with minimal total promises, there exists k_* such that:

- $r_i \geq 0$ for all $i < k_*$,
- $r_j \leq 0$ for all $k_* \leq j$ and,
- $v_i^r \leq v_j^r$ for all $i < k_* \leq j$.

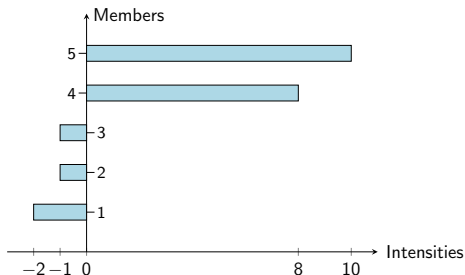
Result:

- Top-down flow of promises.
- The order of inter-coalition *ex ante* intensities is not reversed by the *ex post* intensities.

An example of frustrated minority: $|\mathcal{C}^R| < \kappa$

- A committee with 5 members rules by majority $\kappa = 3$:

$$\mathbf{u} = (u_1, u_2, u_3, u_4, u_5) = (-2, -1, -1, 8, 10).$$

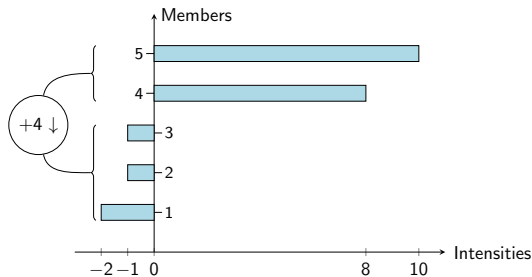


- In any equilibrium, $\mathcal{T}_r = 4$ and the coalition $\mathcal{C}^R = \{4, 5\}$ need to promise a total of 4 to the coalition $\mathcal{C}^S = \{1, 2, 3\}$.

An example of frustrated minority: $|C^R| < \kappa$

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- Examples of minimum payment equilibrium

- ▶ $\mathbf{r} = (2, 1, 1, -2, -2)$ leading to $\mathbf{v}^r = (0, 0, 0, 6, 8)$.
- ▶ $\mathbf{r} = (0, 0, 4, 0, -4)$ leading to $\mathbf{v}^r = (-2, -1, 3, 8, 6)$.

Proposition 5: “Frustrated minority”

The promise profile $\mathbf{r} \in \mathcal{P}$ is an equilibrium if and only if

- 1 $\mathcal{T}_{\mathbf{r}} = \min_{\mathbf{r}' \in \mathcal{S}_0} \mathcal{T}_{\mathbf{r}'} = U_S$
- 2 Members of \mathcal{C}^R send the promises: $r_i \leq 0$ for all $i \in \mathcal{C}^R$. The promises are indeterminate but satisfy the voluntary participation constraint $-u_i \leq r_i$ and $\sum_{\mathcal{C}^R} r_i = -U_S$.
- 3 Members of the coalition \mathcal{C}^S receive the promises: $r_i \geq 0$ with $\sum_{\mathcal{C}^S} r_i = U_S$.
 - 1 When $|\mathcal{C}^R| < \kappa - 1$, $r_i = -u_i > 0$ for all $i \in \mathcal{C}^S$.
 - 2 When $|\mathcal{C}^R| = \kappa - 1$, there are multiple ways of distributing the total payment promises of U^S among the members of \mathcal{C}^S .
- 4 In all cases, the ex post intensity of any reform opponent is smaller than the ex post intensity of any reform supporter

$$u_i + r_i \leq u_j + r_j, \text{ for all } i \in \mathcal{C}^S, j \in \mathcal{C}^R.$$

Equilibrium with strong support for the reform: $|\mathcal{C}^R| \geq \kappa$

- We denote by n the swing voter for the status quo $\mathcal{C}^S = \{1, \dots, n\}$ with $|\mathcal{C}^S| = n \leq l - \kappa$, so that $\mathcal{C}^R = \{n + 1, \dots, l\}$.
- The minority coalition \mathcal{C}^S can “entice” the coalition:

$$\underline{\mathcal{C}}^R = \{n + 1, \dots, l - \kappa + 1\}$$

- The coalition \mathcal{C}^S need to promise a total of $\underline{U}^R := \sum_{\underline{\mathcal{C}}^R} u_i$ to convince members of the coalition $\underline{\mathcal{C}}^R$ to vote against the reform.
- The gains from trade of the coalition \mathcal{C}^S is:

$$G^S = U^S - \underline{U}^R.$$

Proposition 6: No trade equilibrium

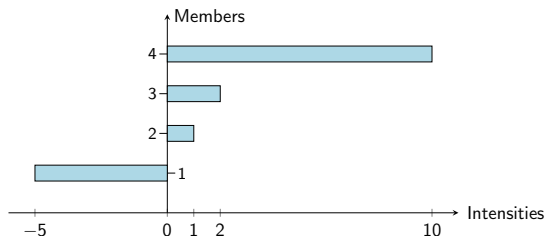
Assume $|\mathcal{C}^R| \geq \kappa$ and $G^S \leq 0$. The only equilibrium is a no trade equilibrium $\mathbf{r} = \mathbf{0}$.

Strong support of the reform, $|C^R| \geq \kappa$ and $G^S > 0$

- Members of the coalition C^R/\underline{C}^R have to promise G^S to preempt members of the coalition C^S from “bribing” the coalition \underline{C}^R into voting for S .
- The total payment promise will be at least G^S .
- The analysis shows that two subcases need to be considered:
 - ▶ The coalition C^R/\underline{C}^R can afford to promise G^S to preempt the enticement of \underline{C}^R from taking place without inducing some of its members to be new subjects of enticements to vote against the reform.
 - ▶ The coalition C^R/\underline{C}^R cannot afford to promise G^S without reversing the natural order to *ex ante* intensities.

The case $|\mathcal{C}^R| \geq \kappa$ and positive but small G^S

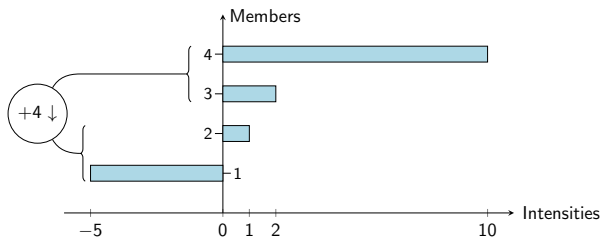
$l = 4$, *ex ante* intensities $\mathbf{u} = (-5, 1, 2, 10)$; majority rule $\kappa = 3$.



- We have $U^R = 13$, $U^S = 5$, $\underline{U}^R = 1$ and $G^S = 4$
- All equilibria require the coalition $\{3, 4\}$ to promise 4 to the members of the coalition $\{1, 2\}$ without reversing the *ex ante* inter coalition ranking of intensities.
- All equilibria have $\mathcal{T}_r = 4$

The case $|\mathcal{C}^R| \geq \kappa$ and positive but small G^S

$l = 4$, *ex ante* intensities $\mathbf{u} = (-5, 1, 2, 10)$; majority rule $\kappa = 3$.



- Indeterminacy occurs again:

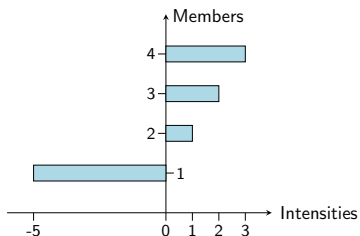
$$\begin{aligned} \mathbf{r} &= (3, 1, 0, -4), & \mathbf{v}^r &= (-2, 2, 2, 6); \\ \mathbf{r} &= (4, 0, -1, -3), & \mathbf{v}^r &= (-1, 1, 1, 7); \end{aligned}$$

- The following \mathbf{r} is not an equilibrium, although its total payment is \$4:

$$\mathbf{r} = (2, 2, -1, -3), \quad \mathbf{v}^r = (-3, 3, 1, 7).$$

The case $|\mathcal{C}^R| \geq \kappa$ and large $G^S > 0$

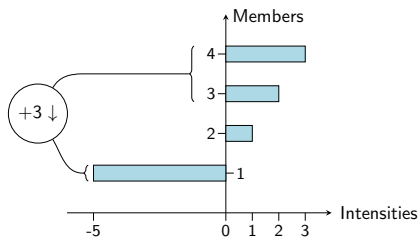
$I = 4$, *ex ante* intensities $\mathbf{u} = (-5, 1, 2, 3)$; majority rule $\kappa = 3$.



- We have $U^R = 6$, $U^S = 5$, $\underline{U}^R = 1$ and $G^S = 4$
- If the members of the coalition $\{3, 4\}$ promise 4 to the members of the coalition $\{1, 2\}$ the *ex ante* inter coalition ranking of intensities cannot be preserved by the *ex post* intensities.
- For example $\mathbf{r} = (4, 0, -2, -2)$ lead to the it *ex post* intensities $\mathbf{v}^r = (-1, 1, 0, 1)$: Member 2 becomes a new target of enticement by member 1.

The case $|C^R| \geq \kappa$ and large $G^S > 0$

$I = 4$, *ex ante* intensities $\mathbf{u} = (-5, 1, 2, 3)$; majority rule $\kappa = 3$.



- To achieve an equilibrium the following algorithm need to be performed:
- **Step 1:** Member 3 and 4 need to promise just enough to align their intensities with that of member 2

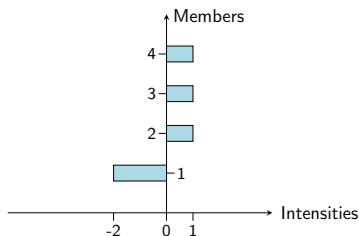
$$\mathbf{r}^{[1]} = (3, 0, -1, -2).$$

- New intensities become

$$\mathbf{u}^{[1]} = (-2, 1, 1, 1).$$

The case $|\mathcal{C}^R| \geq \kappa$ and large $G^S > 0$

$I = 4$, *ex ante* intensities $\mathbf{u} = (-5, 1, 2, 3)$; majority rule $\kappa = 3$.



- New intensities are

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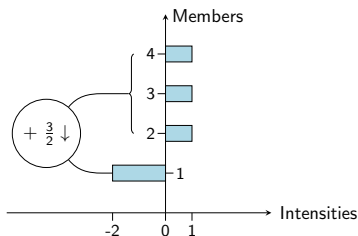
- Gains from trade is

$$G_{[1]}^S = 1$$

- Members of the coalition $\{2, 3, 4\}$ need to promise the same amount otherwise whoever pays more becomes a new target of enticement

The case $|C^R| \geq \kappa$ and large $G^S > 0$

$I = 4$, *ex ante* intensities $\mathbf{u} = (-5, 1, 2, 3)$; majority rule $\kappa = 3$.

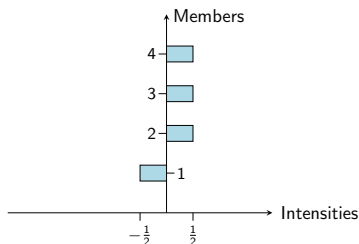


- Each member of the coalition $\{2, 3, 4\}$ promises 0.5 to member 1
- The total payment promises after the two rounds is

$$\mathcal{T}_r = 3 + 3/2 = 9/2 > G^S = 4$$

The case $|\mathcal{C}^R| \geq \kappa$ and large $G^S > 0$

$I = 4$, *ex ante* intensities $\mathbf{u} = (-5, 1, 2, 3)$; majority rule $\kappa = 3$.



- New intensities are

$$\mathbf{u}^{[2]} = \left(-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right).$$

- No more incentives for enticements: $G_{[2]}^S = 0$.

The case $|\mathcal{C}^R| \geq \kappa$ and positive but small G^S

Proposition 7: Preemptive promises of first order

Assume $|\mathcal{C}^R| \geq \kappa$ and $0 < G^S \leq \sum_{j \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R} [u_j - u_{l-\kappa+1}]$.

The payment promises profile $\mathbf{r} \in \mathcal{P}$ is an equilibrium if and only if

- Members of the coalition $\mathcal{C}^S \cup \underline{\mathcal{C}}^R$ receive the promises while the members of the coalition $\mathcal{C}^R \setminus \underline{\mathcal{C}}^R$ send the promises :

$$-u_j \leq r_j \leq 0 \leq r_i \text{ for all } i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R \text{ and } j \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R.$$

- the ex post intensity of any reform opponent is smaller than the ex post intensity of any reform supporter

$$u_i + r_i \leq u_j + r_j, \text{ for all } i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R \text{ and } j \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R.$$

- The minimum total payment achieved in all equilibria is

$$\mathcal{T}_* = \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} r_i = G^S$$

The case $|\mathcal{C}^R| \geq \kappa$ and large $G^S > 0$

Proposition 8: Preemptive promises of higher order

Assume $|\mathcal{C}^R| \geq \kappa$ and $G^S > \sum_{j \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R} [u_j - u_{l-\kappa+1}]$. Define k_* with

$$\sum_{j \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R} [u_j - u_{k_*}] < G^S \leq \sum_{j \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R} [u_j - u_{k_*-1}]$$

The promises $\mathbf{r} \in \mathcal{P}$ is an equilibrium if and only if

$$r_i \geq 0 > r_j, \quad v_i^r \leq u_{k_*} - x_* = v_j^r, \quad \forall i < k_* \leq j.$$

The minimum total promises achieved in all equilibria is,

$$\mathcal{T}_* = \sum_{i < k_*} r_i = \sum_{i \geq k_*} [u_i - u_{k_*} + x_*] > G^S,$$

$$x_* := \frac{G^S - \sum_{j \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R} [u_j - u_{k_*}]}{\kappa - 1}.$$

Conclusion

- We consider a voting model where voters can freely make promises contingent on vote outcome and prior to voting in order to influence the vote of those who receive the promises.
- The promises are decentralized, enforceable and, are only guided by self interest
- Median voter theorem does not hold because the policy set is multidimensional: The political equilibrium is based on stability and total promises minimization.
- We find, that equilibria exist, are indeterminate but satisfy some general properties:
 - ▶ Push toward equality: Top-down flow of payment.
 - ▶ When the reform is defeated under no trade: Frustrated minority coalition compensates a majority coalition to sway their vote in favour of the reform.
 - ▶ When the reform is adopted under no trade: Trading may be needed to preempt the emergence of frustrated minorities