Subjective Survival Expectations and Annuitization Decisions

Tianyu Pang
University of Cambridge

CEPR European Conference on Household Finance

September 2025

Overview

Private Pensions and Annuitization

- Private defined-contribution (DC) pensions are becoming popular worldwide.
- DC schemes expose retirees to the risk of outliving their savings.
- In the UK, 70% private sector employees hold DC pensions (Cribb et al., 2023).
- Annuities convert pension savings to a regular guaranteed income for life.
- The benchmark model predicts full annuitization is optimal, but annuity uptake remains low worldwide (Davidoff et al., 2005; Pashchenko, 2013; Yaari, 1965).

Annuity Purchase in the UK

Annuity Purchase Rate by Age Group

	63-65	66-69	70-74	75-79	80-84	85-90	All
Number of Observations	120	470	413	239	148	69	1459
Fraction Purchasing Annuity in the Past Year	26.7%	16.8%	17.2%	15.5%	18.2%	14.5%	17.5%

- Sample from the English Longitudinal Study of Ageing (2016, 2018 and 2021)
  - Retirees with available DC savings in 2016
- Low annuitization rate with old-age annuity purchase

$\Rightarrow$ in sharp contrast to Yaari's (1965) benchmark model

Biased Survival Beliefs

- The perceived value of annuities depends critically on subjective length of life.
- Individuals have biased beliefs about longevity (Elder, 2013; Wu et al., 2015).
  - Understate the probability of surviving to younger ages (age 70s)
  - Overstate the probability of surviving to advanced ages (age 85 and beyond)
  - Pessimism dominates
- Survival pessimism is a potential explanation for under-annuitization (O'Dea & Sturrock, 2023).

Research Questions

1. How do biased survival beliefs distort the timing and magnitude of annuitization?
2. What is the welfare cost arising from biased survival beliefs?
3. What can be done by the government to mitigate the welfare loss?

Methodology

- Estimate subjective and objective survival models using survey respondents' probabilistic belief elicitation and their actual death records.
- Develop and estimate a dynamic model of annuitization with subjective survival probabilities.
- Conduct counterfactual experiments to study the implications of biased survival beliefs for annuitization decisions and welfare.

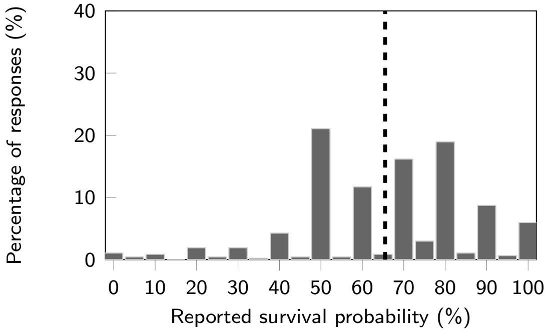
Subjective Survival Expectations

- Sample from the English Longitudinal Study of Ageing (N=1459).
- Question in the survey: *What are the chances that you will live to be X or more?*
- The target age X depends on the respondent's current age:

Age of the respondent	Target age X
60-65	75
66-69	80
70-74	85
75-79	90
80-84	95
85-89	100

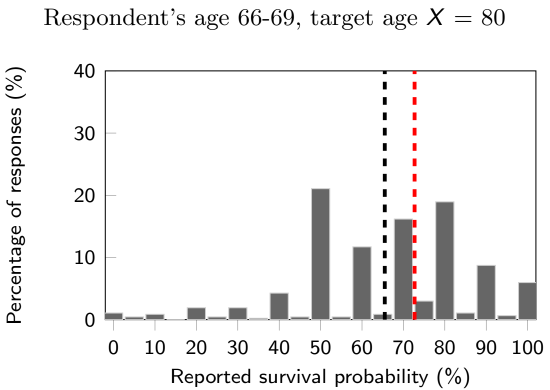
Probabilistic Belief Elicitation

Respondent's age 66-69, target age $X = 80$



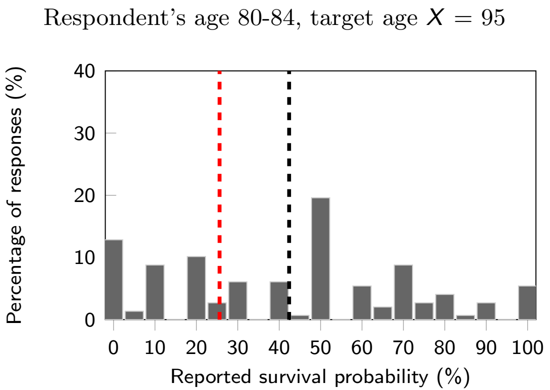
- Black dashed lines: average subjective probabilities

Probabilistic Belief Elicitation



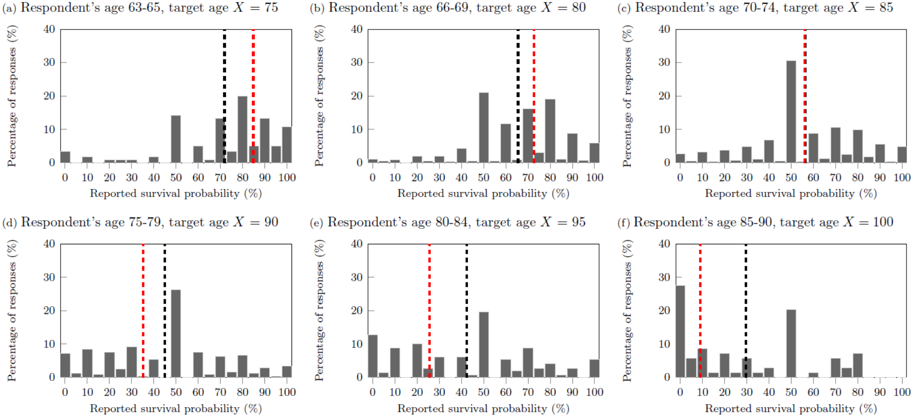
- Black dashed lines: average subjective probabilities
- Red dashed lines: average cohort life table probabilities

Probabilistic Belief Elicitation



- Black dashed lines: average subjective probabilities
- Red dashed lines: average cohort life table probabilities

Probabilistic Belief Elicitation



- Black dashed lines: average subjective probabilities
- Red dashed lines: average cohort life table probabilities

Subjective vs Objective Survival Probabilities

Subjective survival probabilities

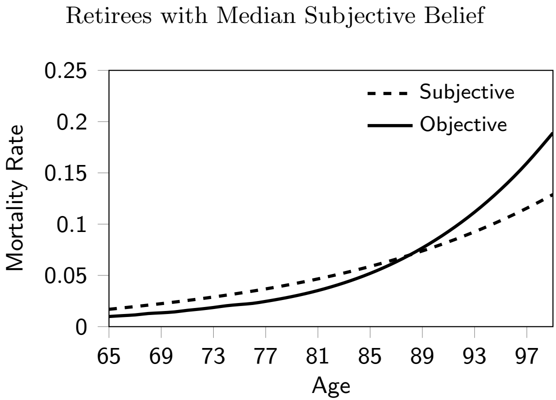
- Capture individual survival beliefs relevant to decision-making
- Modeled as a function of demographics and an individual random effect
- Estimated using survey respondents' probabilistic belief elicitation

Objective survival probabilities

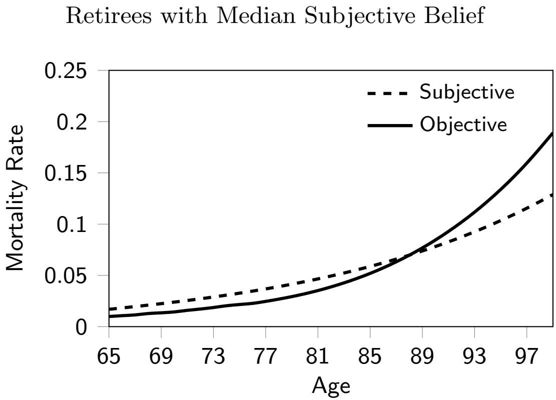
- Capture actual mortality shocks and are relevant only in counterfactual scenarios
- Modeled as a function of the life table, demographics and an individual random effect
- Estimated using survey respondents' actual death records

Model Subjective | Model Objective

Biased Survival Beliefs

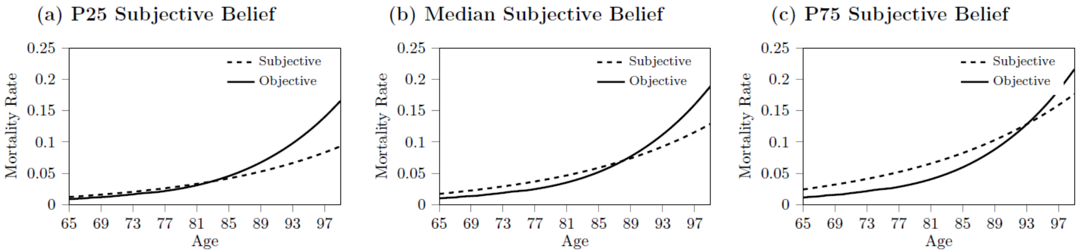


Biased Survival Beliefs



- Age 60s and 70s: pessimistic $\Rightarrow$ 80s and beyond: optimistic

Biased Survival Beliefs: Heterogeneity



- Age 60s and 70s: pessimistic ⇒ 80s and beyond: optimistic

Model Overview

- Individual retirees derive utility from consumption and leaving bequests.
- Retirees make consumption and annuitization decisions every year.
- Survival probabilities are subjective and heterogeneous across retirees (max age $= 100$).
- Annuity price depends on annuitant's age and health at purchase.

Preferences and Health

- Individuals derive utility from consumption and leaving bequests.
  - Utility from consumption:

$$u(c_{it}) = \frac{c_{it}^{1-\gamma}}{1-\gamma}$$

  - Utility from leaving bequests depends on assets at death:

$$b(a_{it}) = \theta \frac{(\kappa + a_{it})^{1-\gamma}}{1-\gamma}$$

- Health status h_{it} can be either good (0) or bad (1) at each age, and health transitions follow a first-order Markov process.

Subjective Survival Probabilities

- The subjective probability of surviving to the next period at each age, s_{it}, depends on age t, health h_{it}, and an individual survival effect δ_i.

- The survival function takes a Gompertz form:

$$s_{it} = e^{-\frac{\phi_{it}}{\alpha}\left(e^{\alpha(t+1)} - e^{\alpha t}\right)}$$

where

$$\phi_{it} = e^{a + bh_{it} + \delta_i}$$

- This is the underlying model that generates the subjective mortality curves introduced previously.

Annuities

- One unit of annuity pays one pound every period until death.
- The price of one unit of annuity, p_{it}, is a function of age and health at purchase.
- Annuity prices are calibrated to the data.
- Equilibrium annuity prices will be calculated in counterfactual cases (will come back to this later).

Income and Assets

- Annuity accumulation equation:

$$n_{it} = n_{it-1} + \Delta_{it}$$

  - n_{it}: total annuity income at age t
  - Δ_{it}: the additional annuity income purchased at age t

- Assets accumulation equation:

$$a_{it+1} = (a_{it} + y_i + n_{it} - \Delta_{it} p_{it} - c_{it})(1 + r)$$

  - y_i: income from public pensions and DB pensions, constant over time
  - a_{it}: total assets at age t, with $a_{it} \geq 0$

The Recursive Problem

Value function at age t:

$$V_t(a_{it}, n_{it}, h_{it}, \delta_i, y_i) = \max_{c_{it}, \Delta_{it}} \Big\{ u(c_{it}) + \beta s_{it} E_t[V_{t+1}(a_{it+1}, n_{it+1}, h_{it+1}, \delta_i, y_i)] + \beta(1 - s_{it}) b(a_{it+1}) \Big\}$$

subject to annuity and assets accumulation equations, where

$$E_t[V_{t+1}(a_{it+1}, n_{it+1}, h_{it+1}, \delta_i, y_i)] = \sum_{h \in \{0,1\}} Pr(h_{it+1} = h | h_{it}, t) V_{t+1}(a_{it+1}, n_{it+1}, h, \delta_i, y_i)$$

Estimation: A Two-step Strategy

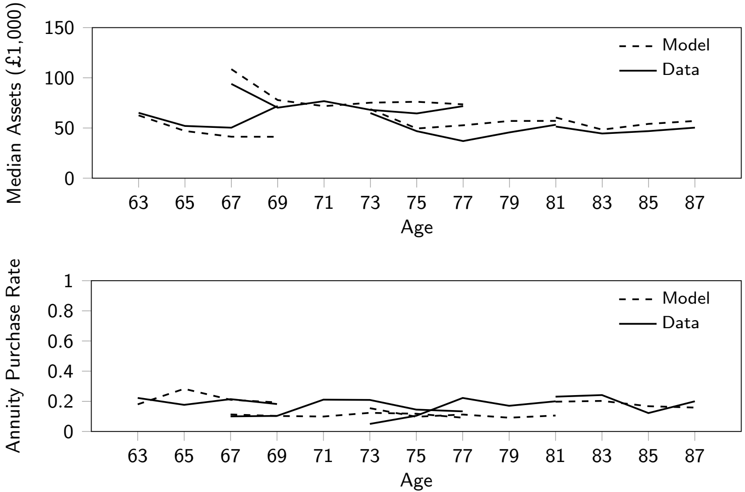
First step: estimation outside of the model

- Health transition matrices
- Subjective survival probabilities
- Annuity prices

Second step: estimate preference parameters $\mathbf{\Theta} = (\beta, \gamma, \theta, \kappa)$

- The Method of Simulated Moments
- Moment conditions: median assets and the fraction of individuals purchasing annuities, by age and birth cohort (38 moments in total)

Model Fit



Parameter Estimates

Annuity Price in Counterfactual Cases

- The price of one unit of annuity is a function of age and health:

$$p_{it} = (1 + \tau(t, h_{it})) \underbrace{\sum_{k=t}^{T} \frac{\widehat{S^{an}}(k|t, h_{it})}{(1+r)^{k-t}}}_{\text{Expected Present Discounted Value}}$$

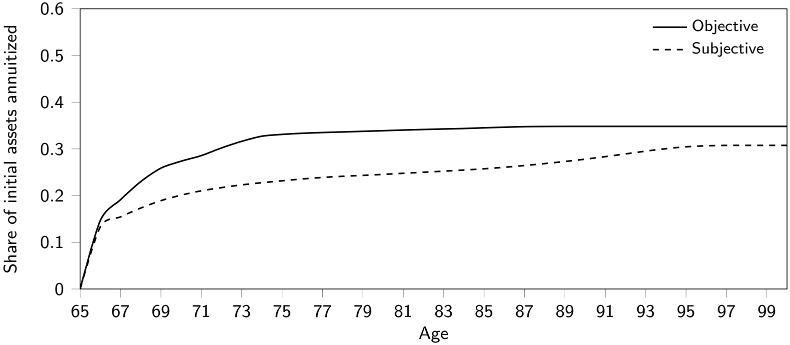
  - $\tau(t, h_{it})$: an annuity load factor
  - $\widehat{S^{an}}(k|t, h_{it})$: the equilibrium annuitant's objective survival probability to age k given age t and health h_{it} at the time of purchase

- In counterfactual cases, $\tau(t, h_{it})$ stays unchanged, and a new $\widehat{S^{an}}(k|t, h_{it})$ is solved through a fixed-point problem.

Annuity Prices

Counterfactual: Objective Survival Probabilities

- Average cumulative annuitized assets as a percentage of initial assets by different ages



Distorted Annuitization

Decision rules	Share of initial assets annuitized by		
	Age 75	Age 85	End of life
Subjective probabilities	23.2%	25.7%	30.8%
Objective probabilities	33.2%	34.5%	34.8%

- Biased survival beliefs reduce annuitization by 4% of retirement-age assets.

Distorted Annuitization

Decision rules	Share of initial assets annuitized by			Weighted avg. age of annuitization
	Age 75	Age 85	End of life	
Subjective probabilities	23.2%	25.7%	30.8%	72.8 years
Objective probabilities	33.2%	34.5%	34.8%	69.6 years

- Biased survival beliefs reduce annuitization by 4% of retirement-age assets.
- Biased survival beliefs delay the average age of annuitization by 3.2 years.

The Welfare Cost of Biased Survival Beliefs

- Retirees incur welfare loss due to their decisions deviating from the optimal paths.
- A monetary measure of welfare
  - Assets equivalent: if individuals had based their decisions on objective survival probabilities, how much initial assets they would need to attain the same level of well-being as with subjective survival probabilities and actual initial assets.
  - Normalized as a percentage of actual initial assets.

The Welfare Cost of Biased Survival Beliefs

	Men	Women	All
Mean	98.34%	98.05%	98.22%
25th	97.82%	97.38%	97.62%
Median	99.09%	98.71%	98.89%
75th	99.48%	99.16%	99.36%

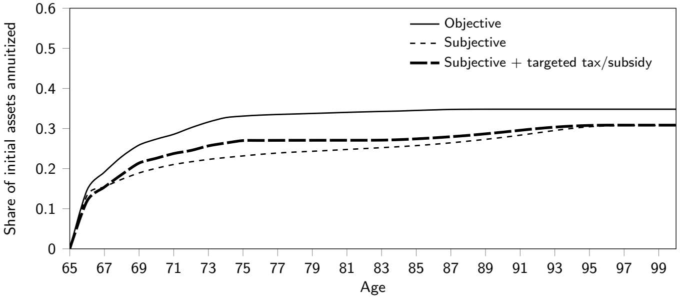
- The average welfare loss is equivalent to 1.8% of retirement-age assets.
- The welfare loss is slightly larger for women.

Policy Implications: A Quantitative Example

- Subsidize 5% of the premiums for annuity purchases prior to age 75
- Tax by 2% of the premiums for annuity purchases made thereafter

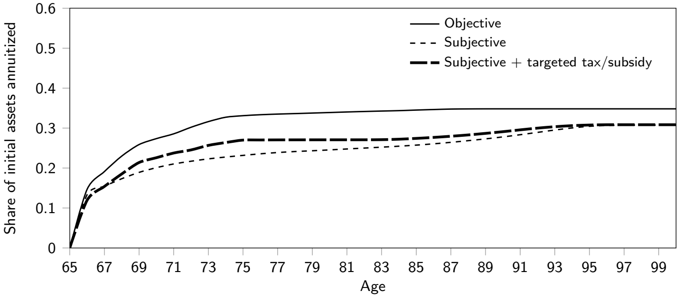
Policy Implications: A Quantitative Example

- Subsidize 5% of the premiums for annuity purchases prior to age 75
- Tax by 2% of the premiums for annuity purchases made thereafter



Policy Implications: A Quantitative Example

- Subsidize 5% of the premiums for annuity purchases prior to age 75
- Tax by 2% of the premiums for annuity purchases made thereafter



- Net costs to the government: -1.28% of total retirement-age assets
- Welfare gains: +1.62% of total retirement-age assets

Conclusion

- Retirees typically underestimate their chance of survival during the early years of retirement and overestimate it after their 80s.
- I develop and estimate a dynamic model of annuitization with subjective survival probabilities.
- Counterfactual analysis shows that, on average:
  - Biased survival beliefs reduce annuitized assets by 11.7% and delay the average age of annuitization by 3.2 years.
  - The welfare loss amounts to 1.8% of retirement-age assets.
  - Government subsidies for early annuity purchases by just-retired individuals financed in part by taxing later annuity purchases would improve welfare.

Thank You for Listening!

Model of Reporting Survival Beliefs

- Individuals report their subjective probability of surviving to a future target age a
- The subjective probability of surviving to the next period:

$$s_{it} = e^{-\frac{\phi_{it}}{\alpha}(e^{\alpha(t+1)} - e^{\alpha t})}$$

where

$$\phi_{it} = e^{a + bh_{it} + \delta_i}$$

- Derive $S(a|t, h_{it}, \delta_i)$, the true subjective probability of surviving to a future target age a

Model of Reporting Survival Beliefs

- Self-reported survival probabilities are subject to recall errors:

$$P_{it}^{*} = S(a|t, h_{it}, \delta_i) + \varepsilon_{it}$$

  - P_{it}^{*}: the recalled subjective survival probability
  - ε_{it}: an i.i.d. recall error

- The latent P_{it}^{*} is rounded before being reported. The probability of using a particular rounding rule r is:

$$Pr(R_{it} = r) = Pr(\mu_{r-1} < \delta_i^{rd} + \varepsilon_{it}^{rd} < \mu_r)$$

  - R_{it}: a random variable representing rounding rules
  - μ_r: $\mu_0 = -\infty$, $\mu_4 = \infty$, and μ_1, μ_2, μ_3 are model parameters
  - δ_i^{rd}: an unobserved individual rounding effect
  - ε_{it}^{rd}: an i.i.d rounding shock

Model of Reporting Survival Beliefs

- The density function:

$$f(P_{it}|t, h_{it}, \delta_i, \delta_i^{rd}) = \sum_{r \in \Omega_{it}} Pr(R_{it} = r|\delta_i^{rd}) * Pr(l_r(P_{it}) \leq P_{it}^* < u_r(P_{it})|t, h_{it}, \delta_i)$$

  - P_{it}: reported survival probability in the survey
  - Ω_{it}: a set of rounding rules compatible with the reported probability
  - $l_r(P_{it})$ and $u_r(P_{it})$: lower bound and upper bound of the latent P_{it}^* given a specific rounding rule r

- Individual effects δ_i and δ_i^{rd} follow a bivariate normal distribution, and the model is estimated using maximum simulated likelihood

Back

Model of Objective Survival Probabilities

- The objective mortality rate at age t depends on cohort c_i, gender g_i, individual survival effect δ_i and age-dependent health status h_{it}:

$$q_{it}^{o} = e^{(\psi_0 + \psi_1 h_{it} + \psi_2 \delta_i)} q^{lt}(t, c_i, g_i)$$

where $q^{lt}(t, c_i, g_i)$ is taken from the cohort life table

Back

Annuity Prices

- The annuity load factor is a linear function of age, health status and their interaction.
- The set of parameters $\boldsymbol{\eta}$ is estimated according to the following:

$$\hat{\boldsymbol{\eta}} = \underset{\boldsymbol{\eta}}{\operatorname{argmin}} \sum_{(t,h_t)\in\mathbb{V}} \left(p^{data}(t,h_t) - (1+\tau(t,h_t;\boldsymbol{\eta})) \sum_{k=t}^{T} \frac{S^{an}(k|t,h_t)}{(1+r)^{k-t}} \right)^2$$

  where $p^{data}(t,h_t)$ and $S^{an}(k|t,h_t)$ are taken from the data, and $\mathbb{V} = \{55,60,...75\} \times \{0,1\}$.

- In counterfactual cases, $\boldsymbol{\eta}$ stays unchanged, and the new $S^{an}(k|t,h_t)$ is solved through a fixed-point problem.

Back

Parameter Estimates

Parameter	Description	Estimate	S.E.
β	Time discount factor	0.962	0.004
γ	Coeff. of relative risk aversion	2.050	0.017
θ	Marginal prop. to bequeath	0.847	0.004
κ	Asset threshold for bequest	749.2	25.1

Back

References

Cribb, J., Emmerson, C., Johnson, P., Karjalainen, H., & O'Brien, L. (2023). *Challenges for the uk pension system: the case for a pensions review* (No. R255). IFS Report.

Davidoff, T., Brown, J. R., & Diamond, P. A. (2005). Annuities and individual welfare. *American Economic Review*, *95*(5), 1573–1590.

Elder, T. E. (2013). The predictive validity of subjective mortality expectations: Evidence from the health and retirement study. *Demography*, *50*(2), 569–589.

O'Dea, C., & Sturrock, D. (2023). Survival pessimism and the demand for annuities. *The Review of Economics and Statistics*, *105*(2), 442–457.

Pashchenko, S. (2013). Accounting for non-annuitization. *Journal of Public Economics*, *98*, 53–67.

Wu, S., Stevens, R., & Thorp, S. (2015). Cohort and target age effects on subjective survival probabilities: Implications for models of the retirement phase. *Journal of Economic Dynamics and Control*, *55*, 39–56.

Yaari, M. E. (1965). Uncertain lifetime, life insurance, and the theory of the consumer. *The Review of Economic Studies*, *32*(2), 137–150.