Giovanni Maria Cocilovo

SAVINGS, HOUSING, FINANCIAL FRICTIONS, AND IDIOSYNCRATIC RISK

STOCKHOLM SCHOOL OF ECONOMICS

SAVINGS, HOUSING, FINANCIAL FRICTIONS, AND IDIOSYNCRATIC RISK

This dissertation investigates the impact of financial frictions and imperfect access to credit in an economy characterized by idiosyncratic income risk. First, the dissertation analyses in detail the traditional case of a single-asset economy, with frictionless financial markets. Then, a two-asset economy is considered, where agents can invest in government bonds and buy housing. The research paper shows that a tightening of financial conditions (i.e. a credit crunch or a rise in intermediation costs) leads to a portfolio reallocation effect, where households with initial low wealth prefer to reduce their debt outstanding and invest in liquid bonds, in order to self-insure against adverse income shocks, while agents with high initial wealth prefer to invest in housing than bonds, as the former becomes a more profitable investment, after the negative financial shock. This, in turn, leads to an increased wealth inequality across households.

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Giovanni Maria Cocilovo

STOCKHOLM SCHOOL OF ECONOMICS

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To my wife and my family

Foreword

This volume is the result of a research project carried out at the Department of Economics at the Stockholm School of Economics (SSE).

This volume is submitted as a Licentiate thesis at SSE. In keeping with the policies of SSE, the author has been entirely free to conduct and present his research in the manner of his choosing as an expression of his own ideas.

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I owe my wife Monika, my parents Maria Antonietta and Francesco, and my sister Beatrice a huge debt of gratitude, for their love, patience and undying support.

Stockholm, June 1 2023

Giovanni Maria Cocilovo

Contents

Abstract

This dissertation investigates the impact of financial frictions and imperfect access to credit in an economy characterized by idiosyncratic income risk. The first part of the analysis describes and investigates in detail the baseline framework developed by Guerrieri and Lorenzoni (2017), and replicate the credit crunch experiment laid out in the original paper. In the following sections I also extend the analysis to a two-asset economy, with liquid bonds and illiquid housing, also characterized by transaction costs. I will show that a tightening of financial conditions (i.e. a credit crunch or a rise in intermediation costs) leads to a portfolio reallocation effect, where households with initial low wealth prefer to reduce their debt outstanding and invest in liquid asset, in order to self-insure against adverse income shocks, while agents with high initial wealth prefer to invest in housing than liquid bonds, as the former becomes a more profitable investment, after the negative financial shock. This, in turn, leads to an increased wealth inequality across households.

Introduction

In their well-known paper from 2017, Veronica Guerrieri and Guido Lorenzoni address the pivotal questions on how consumers' choices (in terms of consumption, investments, and labour supply) are affected by a sudden tightening of the financial markets, and how such changes impact the aggregate economy. By using a Bewley-Hugget-Aiyagari setup, with idiosyncratic income shocks and a single (liquid) financial asset b, the authors find out that a credit crunch yields a heterogeneous effect on households, depending on their initial conditions[1]. More specifically, agents with low initial wealth reduce their debt size and increase their precautionary savings, for an eventual negative income shock (the so called partial equilibrium or precautionary channel), while individuals with high initial wealth reduce their investment portfolio and increase their standard consumption, as a response to the negative adjustment of the economy's interest rate[2]. At the aggregate level, the reduced availability of resources for households yields two opposite effects on output: on the one hand, it depresses consumption, both in the short and in the long run, thus pushing aggregate income down; on the other hand it leads to higher labour supply, to compensate for the lost funds, hence yielding a positive effect on output. Under the proposed calibration, the consumption channel is stronger, therefore the credit crunch has an overall negative effect on output.

In this dissertation I extend the setup of Guerrieri and Lorenzoni (2017), by modelling asset markets so that they include additional and more realistic elements. The aim is to investigate how households' investment and borrowing decisions are formed under the assumption that there are multiple assets on the market, both liquid (represented by loans and bonds) and illiquid (represented by housing), and by taking into account transaction costs and credit restrictions. More specifically, I will consider the case in which the interest rate on loans is higher than the real interest rate, due to intermediation, and borrowing is only possible when covered by collateral. The inclusion of illiquid housing in the model yields several discrepancies from the baseline setup of Guerrieri and Lorenzoni (2017). Firstly, the presence of market frictions generate sizable inequality in the

[1] In Guerrieri and Lorenzoni (2017) a credit crunch is modelled as an drop in the absolute value of the economy's exogeneous borrowing limit, denoted by ϕ.

[2] In Guerrieri and Lorenzoni (2017), the increased demand for assets b, stemming from the low-wealth individuals, negatively affects interest rates.

cross-sectional distribution of wealth of the economy and, as market conditions become tighter, the difference in asset-holding between high- and low-wealth individuals becomes wider. Secondly, while in Guerrieri and Lorenzoni (2017) a credit crunch prompts low-wealth individuals to increase their savings, and high-wealth individuals to decumulate their bond holdings, in a two-asset economy, a drop in the borrowing limit yields to a portfolio re-allocation effect. Motivated by precautionary reasons, low-wealth agents look for increased liquidity, by selling assets and purchasing bonds, which can be disposed of more easily in case of a negative income shock, thus driving the interest rates down. At the same time, following the drop in rates, high-wealth individuals prefer to sell liquid assets and invest in housing, now a more profitable investment asset.Such dynamics generate even more wealth inequality across households. Thirdly, at aggregate level, a credit crunch does not necessarily lead to a drop in output: I find that, while aggregate labour supply increases due to the reduced available resources, the drop in consumption is relatively small, compared to the baseline economy in Guerrieri and Lorenzoni (2017), thus the final effect on output is positive.

Literature Review. The dissertation is related to the vast macroeconomic literature that analyses household consumption and saving choices, in an economy with idiosyncratic income risk. More specifically, my work is related to the pivotal contributions of Bewley (1977), Huggett (1993), and Aiyagari (1994). These authors move away from the classical representative-agent assumption, which posits that the whole economy can be represented by a single (type of) agent, to characterize an economy populated by an ensemble of heterogeneous households, each one with idiosyncratic characteristics (e.g. in terms of labour productivity, preferences or expectations), which change according to a random process. Throughout the analysis, I assume that each agent has an idiosyncratic labour productivity, which evolves over time according to a Markov process, and that agents cannot fully self-insure against the eventuality that the labour productivity drops to a worse state (uninsurable income risk). The adoption of the heterogeneous-agent paradigm in my analysis is pivotal, as it allows to understand how different initial conditions lead to different investment and borrowing choices, and how a tightening of financial conditions in the economy affects the distribution of wealth across households.

Additionally, the dissertation investigates how a tightening of financial markets affects individual's decisions in terms of investment and borrowing. To do so, I follow the methodology laid out in Boppart et al. (2018). More specifically, I solve the economy for the stationary equilibrium, calibrating it to match some long-term targets, and then I model the financial contractions as one-time MIT shocks. I study the impact of such shocks on the initial equilibrium, and I compute the transitional dynamics of the aggregate economy, to understand how it converges to a new final stationary equilibrium. The main advantage of this method is that, unlike alternative methods, such as the one in Krusell and Smith Jr. (1998), it allows the researcher to fully define and analyse the effect of one shock at a time on the economy, without the need of specifying further assumptions to disentangle the effect of the shock under analyses, from the impact of alternative concurring phenomena. This way, it is possible to have a clear and complete picture of how the shock under analysis affects both on the aggregate economy and on the wealth distribution of the households. In addition to this, it keeps the computational time required to solve the model at a manageable level, when the simulation is performed using a computer software.

The analysis is also related to the branch of the literature that analyses the effects of financial constraints and frictions on households' and firms' portfolios and net worth. In their well-known paper, Kyotaki and Moore (1997) show that a tightening of credit constraints has an adverse effect on firms' net worth. When borrowing is constrained, firms that are hit by a negative productivity shock may need to cut back on their investment expenditure, since they are no longer able to borrow more to sustain their production activities. This in turn leads to a lower demand for investment assets, and to a drop in asset prices. Lower production implies less revenues and less financial resources to invest in future times, thus leading to a protracted period of low investments and asset prices. Hall (2011) study the impact of financial frictions on household investment and consumption decisions, in a New Keynesian framework. The author find that tight financial conditions systematically affect individual decisions, and a rise in credit spread may depress consumption choices (and hence aggregate output) and reduce investments, given the imperfect access to credit. Therefore, tighter financial frictions may greatly affect credit and investment decisions across the business cycles and lead to the insurgence of financial crises (Hall, 2013). Using panel data at the household level, from 30 countries in the period 1960-2012, Mian, Sufi, and Verner (2017) study the impact of lowering mortgage rates on households' finances. They find that a systematic drop in mortgage rate generate a credit boom for agents, leading to an sizeable increase in the aggregate debt-to-GDP ratio and to the aggregate consumption levels. Furthermore, Guerrieri and Lorenzoni (2017) find that a drop in the economy's borrowing limit (i.e. a credit crunch) leads to a forced deleveraging for constrained individuals, and pushes the real interest rate of the economy down, due to an increase in precautionary savings. The dissertation's main contribution in this field is to extend the aforementioned investigations to analyse the dynamics of household's portfolio, in an economy with two assets - liquid bonds and illiquid housing - and financial frictions, when access to credit is further restrained.

Finally, the dissertation is also related to that part of the macroeconomic literature that studies the role of credit and housing markets in the transmission of shocks to the aggregate economy, and to determine the effectiveness of monetary policies. Among these, Iacoviello (2005) develops and estimate a DSGE models with nominal loans, housing, and collateral constraints, to study the role of the asset market channel in the transmission of demand and supply shocks, and how the economy reacts to sudden variations in asset prices. Iacoviello (2005) shows that the presence of a credit market channel amplifies the positive effect of demand shocks on net worth and output, while it dampens the negative effects of a positive shock to inflation. Additionally, Iacoviello (2005) studies whether it would be beneficial, in terms of output and inflation stabilization, for the monetary authority to target asset prices, but this yields negligible effects. Furthermore, using microfinancial data for the US for the early 2000s, Mian and Sufi (2011) show that the sharp decrease in the nominal value of home equity, during the 2007-2008 financial crisis, was the main factor responsible for the decline in household demand for loans and for the increase in defaults rates across the US. This was principally due to the fact that individual borrowing possibilities are strictly related to the value of the collateral provided. In addition to this, Justiniano et al. (2019) construct a heterogeneous-agent macroeconomic model (although with no income risk) with a housing market and nominal loans. The authors introduce a borrowing constraint for debtors and a lending constraint for credit suppliers in the economy, and they show that a combination of looser lending and borrowing standards in the years preceding the 2008 crises lead to a housing price bubble and to the explosion of private debt in the US. Fianlly, Alpanda and Zubairy (2019) investigate the housing channel for the transmission of monetary policies to the real economy. The authors build a heterogeneous-agent model with a housing an a loan market, although with no income risk, to show that when overall indebtedness is high in the economy, credit and housing are ineffective as transmission channels for monetary policies. The main contribution of my dissertation to this area of research is that it takes into account the role of idiosyncratic income risk in a model with housing and it shows that precautionary reasons constitute a pivotal element for the household's decision process in terms of borrowing and investments. In particular it shows that a financial tightening yields a heterogeneous impact on individual agents, depending on their initial conditions in terms of income and wealth, and affects more those who are more exposed to income risk. The impact of the shock at micro level then determines the dynamics of asset markets, and the overall effect on the aggregate economy.

Dissertation Structure. The structure of the dissertation is the following. In Chapter 1, I will illustrate the baseline setup developed by Guerrieri and Lorenzoni (2017), and I will solve the model for the stationary equilibrium, using the same targets and calibration of the original paper. In Chapter 2, I will replicate the credit crunch experiment of the original paper, and illustrate how changes in the borrowing limit affect households' investment and borrowing decisions, and discuss how different shock sizes may not guarantee the existence of an equilibrium. In Chapter 3 I will explore how different assumptions made on the income process affect the results observed in Chapters 1 and 2. In Chapter 4, I will present an extension of the model in Guerrieri and Lorenzoni (2017), by introducing transaction costs. I will firstly solve the model for the stationary equilibrium, and analyse the impact of two MIT shocks, one to the credit limit and the other one to the transaction cost parameter. In Chapter 5, I will introduce the two-asset economy: I will solve it for the stationary equilibrium and reproduce the same experiments as in Chapter 4. Finally, conclusions will follow.

Chapter 1

The Baseline Model

Following Guerrieri and Lorenzoni (2017), in this chapter I consider a Bewley-type economy, populated by a continuum of infinitely lived agents, of aggregate size 1, who face uninsurable income risk. Individuals maximize an intertemporal utility function over consumption and leisure, and can save or borrow through an incomplete bond market. Each agent provides work to the production sector, which consists of infinite firms that produce aggregate consumption good Y and operate under perfect competition, so no profit is made at any time t. For each unit n_t of work provided, an agent i has individual productivity w_t^i, which is assumed to be subject to an idiosyncratic shock. Agents can borrow up to an exogenous limit ϕ. Bonds on the market are issued by the government of the economy, in fixed supply. The economy is closed.

1.1. The Economy

In this section I explain in detail the principal features of the economy under analysis.

1.1.1. Consumer's problem

Each agent i maximizes the following intertemporal utility function:

$$\mathcal{U}(\mathbf{c}, \mathbf{n}) = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta u(c_t^i, n_t^i) \right] \tag{1.1}$$

where c represents standard consumption and n is work[1]. I make the following assumption regarding u:

[1]Here it is assumed that agents have a total time endowment normalized and equal to 1. Hence work supply n is modelled so that it can take values only in an interval between 1 and 0.

Assumption 1 *The period-t utility function $u(c, n)$ is continuous, continuously differentiable and concave in both c and n. The partial derivative for c satisfies the Inada conditions:*

$$\lim_{c \to 0} \frac{\partial}{\partial c} u(c, n) = \infty$$
$$\lim_{c \to \infty} \frac{\partial}{\partial c} u(c, n) = 0$$

while the partial utility for n satisfy the following conditions:

$$\lim_{n \to 0} \frac{\partial}{\partial n} u(c, n) = -\Psi$$
$$\lim_{n \to 1} \frac{\partial}{\partial n} u(c, n) = -\infty$$

where Ψ is some positive constant value.

Moreover I assume that u is separable in consumption and leisure and it is of CRRA type:

$$u(c, n) = \frac{c^{1-\gamma}}{1-\gamma} + \Psi \frac{(1-n)^{1-\eta}}{1-\eta} \tag{1.2}$$

where γ and η are, respectively, the elasticities of consumption and leisure, and Ψ is a parameter that regulates the utility from leisure. Given that this is a closed economy, the budget constraint each agent i faces is the following:

$$p_t b^i_{t+1} + c^i_t \leq w^i_t n^i_t + b^i_t + z^i_t \tag{1.3}$$

where $p_t \equiv R_t^{-1}$ is the real discount factor of the economy, equal to the inverse of the real gross interest rate, which determines the real market value of government bonds b_{t+1} issued at time t with a payoff value next period equal to their face value in real terms. Furthermore, z^i_t is the individual's specific tax or transfer: if the agent is unemployed, they will receive a transfer (z positive), otherwise they will pay a lump-sum tax to the government (z negative). The term $w^i_t n^i_t$ represents agent i's total labour income, for a given productivity level w.

Agents receive a salary that, at equilibrium, should match individual productivity w. Productivity is assumed to be subject to an idiosyncratic shock, and the following assumption holds:

Assumption 2 *Individual productivity w evolves according to the following AR(1) stationary process:*

$$\ln(w_{t+1}) = (1-\rho)\mu + \rho \ln(w_t) + \varepsilon_{t+1} \tag{1.4}$$

where $\varepsilon_{t+1} \sim N(0, \sigma_\varepsilon^2)$ and iid. μ, ρ and σ_ε^2 are, respectively, the unconditional mean, the persistence, and the standard deviation of the process. Following Guerrieri and Lorenzoni (2017), I use Tauchen's (1986) method to discretize the process above in J states, whose transition dynamics can be described by a $J \times J$ Markov transition matrix P. In the following sections, I will discuss how using a different discretization method may potentially affect the results. Additionally, I assume that there is a further income status, characterized by zero individual productivity, i.e. $w = 0$, which I defined as unemployment[2]. The probabilities of becoming employed after being unemployed, and of losing a job are given, respectively, by $\pi_{e|u}$ and $\pi_{u|e}$.

Agents face the following borrowing constraint:

$$b_t^i \geq -\phi \tag{1.5}$$

Moreover, I assume that financial markets are frictionless, there is perfect information, and the only tradable instrument is the (safe) government bond b.

1.1.2. The Government and the Production Sector

The government provides unemployment benefits to jobless agents. To collect money, taxes are raised over employed individuals, and bonds are issued on the market. Let us define with ν and τ, respectively, the unemployment benefit and the lump-sum tax. Then, if at any time t the share of individuals that are unemployed is given by u_t, the government budget constraint is the following:

$$u_t \nu_t + B_t = p_t B_{t+1} + \tau_t \tag{1.6}$$

where B_t is the time-t supply of bonds in the economy. All agents are subject to the lump-sum tax τ_t: if the agent has positive productivity w_t^i, they will pay τ_t units to the government, otherwise they receive a net subsidy $\nu_t - \tau_t > 0$. Agents can invest in the market by buying government bonds, while individuals who want to loan can short-sell them to other agents[3].

The production sector consists of a continuum of firms of aggregate size 1, which produce a standard consumption good and operate in perfect competition and have constant returns to scale in inputs, thus firms make zero profits. The sole input in the process is labour provided by the households, whose aggregate supply at time t is denoted

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[2] Following Guerrieri and Lorenzoni (2017), an agent is defined unemployed if their individual productivity is 0, an agent with positive productivity, but zero labour supply, i.e. $n_i^t = 0$, is still defined as employed. The rationale here is that an agent with positive productivity is able to receive a labour wage for any unit of labour supplied to the economy, but willingly decides not to do so.

[3] In short: an agent at time t can sell a unit of government bond for $p_t < 1$ to another agent, under the promise of repurchasing it at $t + 1$ for 1. The net profit of the lender would be $(1 - p_t)/p_t = r_t$, which is the implicit real net interest rate of this economy.

by N_t. Moreover, the production function is assumed to be linear in the efficiency unit of labour $w_t^i n_t^i$: as aforementioned, each agent i produces $y_t^i = w_t^i n_t^i$ units of consumption goods, depending on their individual productivity w_t^i.

1.2. Stationary Equilibrium

In this section I am going to solve for the recursive equilibrium. Each agent i faces the following optimization problem (written in recursive form):

$$V(w, b) = \max_{c,n,b'} u(c, n) + \beta \mathbb{E}[V(w', b')|w]$$

subject to:

$$\begin{aligned}
&c \leq wn - z + b - pb' \\
&b \geq -\phi \\
&c \geq 0 \\
&n \in [0, 1]
\end{aligned}$$

which together describe a compact set.

1.2.1. Definition of Recursive Equilibrium

Let us denote with $\mathbf{B}$ and $\mathbf{W}$, respectively, the sets of all the possible values for b and w. Define $\mathbf{S}$ as the set of all the possible states of the world in this economy, i.e. $\mathbf{S} = \mathbf{B} \times \mathbf{W}$, and with $\mathcal{B}$ its associated Borel σ-algebra. Let us denote with Λ the set of all the possible probability measures over the measurable space $(\mathbf{S}, \mathcal{B})$ and, for any subset $\mathcal{S} \in \mathcal{B}$, let us denote with $\lambda(\mathcal{S})$ the stationary measure of agents in $\mathcal{S}$. Therefore:

Definition 1 *A **stationary recursive competitive equilibrium** in this economy is a value function V, an interest rate r, a set of policy functions $\{c, n, b'\}$, a set of government policies and external bond supply $\{\tau, \nu, B\}$, and a stationary probability measure $\lambda \in \Lambda$, such that:*

I. *Given r and $\{\tau, \nu, B\}$, and for any $(w, b) \in \mathbf{S}$, policy functions $c : \mathbf{S} \to \mathbb{R}_+$, $n : \mathbf{S} \to [0, 1]$, and $b' : \mathbf{S} \to \mathbb{R}$ solve household's problem, and $V : \mathbf{S} \to \mathbb{R}$ is the associated value function;*

II. *The financial market is in equilibrium, i.e. the aggregate bond demand is equal to the exogenous supply B*

$$\int_{\mathbf{S}} b'(w, b) d\lambda(w, b) = B$$

III. *Government policies* $\{\tau, v, B\}$ *satisfy the government's budget constraint:*

$$pB + uv = \tau + B$$

where $p = R^{-1} = (1+r)^{-1}$.

IV. *The market for goods is in equilibrium. Aggregate labour supply at equilibrium is:*

$$N = \int_{\mathbf{S}} n(w, b) d\lambda(w, b)$$

while aggregate output is given by:

$$Y = \int_{\mathbf{S}} y(w, b) d\lambda(w, b)$$

with $y(w, b) = w \times n(w, b)$ *being the optimal efficiency unit of labour at equilibrium, for a given initial* w *and* b*. Consumption then must be:*

$$C = \int_{\mathbf{S}} c(w, b) d\lambda(w, b) = Y$$

V. *For all subsets* $\mathcal{S} \in \mathcal{B}$*, the probability measure* λ *is such that:*

$$\lambda(\mathcal{S}) = \int_{\mathbf{S}} P((w, b), \mathcal{S}) d\lambda(w, b)$$

where $P((w, b), \mathcal{S})$ *is the Markov transition function associated to the household's problem, which maps the transition from the current state* (w, b) *to the set* $\mathcal{S}$ *in the next period.*

The consumer's problem, at the stationary recursive equilibrium, can be characterized by the following system of equations:

$$u_c(c, n) \geq \beta R \mathbb{E}[u_c(c', n')] \tag{1.7}$$

$$w u_c(c, n) \leq -u_n(c, n) \tag{1.8}$$

$$c = wn - z + b - pb' \tag{1.9}$$

$$c > 0 \tag{1.10}$$

The Euler equation (1.7) holds with equality if the constraint in (1.5) is slack, while intratemporal optimality condition (1.8) holds with equality if $n > 0$.

Variable	Value	Target/Source
Average employment	40% of total time endowment	Nekarda and Ramey (2020)
Unemployment benefit	40% of GDP (quarterly)	Nekarda and Ramey (2020)
Total Liquid Asset	178% of GDP (annualized)	US economy in 2006
Household Debt	18% of GDP (annualized)	US economy in 2006
Real Interest Rate	2.5% (annualized)	

Table 1.1. *Targets for the Calibration of the Stationary Equilibrium*

1.2.2. Calibration

I solve for the economy's equilibrium using numerical simulations. To do so I use the parameter calibration laid out by Guerrieri and Lorenzoni (2017) for their baseline model. The calibration targets are summarized in Table (1.1). The time period in the model is equal to 1 quarter.

For what concerns employment, they follow the empirical results in Nekarda and Ramey (2020), who show that, on average, household work accounts for 40% of their total time endowment (which, in this economy, is normalized to 1). Additionally, they set unemployment benefits to be equal to 40% of the total quarterly labour income, on an annual basis. As regards the financial markets, the authors set 178% and 18% as, respectively, the long-run targets for the liquid-assets-to-income ratio and for the private-debt-to-income ratio (annualized). Such targets are chosen to replicate the US economy in 2006: the aggregate amount of liquid assets in 2006, defined as the sum of external bond supply and private household debt, was circa 178% of GDP, while consumer credit was circa 18% of US GDP. Furthermore, the real interest rate is set to be equal to 2.5% *per annum*. To attain these targets, Guerrieri and Lorenzoni (2017) implement the quarterly parameter calibration summarized in Table (1.2). In regard to the wage process, it is calibrated to match the empirical findings in Flodén and Lindé (2001), who estimate that (annually) wages have persistence and variance equal to, respectively, 0.913 and 0.0426, which are approximately equivalent to the quarterly calibration in Table (1.2).

1.2.3. Results

Figure (1.1) displays the policy functions at the stationary equilibrium, as function of the initial liquid wealth b, and for different levels of wage[4]. Policy functions for consumption are increasing in both b and w, and strictly concave for low levels of wealth, whereas they become more linearized as b grows. As in Guerrieri and Lorenzoni (2017), when b is low, agents are more cautious: high risk aversion γ and income uncertainty push agents to

[4]For simplicity, the different wage levels are labelled as w^0, w^1, ..., w^{12} in growing order, with w^0 being the unemployment case and w^{12} being the highest possible wage level.

Parameter	Description	Value	Target or Source
β	Rate of time preference	0.9775	Int. rate = 2.5% annualized
γ	Constant Risk Aversion	4	
η	Concavity disutility from labour	1.5	Frisch elasticity = 1
Ψ	Coefficient disutility from labour	15.82	Avg. employment 40% of annual GDP
ϕ	Borrowing limit	1.65	Equal to 99% of annual GDP
ν	Unemp. benefits	0.167	Target = 40% of quarterly GDP
B	Bond supply	2.67	Target = 160% of annual GDP
μ	Uncond. mean wage process	0	Set mean wage $w = 1$
ρ	Persistence wage process	0.967	Flodén and Lindé (2001)
σ^2	Variance wage process	0.017	Flodén and Lindé (2001)
J	Number of wage states (unemployment excluded)	12	
π_{ue}	Prob. of finding a job	0.882	Shimer (2005)
π_{eu}	Prob. of losing a job	0.057	Shimer (2005)

Table 1.2. *Calibration of the Stationary Equilibrium*

consume more, rather than save, when their wealth increases marginally[5]. On the other hand, as b grows, the marginal propensity of households to consume, with respect to changes in wealth, becomes smaller, and differences in consumption among wage levels tend to disappear. According to Guerrieri and Lorenzoni (2017), this is coherent with the permanent income hypothesis: increases in wealth do not automatically translate into higher consumption expenditure, which becomes proportional to the agent's expected value of future wealth. Thus any precautionary behaviour, and hence concavity of the policy function tend to fade away as wealth grows (Carroll, 1992).

For a given w, Figure (1.1) shows that agent's labour supply (as well as total income) is decreasing in b. The higher is the level of agents' initial resources, the less they will need to work to maximize their utility from consumption. Individuals with sufficiently high wealth (in the simulation for $b > 22$ circa) will not need to supply labour at all, and hence their labour income would drop to zero[6]. Guerrieri and Lorenzoni (2017) argue that agents in this economy are subject to both a substitution and income effect. For very low values of b the income effect is stronger: low-wage individuals supply more labour than their high-wage counterparts - showing that agents prefer utility from consumption rather than from leisure. As b increases, substitution effect dominates and supply n grows with wage w. Figure (1.2) displays the asset distribution at the stationary equilibrium. Coherently with the relevant macroeconomic literature, wealth distribution is skewed towards the left: the majority of wealth is distributed across a multitude of individuals,

[5] See Carroll and Kimball (1996) and Berger et al. (2017)

[6] The inverse relationship between work income and wealth is well documented in the economic literature, both at the theoretical and empirical level. See for example Cesarini et al. (2015).

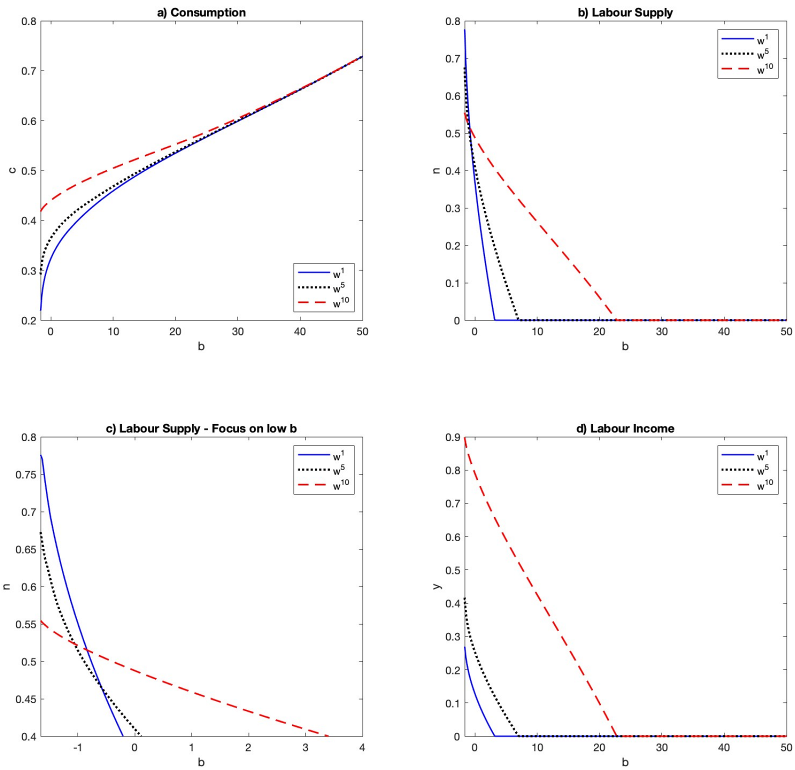

Figure 1.1. *Policy functions at the stationary equilibrium, for different productivity levels*

each one of them holding a small fraction of total assets in the economy. Constrained individuals (i.e. those subject to the binding version of equation (1.5), are located at the very left of the distribution.

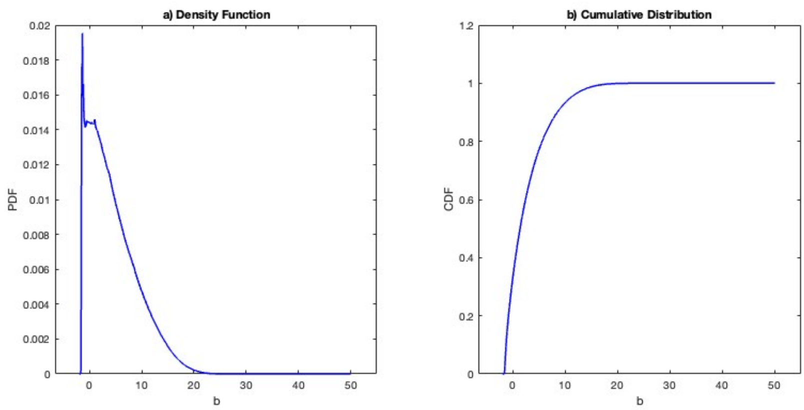

Figure 1.2. *Density and cumulative distributions of asset holdings in the economy at the stationary equilibrium*

Chapter 2

A Credit Crunch Shock

In this chapter I am going to replicate the experiment run by Guerrieri and Lorenzoni (2017): I assume that the economy at the steady state is hit by a credit crunch, i.e. an MIT shock that reduces the (absolute value of) the borrowing limit ϕ. As in the original paper, I set that ϕ drops to 0.876, a level that is consistent with a 10% drop in the (annual) private debt-to-income ratio, once the economy has converged to a new steady state. As in Guerrieri and Lorenzoni (2017), I assume that the decrease in ϕ is not immediate, but takes place over a set number of h quarters, along a linear law of decay. More specifically, let us denote with ϕ' the initial value of the borrowing limit, and with ϕ'' its terminal value after h quarters. Let us assume that the shock first hits the economy at time $t = 0$, then:

$$\phi_t = \begin{cases} \phi' & \text{if } t = 0 \\ \phi'' & \text{if } t = h \\ \phi' - \Delta\phi \times t > \phi'' & \text{if } t = 1, ..., h-1 \end{cases}$$

where $\Delta\phi$ is the chosen rate of decay of the borrowing limit. Such assumption is necessary given how the model is constructed. When the crunch hits, constrained individuals must repay part of their debt, which is now exceeding the new borrowing limit. If at any time t the difference between old and new borrowing limit is too big, constrained agents would not be able to cover the repayment with their available resources, and they will be forced to default. As a matter of fact, if agent i is constrained, then, according to the budget constraint we have:

$$-p_t\phi_{t+1} + c_t^i = w_t^i n_t^i - z_t^i - \phi_t \tag{2.1}$$

Then, the repayment Φ_t that the agent is forced to make, because of the drop in ϕ, is:

$$\Phi_t \equiv \phi_t - p_t\phi_{t+1} = w_t^i n_t^i - z_t^i - c_t^i \tag{2.2}$$

If $\Phi_t \geq w_t^i n_t^i - z_t^i$, then there would be zero or negative consumption, and this would break the model. Allowing a gradual impact of the shock, which is protracted for h

quarters, ensures that agents would always be able to cover the repayment Δ_t for any t. Following Guerrieri and Lorenzoni (2017), I initially set $h = 6$. I will discuss the possibility of reducing this time window in the next sections.

2.1. Transitional Dynamics

Figure (2.1) displays the transitional dynamics to the aforementioned MIT shock. The first noticeable consequence of the shock is the sharp drop of the interest rate in the short run: in just 5 quarters it falls to -2%. Guerrieri and Lorenzoni (2017) argue that such decrease is due to the heterogeneous effect that the credit shock yields to different agents in the economy. With the drop in ϕ, individuals closer to the original borrowing limit (on the left-hand side of Figure 1.2) need to repay parts of their stock of debt, as ϕ converges to its new equilibrium value, or, if the initial b is positive, to increase their asset holding. Such tendency of accumulating wealth, also motivated by the precautionary rationale of not facing a binding borrowing constraint in the next period, leads to an increased demand for assets from the lower part of the distribution. At the same time, high-wealth agents are not directly affected by the drop in ϕ; however, they face a reduced demand for 'loans' from low-wealth individuals. Since the external supply of bonds to the economy is fixed, wealthy agents are pushed by such market forces to decrease their asset exposure. Since Figure (1.2) shows that the proportion of low-wealth agents is greater than the proportion of high-wealth ones, then there is an excess demand for bonds on financial markets. Therefore, to curb such excess, the interest rate on bonds must drop. In the long-run, interest rate adjusts to its new (annualized) equilibrium value, which is approximately 1.5%.

The transition path of output is similar to the one of the interest rate. In the short run, aggregate income suddenly drops by more than 1%, compared to its original equilibrium level. Afterwards, aggregate income grows until convergence to its new equilibrium value, which is 0.2% lower than the pre-shock one. To explain the evolution in output, Guerrieri and Lorenzoni (2017) argue that the final impact on aggregate income depends on the combination of two separate and opposite effects on consumption and labour supply. On the one hand, Guerrieri and Lorenzoni (2017) point out that the overall impact of the crunch on consumption is negative, mainly due to the reduced available resources for consumers. On the other hand, the effect on labour supply is positive: Figure (2.1) shows that labour supply grows in the short run, following the shock, and it converges to a new equilibrium value, which is circa 0.75% higher than in the original equilibrium.

As regards the consumption, there are two forces at play: Figure (2.2a) splits the dynamics of consumption into its two main components. On the one hand, we have a partial equilibrium channel (dashed line), which represents the mere effect of a change in ϕ, epurated from the impact of a change in interest rate (which is kept constant at the original level). On the other hand (dotted line), we have the change in consumption

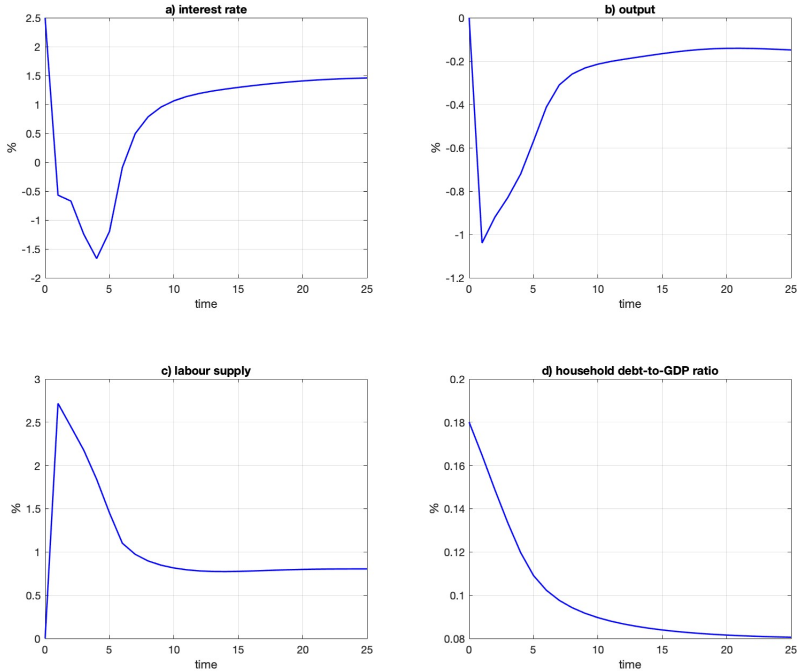

Figure 2.1. *Transitional dynamics to an MIT shock to the borrowing limit ϕ ('credit crunch'). Top left - borrowing limit to income. Top right - debt to income ratio. Bottom left - Annualized interest rate. Bottom right - Output, % deviation from the initial steady state.*

due to the adjustment of the interest rate, keeping ϕ constant at its original level (i.e. the interest rate channel). The partial equilibrium effect is negative: the crunch depletes individuals' savings, especially for those closer to the borrowing limit, and reduces available resources - therefore, consumption drops. Interest rate dynamics have the opposite effect: following equation(1.7), a drop in interest rate is, ceteris paribus, matched by a decrease in marginal utility of consumption, because of Inada conditions, this prompts an increase in individual consumption, for all i, and, hence, in aggregate consumption. Given that the overall effect of the crunch on consumption is negative (solid line), the partial equilibrium channel dominates the interest rate one. However, the wide short-term difference between the general and the partial equilibrium result shows that the adjustment in interest rates is strong enough to mitigate the impact of the credit crunch on the economy.

Figure (2.2b) decomposes the dynamics for labour supply N into the partial equilibrium channel and the interest rate channel. The picture for labour supply mirrors

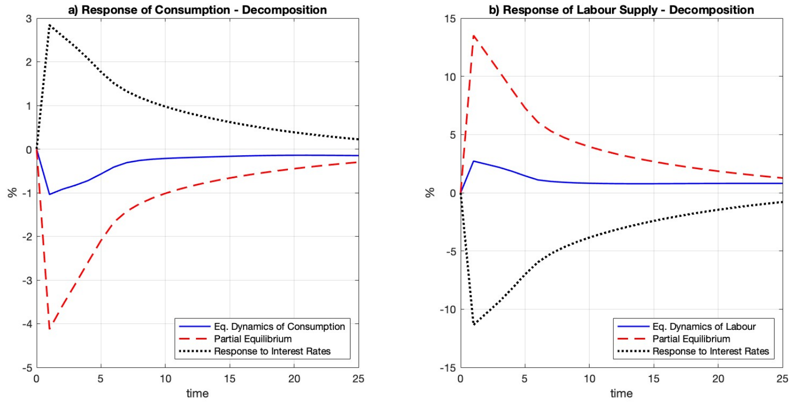

Figure 2.2. *Decomposition of the transitional dynamics of aggregate consumption (left) and labour supply (right) into the partial equilibrium channel (dashed line) and the interest rate channel (dotted line). The equilibrium transition path is represented by the solid line.*

the one for consumption. The partial equilibrium effect here is positive: as resources from financial investments are scarcer, individuals are pushed to work more in order to sustain their consumption, therefore the mere change in ϕ prompts aggregate labour supply to increase. On the other hand, if the marginal utility of consumption decreases as r diminishes, assuming ϕ constant, the right-hand side of the intratemporal equilibrium condition (1.8) decreases too. As a result, individual labour supply n increases for any agent i and aggregate labour raises too. Even in this case, the partial equilibrium effect is stronger, as the cumulative effect of the two channels on aggregate labour supply is positive, both in the short and long run. Taking all this into account, the final impact of the credit crunch on consumption is stronger than the one on labour, thus leading to the drop in output that is observable in Figure (2.1).

2.2. New Stationary Equilibrium

Figure (2.3) compares the agents' policy functions, for a given income level, at the original and at the final stationary equilibrium. The macro effects of the shock are also reflected at the micro level. As regards the consumption, the difference between the policy functions is very small, however it can be noticed that individuals who start with lower b slightly decrease their consumption, while their more wealthy counterparts consume more. At the same time, low-wealth individuals increase their labour supply, for higher b, the agents' labour supply at the final steady state is lower than before the shock (Figure 2.3b). Such difference in results depends on whether the agents are more sensitive to the drop in ϕ

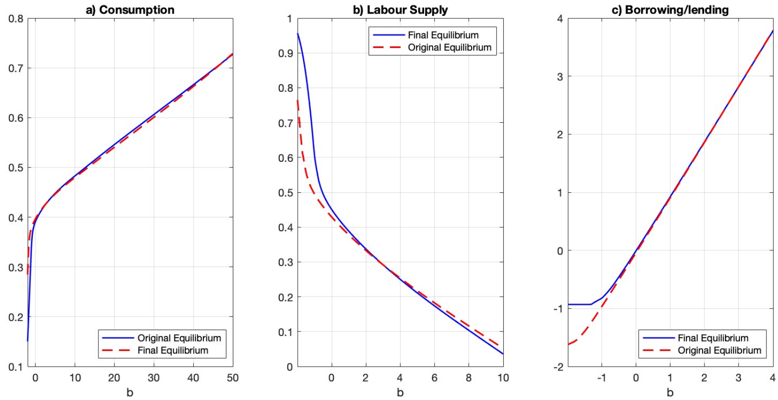

Figure 2.3. *Comparison between the policy functions at the original stationary equilibrium (dashed lines) and at the new stationary equilibrium (solid lines), for different levels of initial wealth b and a given level of income $w = w^6$.*

(partial equilibrium effect) or to the interest rate adjustment. Not surprisingly, agents on the left-hand side of the graph are closer to the borrowing limit, hence, as the credit crunch hit, they modify their consumption and labour decisions based principally on their reduced ability to borrow. On the other hand, high-wealth agents are more sensitive to the drop in the interest rate.

For what concerns the asset distribution, Figure (2.4) shows the difference between the original (solid line) and the final (dashed line) stationary equilibrium. Given that the bond supply remains unchanged, Guerrieri and Lorenzoni (2017) point out that the new distribution has the same mean as the initial one, but the spread of assets across the grid is more concentrated at the new equilibrium. Agents' response to the credit crunch depends on their initial conditions. Low-wealth agents, especially those closer to the borrowing limit, increase their asset holding, thus leading to the spike on the left-hand side, while the converse holds for high-wage individuals, who reduce their asset stock. Such behaviour is consistent with the dynamics of interest rate seen in Section 2.1: since the equilibrium rate is lower in the new steady state, individuals on the very left of the distribution either borrow more (within the new limit), if b is negative, or accumulate bonds faster, if b is negative; high-wealth individuals, on the other hand, are prompted to reduce their asset holdings, given the new low rates.

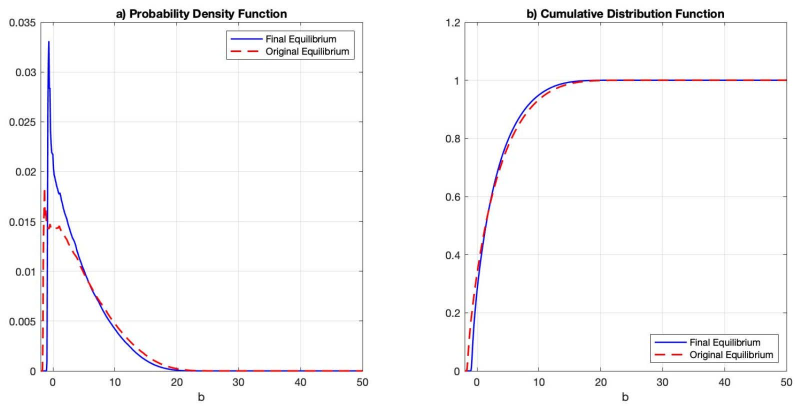

Figure 2.4. *Comparison between the cross-sectional asset distributions at the original equilibrium (dashed lines) and at the final equilibrium (solid lines).*

2.3. The Adjustment of Borrowing Limit

Until now we have assumed that ϕ adjusts gradually to its new equilibrium value, following a linear decay. More specifically, with ϕ' and ϕ'' being, respectively, the initial and the terminal values of the borrowing limit, its decay is given by:

$$\phi_t = \phi' - \Delta\phi \times t$$

for $t = 1, ..., h$, while the constant rate of decay is given by:

$$\Delta\phi = \frac{\phi' - \phi''}{h} \tag{2.3}$$

where h is the number of quarters in which the adjustment takes place. While in the previous sections I set $h = 6$, now I am going to study what happens to the equilibrium dynamics seen so far, when the horizon h is shortened. Let us assume that the economy is at equilibrium at $t = 0$ and that is hit by the shock at time $t = 1$. Under perfect foresight, the $t = 0$ budget constraint of an agent that was constrained at equilibrium is given by equation (2.2):

$$-p_0\phi_1 + c_0^i = w_0^i n_0^i - z_0^i - phi' \tag{2.4}$$

The size of the repayment at $t = 1$ is:

$$\Phi_1 = \phi' - p_0\phi_1 = \phi'(1 - p_0) + p_0\Delta\phi \tag{2.5}$$

i.e. it is proportional to the quarterly rate of change $\Delta\phi$. Hence, the larger is the decay of the borrowing limit, the more burdensome it is for constrained agents to repay the excessive

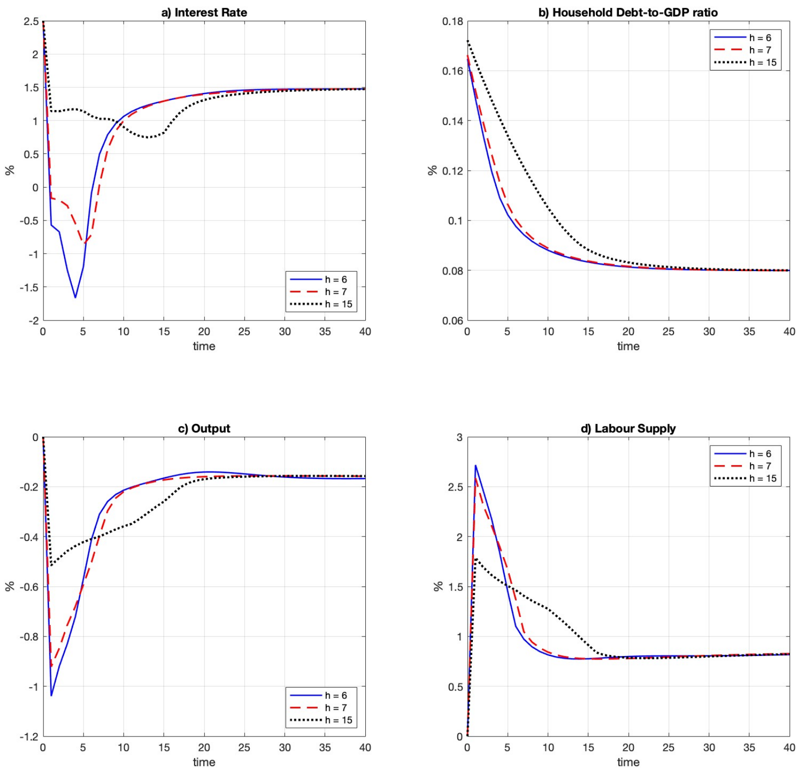

Figure 2.5. *Equilibrium transitional dynamics with* $h = 6$ *(solid lines),* $h = 7$ *(dashed lines),* $h = 15$ *(dotted lines).*

part of their debt after the shock. One can see from the equation (2.3), that $\Delta\phi$ is inversely related to h: thus, if the adjustment horizon reduces, the rate of decay $\Delta\phi$ increases, and so does the repayment Φ_1, which constrained agents must make at time 1. Therefore, for given ϕ', ϕ'', if h is too small, the repayment Φ_1 may become too big and exceed the resource available to the agents. In the limit case in which $\Phi_1 \geq w_0^i n_0^i - z_0^i$, consumption would need to be zero or negative for the budget constraint to be in equilibrium. Therefore, in such situation, agents would need to default on (part of) their debt in order to fulfill the equilibrium condition of having positive consumption at any time.

To illustrate the effect of a change of h on the equilibrium dynamics of this economy, I repeat the credit crunch experiment, for different values of the horizon h, while keeping the remaining parametrization as above. If I shorten h, i.e. for $h \leq 5$, the economy no longer converges to a new stationary equilibrium as before, and no transitional path can be computed using the same simulation method as above. When $h = 5$, for instance, the decay term $\Delta\phi$ becomes too big and the repayment Φ_1 that is due right after the shock hits the economy exceeds the available resources of the constrained agents, i.e. $\Phi_1 \geq w_0^i n_0^i - z_0^i$. Therefore, in order to solve for the transitional equilibrium path, the model should allow for the possibility of defaulting, at least partially, on the debt. If we consider longer horizons h the short-term response of the economy to the shock changes, while convergence to the new equilibrium remains unaffected. Figure (2.5) plots the transitional dynamics for $h = 7$ (dashed lines) and $h = 15$ (dotted lines), other then the standard case (solid lines). Overall, the transition dynamics, for larger h, follow a pattern that is similar to the benchmark 6-quarter case. However the short term fluctuations are less acute, due to the fact that the per-quarter drop in ϕ is smaller as h increases. The top-left panel shows that increasing h by just 1 quarter greatly reduces the drop in r_t.Since constrained agents need to repay a smaller quota of their existing debt, compared to the benchmark case, the impact observed on the interest rate in Section 2.1 is now reduced. The demand for 'loans' (i.e. negative asset holding positions) decreases slower, and the bond accumulation of low-wealth individual is now lower than in the benchmark case. Therefore, the adjustment in r_t is not as dramatic as for the benchmark case. When $h = 15$, the real interest rate does not even enter the negative region after the shock. When h is larger, and repayments are reduced, there is an increased availability of resources for agents, compared to the benchmark case. Consumption (and hence output) is less affected, and individuals do not need to supply as much labour as in the benchmark case with $h = 6$. Therefore, as h increases, the drop in Y_t and the rise in N_t are less dramatic. However, when $h = 7$ or 15, the number of repayments that needs to be done increases. Therefore, in the long run, for lower values of h, the economy converges to the equilibrium faster.

Chapter 3

Sensitivity Analysis for the Income Process

In this Chapter, I study how the results seen in Chapters 1 and 2 change with the calibration of the income process. In the previous sections the productivity w followed a Gaussian AR(1) process with zero unconditional mean, persistence $\rho = 0.967$ and unconditional error variance $\sigma^2 = 0.017$. In the following paragraphs I will explore how equilibrium results and transition dynamics change with different values for ρ and σ^2.

3.1. Sensitivity Analysis for Persistence

Let us consider the persistence ρ first. While in the original model $\rho = 0.967$, I now explore two alternative settings in which, respectively, $\rho = 0.98$ (high-persistence calibration) and $\rho = 0.936$ (low-persistence calibration). Figure (3.1) compares the equilibrium policy functions and stationary distributions across the three cases. Persistence affects the slope of policy functions, and its effect depends on agents' initial wealth. As ρ rises, borrowers consume less than in the standard economy, while they supply additional labour. At the same time, individuals borrow less and accumulate more bonds. For high values of b the converse holds true. The variance of AR(1) process rises with persistence ρ. Therefore wage states, once the process has been discretized, are increasingly dispersed around the unconditional mean. Additionally, variations of ρ also affect the transition probabilities in the Markov matrix P. When persistence is higher, the value of diagonal elements of P increases, while the value of off-diagonal entries decreases. This implies that the probability of remaining in the current state rises with ρ. Because of this, agents that rely more on labour income (those with low wealth b) are increasingly cautious when ρ is higher, while wealthier individuals, who provide little or zero labour to the economy, are not. For lower values of ρ, the opposite effect occurs: since the variance of the process and the probabilities of remaining in the same states are lower, wealthier individuals are more cautious, while individuals with lower wealth b consume more and work less.

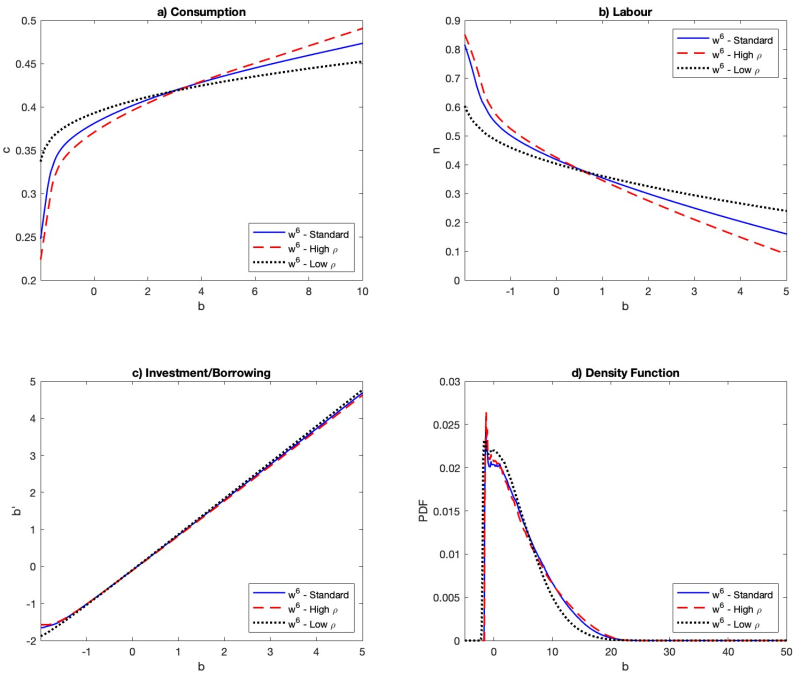

Figure 3.1. *Policy functions and stationary distributions at the stationary equilibrium (for a given level of wages $w = w^6$) for the three cases in which the economy follows the standard calibration (solid line) of the wage process, the calibration with high $\rho = .98$ (dashed line), and the the calibration with low $\rho = .936$ (dotted line).*

Figure (3.2) displays the transition dynamics, following a credit crunch shock of the same magnitude as in Chapter 2, for the three different calibrated specifications under analysis. Results show that, as ρ increases, the economy becomes more sensitive to the shock[1]. In the short-run, the drop in interest rates is similar across the three cases, since individuals deleverage and increase their bond holdings in all the three specifications, although for high values of ρ the decline is more evident. For output and labour supply the difference is more striking: when $\rho = 0.98$, N_t rises by 3.5% (1% higher than in the standard case) and Y_t drops by 1.2%. This is a consequence of the increased precautionary behaviour of agents described above: agents are more willing to increase their resources and cut their spending. In the long run, differences in terms of N_t and Y_t are very small

[1] In this exercise I set that ϕ converges to its new value after $h = 7$ periods, for all the three cases under analysis, to prevent agents from defaulting on their debt and ensure comparability across the three cases. As a matter of fact, when $\phi = 0.98$, and if $h = 6$, constrained agents default.

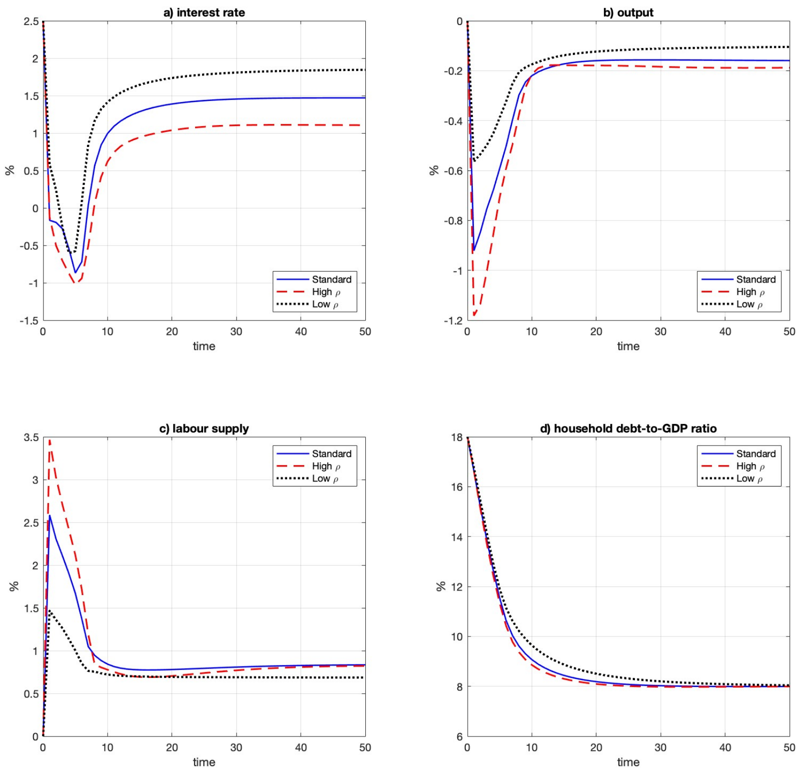

Figure 3.2. *Transitional dynamics after a credit crunch shock for the three cases in which the economy follows the standard calibration (solid line) of the wage process, the calibration with high $\rho = 0.98$ (dashed line), and the the calibration with low $\rho = 0.936$ (dotted line).*

across the three cases, but interest rates converge to three different values. As ρ increases, the terminal value for r_t is lower, since the demand for bonds at the new equilibrium is higher in the set up with higher wage persistence.

3.2. Sensitivity Analysis for Innovation Variance

In the baseline setup, innovation variance $\sigma^2 = 0.017$. Now I explore two alternative calibrations in which, respectively, $\sigma^2 = 0.024$ (high-variance calibration) and $\sigma^2 = 0.01$ (low-variance calibration), by keeping the persistence at the standard level $\rho = 0.967$. Therefore, the sensitivity analysis on σ^2 produces similar results to the one on ρ. However,

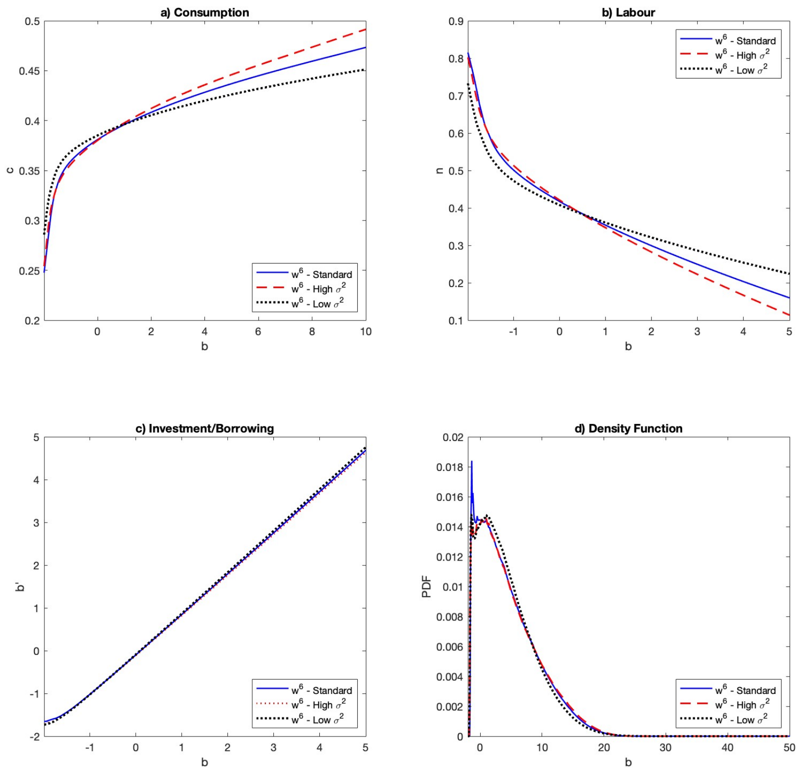

Figure 3.3. *Policy functions and stationary distributions at the stationary equilibrium (for a given level of wages $w = w^6$) for the three cases in which the economy follows the standard calibration (solid line) of the wage process, the calibration with high $\sigma^2 = 0.024$ (dashed line), and the the calibration with low $\sigma^2 = 0.01$ (dotted line).*

the differences across the three calibrations are now less evident. As σ^2 rises, the variance of the AR(1) process increases as well, therefore, as above, wage states are more dispersed around the mean. However, movements in σ^2 have a negligible effect on the transition probabilities in the matrix P. Therefore, as σ^2 increases, low-wealth agents are slightly more cautious, while agents with higher initial levels for b consume more and work less (Figure 3.3). As above, the effect of the credit crunch shock is more severe when the variance is higher (Figure 3.4).

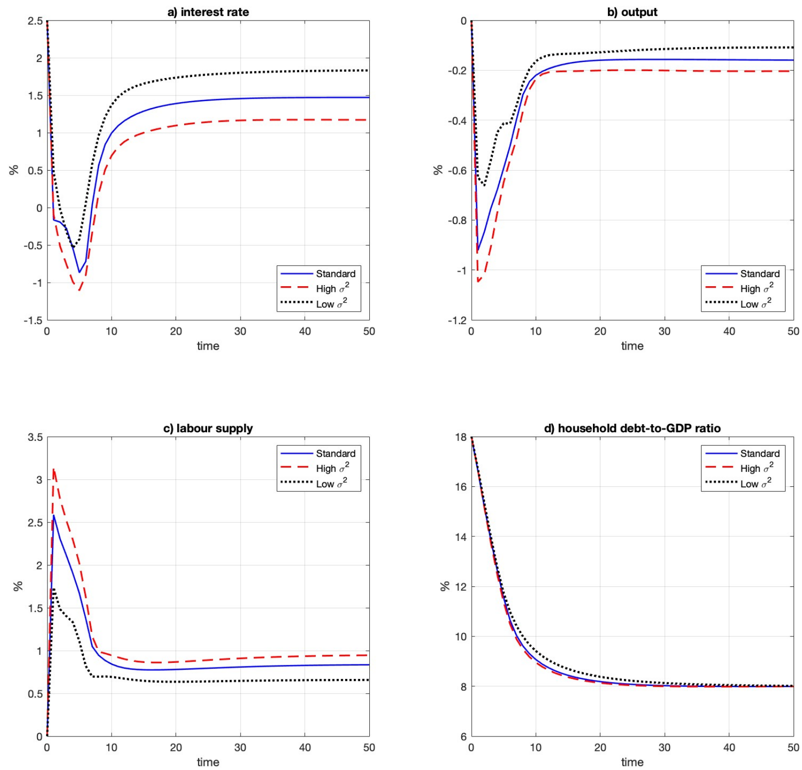

Figure 3.4. *Transitional dynamics after a credit crunch shock for the three cases in which the economy follows the standard calibration (solid line) of the wage process, the calibration with high* $\sigma^2 = 0.024$ *(dashed line), and the the calibration with low* $\sigma^2 = 0.01$ *(dotted line).*

Chapter 4

A Model with Financial Frictions

In this chapter, I consider a modified version of the framework in Chapters 1 and 2. I assume now that lending and borrowing are no longer frictionless, but take place through intermediation, and transaction costs arise.

4.1. The Economy

Let us assume that the economy is populated by a continuum of intermediaries of aggregate size 1, which operate under perfect competition. Their role is to transfer funds from the hands of the lenders to the ones of the borrowers, but by doing so, intermediaries incur costs that need to be covered. Such costs are proportional to the amount borrowed and passed down onto the borrower. Let us denote with ε the transaction cost per unit of funds borrowed[1]. Suppose that agent i at time t lends b to agent j for a price p_t. However, agent j only receives $p_t b(1-\varepsilon)$, once the transaction costs are deducted. However, at time $t+1$, agent j must repay the full value of the loan b. Hence, the effective gross interest rate paid by j for the loan is:

$$R_t^b \equiv \frac{1}{p_t(1-\varepsilon)} = \frac{R_t}{(1-\varepsilon)} \tag{4.1}$$

where $R_t = p_t^{-1}$ is the real interest rate of the economy at time t. Therefore, the presence of such intermediation costs drives a spread between the interest rate of the economy, and the effective interest rate at which individuals can borrow, and such spread is equal to the inverse of $1-\varepsilon$. For now, I assume that the transaction costs ε are constant.

[1]Such modelling of transaction costs is borrowed from the international economics literature, from the notion of 'iceberg costs'. Samuelson (1954) defines iceberg costs as that part of the value of a good that is depleted when the good is transported from a place to another. According to such metaphor, the good acts like an iceberg, whose value partially 'melts' and gets wasted during the journey.

In this economy, agents still maximise the utility function given in equation (1.1), this time under the new constraint:

$$pb' - \mathbb{I}^{(-)} p\varepsilon b' + c \leq wn + z + b \tag{4.2}$$

where $\mathbb{I}^{(-)}$ is an indicator variable that takes value 1 when b is negative, and zero otherwise. Agents' borrowing constraint still retains its form in (1.5). Taking all this into account, the household's Euler equation keeps its form in (1.7) if the household is a net investor (i.e.if $b \geq 0$ at equilibrium). Furthermore, the Euler equation will always be satisfied with equality, since agents with positive or null bond holding are never constrained. However, the Euler equation of agents who borrow becomes:

$$u_c(c, n) \geq \beta \frac{R_t}{1 - \varepsilon} \mathbb{E}[u_c(c', n')] \tag{4.3}$$

Notice that, if transaction costs become extremely high ($\varepsilon \to 1$) the right-hand side of equation (4.3) goes to infinite and, from the Inada conditions, present consumption goes to zero. In the following sections I am going to solve for the stationary equilibrium, and perform two numerical experiments: first, I am going to perform the same credit crunch exercise as above, secondly, I am going to analyse how the economy reacts when the transaction costs suddenly increase.

4.2. Stationary Equilibrium

The definition of stationary equilibrium in this economy is not dissimilar to Definition (1). In this economy, the equilibrium on the market for consumption goods requires that:

$$C + \varepsilon D = Y \tag{4.4}$$

where D is the aggregate debt outstanding at equilibrium, i.e.

$$D \equiv \int_{\mathbf{S}} \min\{0, b'(x, h, y)\} d\lambda(w, b) \tag{4.5}$$

for a stationary probability measure λ that satisfies point V of Definition (1). Additionally, note that there are two equilibrium gross interest rates R and $R^b \equiv R(1 - \varepsilon)^{-1}$: this second one is the rate that the borrowers face.

To solve for the equilibrium using numerical methods, I set the same economic targets used for the baseline model, summarized in Table (1.1). To meet such targets, I use the calibration laid out in Table (4.1). More specifically, transaction costs ε are set to be 2% on annual basis. Some parameters in Table(4.1) differ considerably from their counterparts in Table (1.2). The presence of intermediation costs depletes the available

Parameter	Description	Value	Target or Source
β	Rate of time preference	0.9773	Int. rate = 2.5% annualized
γ	Constant Risk Aversion	4	
η	Concavity disutility from labour	1.5	Frisch elasticity = 1
Ψ	Coefficient disutility from labour	16.15	Avg. employment 40% of annual GDP
ϕ	Borrowing limit	1.81	Equal to 105% of annual GDP
ν	Unemp. benefits	0.167	Target = 40% of quarterly GDP
B	Bond supply	2.67	Target = 160% of annual GDP
μ	Uncond. mean wage process	0	Set mean wage $w = 1$
ρ	Persistence wage process	0.967	Flodén and Lindé (2001)
σ^2	Variance wage process	0.017	Flodén and Lindé (2001)
J	Number of wage states (unemployment excluded)	12	
π_{ue}	Prob. of finding a job	0.882	From Shimer (2005)
π_{eu}	Prob. of losing a job	0.057	From Shimer (2005)
ε	Transaction Costs	2%	Annualized Value

Table 4.1. *Calibration of the Stationary Equilibrium with Transaction Costs.*

resources of agents who borrow on the market. Such individuals are, therefore, prompted to work more than in the baseline case, to make up for the resources wasted. Hence, in order to match the same labour target as in Table (1.1), the calibrated Ψ, which represents the disutility from labour, needs to be higher, to deter agents from supplying excessive work at equilibrium. At the same time, the (annualized) debt-to-GDP ratio is still targeted to be 18% at equilibrium. Given the presence of intermediation costs, which reduce the borrowing possibilities compared to the baseline setup, the borrowing limit must be looser in this case in order to allow for the same equilibrium debt-to-GDP ratio: therefore, the new ϕ is set to be 1.76, which is equal to circa 105% of annual GDP at equilibrium. The remaining parameters are equal, or very close, to those calibrated in the baseline model.

The policy functions for c, n and b' at the stationary equilibrium are plotted in Figure(4.1), for three different levels of individual wage w. The shape of the policy functions, *per se*, is not dissimilar to the ones in the baseline framework. Consumption function is strictly concave for low b, due to the precautionary rationale of low-wealth agents, while they are linearized for high b, at which point the differences among wage levels disappear. By looking at the labour supply policies, one can notice that the income supply is still stronger than the substitution effect for low-wage individuals, when b is low, as they supply more labour than their high-wage counterparts. As b increases the converse holds true and substitution effect dominates.

Figure (4.2) compares the policy functions of this latter model with transaction costs to those of the baseline model, for a given wage level $w = w^8$. The first row displays policy functions for consumption, the second one policy functions for n, and the third

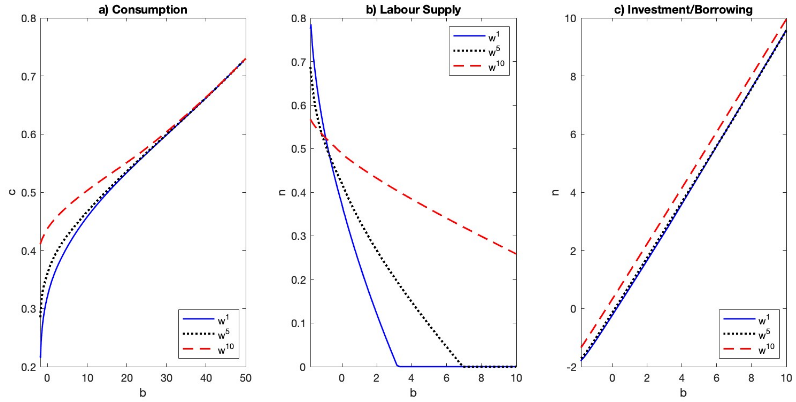

Figure 4.1. *Policy functions for consumption, labour supply and investments/borrowing in the model with transaction costs, for different levels of w.*

one policy functions for b'. The presence of transaction costs modifies agents' optimal choices depending on their initial wealth conditions and their ability to access credit. In the current specification, the borrowing limit ϕ is higher than in the baseline calibration, as a result of imposing the same debt-to-GDP target in both cases. Therefore, the presence of transaction costs on financial markets yield two effects: on the one hand, they increase the cost of debt, since the effective interest rate on loans is higher; on the other hand, they make the economy 'looser', by allowing agents to borrow more than before. Let us denote with ϕ^{B} and ϕ^{TC}, respectively, the borrowing limits for the baseline model and for the economy with transaction costs. Since $\phi^{\mathrm{B}} < \phi^{\mathrm{TC}}$, constrained agents can now borrow and consume more than what they could in the baseline setup. In fact, Panels (b) and (g) in Figure (4.2) show that, for $\phi^{\mathrm{TC}} < b < -\phi^{\mathrm{B}}$, consumption and borrowing levels are higher than in the baseline model. For what concerns n, as $b < -\phi^{\mathrm{B}}$, labour supply is lower than in the baseline model: the availability of extra resources on financial markets due to the higher ϕ, and the higher disutility from labour Ψ, compared to the baseline model, push agents to work less (Figure 4.2e). As agents move away from the borrowing limit, the positive effect stemming from the increase in ϕ disappears. When $-\phi^{\mathrm{B}} < b < 0$, the borrowing constraint of the agents would be slack in both economies, hence, they would not benefit from the looser credit access - consumption, labour and financial choices are solely affected by ε. Therefore, intermediation costs deplete financial resources, and higher labour supply is required to cope with such loss, at least in part (Figure 4.2g). By consequence, overall consumption is also reduced (Figure 4.2b).

For what concerns agents with positive bond holdings, Figure (4.2) shows that there are little or no differences between the policy functions of the two models. Consumption and labour supply at equilibrium appear to be slightly reduced in the setting with transi-

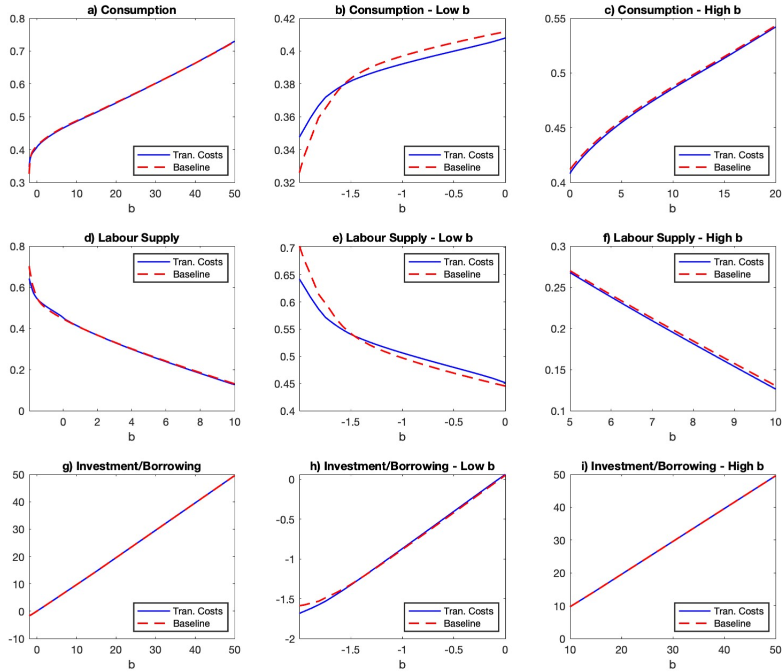

Figure 4.2. *Policy functions for consumption, labour supply and investments/borrowing - Comparison between the baseline model (dashed line) and the model with transaction costs (solid line) -* $w = w^7$

tion costs, for $b > 0$, however, the shape of the policy functions seem not to have changed. The investment function b' is identical in the two economies. While ε reduces the demand for credit from unconstrained borrowers, the increase in ϕ counterbalances it by allowing constrained agents to borrow more funds. Therefore, the aggregate quarterly size of debt is unchanged and equal to 0.3 in both economies. Given that the external supply of government bonds is still the same as in the baseline setting, and that investors are not affected by intermediation costs, the investment possibilities of individuals with positive bond holdings have not changed. For what concerns labour supply, the policy function for n is slightly lower in the transaction cost case then in the baseline set-up. With higher Ψ, *ceteris paribus*, agents are less willing to supply work. This has a negative impact on output as well: compared to the baseline case, aggregate quarterly output is 0.13% lower than in the baseline case at equilibrium. Figure (4.3) compares the asset distribution of the current specification with the one of the baseline model. The increase in the borrowing limit, from ϕ^{B} to ϕ^{TC}, expands the distribution more on the left-hand side of the grid on

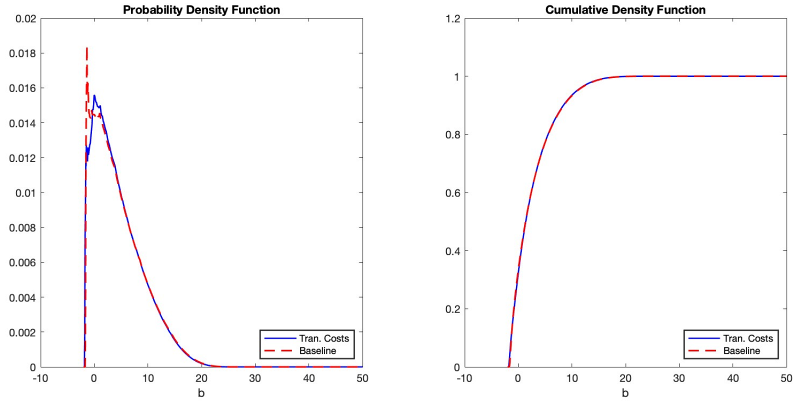

Figure 4.3. *Asset distribution at the stationary equilibrium - Comparison between the baseline model (dashed line) and the model with transaction costs (solid line).*

b. However, the presence of transaction costs reduce the aggregate amount of (negative) assets in the hands of borrowers, thus making the distribution less concentrated than in the baseline model. The distribution of positive bond holdings is close to the baseline case.

To fully appreciate the difference between the standard setting of Chapter 1 and the new setting, and to isolate the impact of ε on individuals' optimal decisions, I now simulate the model at the stationary equilibrium using the calibration of Table (1.2), disregarding the targets of Table (1.1) for the moment. I still maintain that transaction costs ε are equal to 2% on annual basis. Figure (4.4) displays the comparisons between the two settings, for a given wage level $w = w^8$. As in Figure (4.2), the first row displays policy functions for consumption, the second one policy functions for n, and the third one policy functions for b'. Results show that the presence of ε has the effect of depleting part of the resources available for the borrowers. In fact, all the individuals with $b < 0$ consume less than their counterparties in the baseline model (Figure 4.4b). At the same time, borrowers provide additional labour in order to increase their individual work income and compensate for the resources 'wasted' by the transaction costs (Figure 4.4e). However, little or no difference can be observed for individuals with positive savings (Figure 4.4c and f). This further confirms that the presence of ε has a globally negative effect on consumption, when $b < 0$, and that the higher policy function for c that we observed for constrained agents in Figure (4.2b) was due to the higher value for ϕ, which in turn expands the borrowing possibilities. Similarly, the higher values for ϕ and Ψ were responsible for the lower labour supply observed in Figure (4.2c), while the sole effect of ε on n is positive.

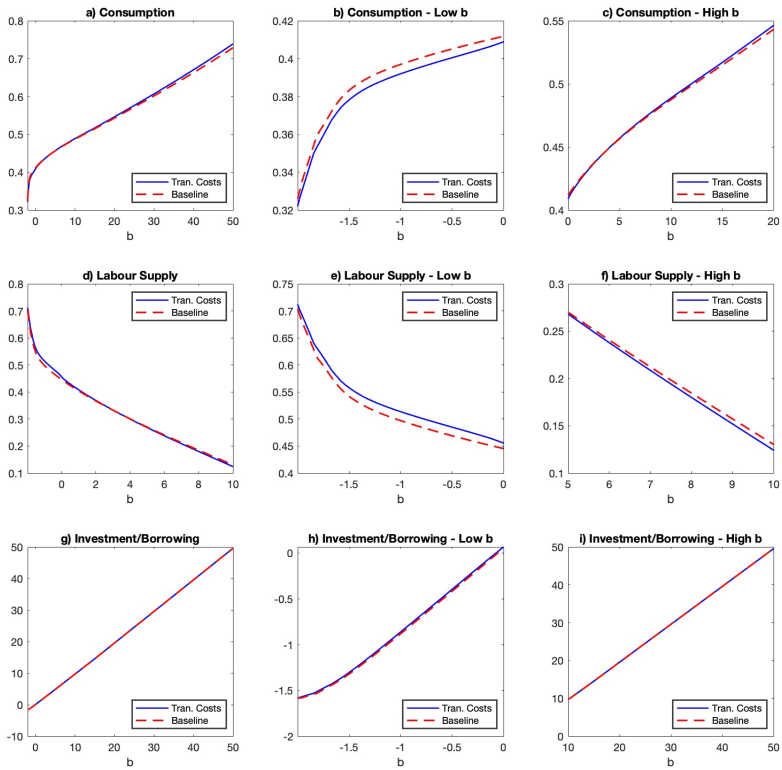

Figure 4.4. *Policy functions for consumption, labour supply and investments/borrowing - Comparison between the baseline model (dashed lines) and the model with transaction costs (solid lines) by keeping the calibration of the baseline model in Table (1.2) - for $w = w^7$.*

4.3. Credit Crunch

In this part, I am going to replicate the same credit crunch experiment performed in Chapter 2. As before, I consider the impact of an MIT shock on ϕ that is consistent with a drop from 18% to 8% in the debt-to-GDP ratio, once the economy has converged at a new steady state after the shock. As above, in order to avoid the possibility of defaults, I assume that the decrease in ϕ takes place over a set number of h quarters, according to a linear decay law:

$$\phi_t = \phi' - \Delta\phi \times t$$

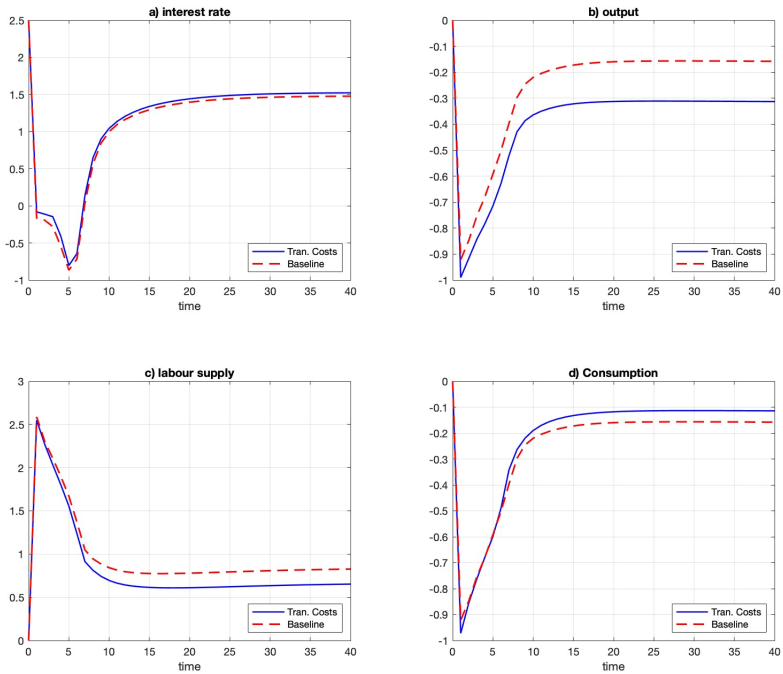

Figure 4.5. *Transition dynamics to a credit crunch - Comparison between the baseline model and the model with transaction costs ($h = 7$).*

for $t = 1, ..., h$. To ensure that the condition above is satisfied, the horizon h is set at 7:In contrast with Chapter 2, the no-default condition does not hold for $h \leq 6$. At the new stationary equilibrium, the final value of ϕ is 0.999. Transition dynamics are shown in Figure (4.5): the solid line represents the model with transaction costs, the dashed line the baseline framework, the responses of aggregate output and labour supply are given as percentage deviations from the initial stationary equilibrium. To ensure the comparability of results among the two cases, transitional dynamics for the baseline model have been re-computed by considering $h = 7$ as well.

The pattern of the transition dynamics is close to the one of the baseline specifications. Within 5 quarters from the initial shock, the equilibrium interest rate drops to almost -1% in both cases. As above, the short-term fall in r is due to the drop in demand for loans and the increased bond accumulation coming from the low-wealth individuals, which drive the interest rate in the negative region. Since investing is no-longer as profitable as before, high-wealth agents reduce the size of their portfolio by disinvesting. The short-term fall in interest rate is slightly less acute than in the baseline model. The distribution of loans is less concentrated around the borrowing limit, compared to the baseline case. Moreover, the initial and the final values of ϕ are both higher than in the baseline model. Therefore, the adjustment in r that is required to keep the asset market in equilibrium, is smaller. In the long-run the rate grows back to a positive level and converges to the new equilibrium value of 1.5% (annualized), which is approximately the same as in the baseline model.

For what concerns aggregate output, the short-term fall following the shock is comparable in sign and magnitude to the one in the baseline setup: in both cases there is a contraction of circa 1%. In the long-run, however, the drop in output in the model with transaction costs is greater. In Chapter 2, the contraction of output was a result of the partial equilibrium effect of credit crunch: the fall in available resources forced agents to reduce consumption and increase labour supply - with the drop in consumption being stronger and thus leading to the contraction of output. In this new framework we observe a similar effect. Figure (4.6) shows the general and partial equilibrium effect of the credit crunch on consumption and labour supply (solid lines), and compares them with the ones computed in the baseline setup (dashed lines)[2]. In the short-run, the drop in ϕ has a more relevant impact on consumption than on N - as a result, output contracts within 1 quarter from the initial shock. At the same time, N increases. Figure (4.6) shows that partial equilibrium effects in the short-run are stronger in the model with transaction costs than in the baseline one. In the long run, on the other hand, the partial equilibrium channel is stronger for N. Aggregate labour supply converges to a new equilibrium value that is strictly lower than the one in the baseline model. At the same time, the new equilibrium value of consumption is strictly higher than the one in the baseline model. The combine effect on output is negative: the new long-term value of Y is lower than in the baseline model.

[2]In this section, I decided to ignore the financial channel of the shock, i.e. the one stemming from the variations in interest rates. As in the previous setting, the effect of the interest rate channel is opposite in sign, compared to the partial equilibrium channel. It is not relevant in determining the direction of the responses, as the partial equilibrium effect is stronger, but it contributes to mitigate the negative (positive) impact of the drop in ϕ on consumption (labour supply).

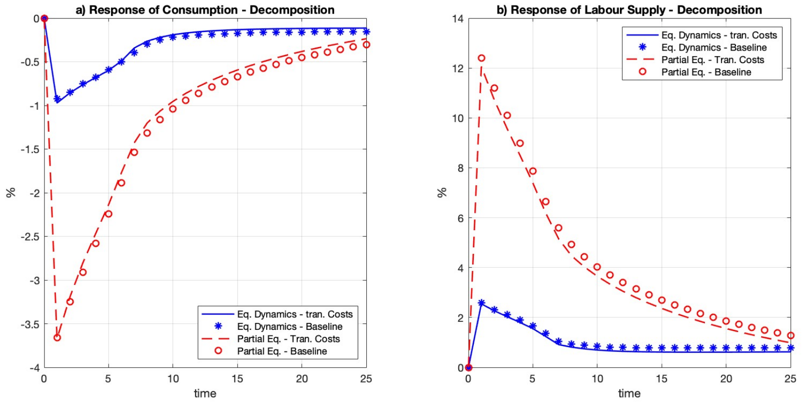

Figure 4.6. *General and partial equilibrium responses of consumption and labour supply - Comparison between the baseline model and the model with transaction costs.*

4.4. Intermediation Shock

In this section I consider the case in which the intermediation cost ε is hit by an MIT shock and, more specifically, I set that it grows up to 6% on an annual basis. This chosen value is consistent with a 20%-decrease in the debt-to-GDP ratio (annualized), from 18% to 14.4%[3]. As opposed to the credit crunch case, I now assume that, when the shock hits the economy at time $t = 0$, ε adjusts to new value immediately. In our previous case, the gradual adjustment was necessary to ensure that all the agents had enough resources to pay off their debt without defaulting. In this new case, the size of the repayment that a constrained agent must make after the shock at $t = 1$ is:

$$\Phi_1 = \frac{R_1 - 1 + \varepsilon''}{R_1}\phi \tag{4.6}$$

where ε'' is the value of the transaction cost after the shock. The repayment depends on the size of ε after the shock, not on its period-by-period variation, as for ϕ in the previous sections. For this reason, in order to prevent defaults, it is sufficient to restrict ε not to grow excessively after the shock. Under the current calibration, the consumer's problem does not explode and an equilibrium solution exists. The transition dynamics to the MIT shock are given in Figure (4.7).

Figure (4.7a) displays the transition dynamics for the net interest rates on investments and loans, $R_t - 1$ and $R_t^b - 1$, in the economy. As the shock hit, in the first quarter, debt

[3]I set a milder debt target for this section, instead of the 8% one for the previous one. The reason behind my choice lies in the fact that an after-shock long-run target of 8% is corresponds to an unrealistically high value for ε, equal to circa 12.7% on an annual basis.

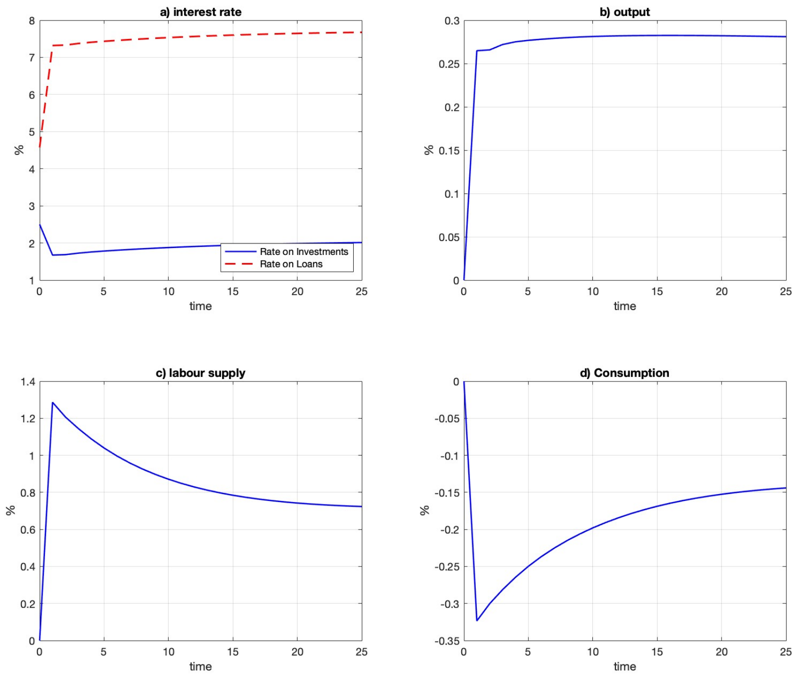

Figure 4.7. *Transition dynamics to an MIT shock to the transaction costs ε. Output, labour supply and consumption are expressed as % deviations from the original stationary equilibrium.*

is more costly, thus agents with negative bond holding are pushed to reduce the size of their loans or to invest rather than borrow. Consequently, as above, the demand for credit decreases, while the demand for positive quantities of bonds increases, thus pushing the interest rate R_t down. What is more, Figure (4.7a) shows that the short-term variation in R_t and R_t^b is asymmetrical: the drop in R_t is much smaller in size than the rise in R_t^b. Such discrepancy can be explained by looking at the different (relative) size of the two asset classes in the economy: debt instruments and government bonds. At the initial equilibrium, aggregate debt instruments are worth only 18% of total GDP (per annum), while government bonds amount up to 160% of yearly GDP. The reduction in the demand for debt instruments, in relation to their supply level, is greater than the contemporaneous increase in the demand for government bonds, in relation to their external supply level. Therefore, to accommodate the change in market dynamics, the interest rate on investment R_t adjusts less than the cost of debt R_t^b. In an economy with zero external supply of assets, the adjustment in interest rate would be perfectly symmetrical. In the long-run, interest rates on investments and loans converge to their new equilibrium value, which are 2% and 7.5%, respectively.

For what concerns output, Figure (4.7b) shows that, in contrast to the previous experiment, an increase in ε yields a positive effect on aggregate income. As discussed above, higher intermediation costs negatively affect agent's access to credit, hence, the available resources for those with negative and low bond holdings are reduced. By consequence, agents need to supply additional labour in order to sustain their optimal choices in terms of consumption and investments. For this reason, as soon as the shock hits the economy, aggregate labour supply increases by 1.2% in one quarter. At the same time, the fall in available resources, due to the shock, reduces agents' consumption possibilities, thus translating into a short-run drop of 0.3% in aggregate consumption (Figure 4.7d).

Chapter 5

The Model with Housing

5.1. The Economy

In this chapter, I enrich the economy analysed so far with an additional housing sector. I assume that, in the new framework, households derive their utility from standard consumption (as per usual denoted by c) and housing, which is denoted by h, and supply labour n, which generates disutility for the workers. Until now, I have assumed that consumption is a non-storable resource, i.e. its utilization cannot be deferred to the future. While keeping this assumption, I postulate that housing, on the other hand, can be stored and transferred to forthcoming periods, acting as a durable consumption good. In terms of functional form, I assume that household i's intertemporal utility is non-separable in consumption and housing, but separable in labour n:

$$\mathcal{U}(\mathbf{c}, \mathbf{h}, \mathbf{n}) = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta u(c_t^i, h_t^i) + d(n_t^i) \right]$$

where u represents Moreover, I assume that agents use a Cobb-Douglas aggregator to evaluate their choices in terms of consumption and housing:

$$u(c, h) = c^{\theta} h^{1-\theta} \tag{5.1}$$

where θ represents the relative elasticity of consumption, with respect to housing. Such formulation yields several benefits for the computation of the stationary equilibrium. Firstly, at equilibrium no household would want to spend all their available resources on just one of the two goods. Secondly, at equilibrium agents are better off when they purchase fixed proportions of c and h, and when such proportions are closer to θ and $1 - \theta$, respectively. Thirdly, the homogeneity of degree one implies that, if an agent want to increase their utility u by a factor x, they must increase both h and c by x. Additionally, previous empirical studies have highlighted that the elasticity of substitution between nondurable goods and durable goods (such as housing) is close to the unity[1]. Disutility from work d has the same functional form as in the previous chapters. I assume that the aggregate stock of housing H_t is fixed and constant at every period of time t, i.e. there is no production of new dwellings or destruction of existing ones. Houses are therefore sold and purchased at a price q_t, which varies over time depending on the aggregate demand.

Adding housing, the individual budget constraint becomes the following:

$$p_t^* b_{t+1}^i + c_t^i + q_t h_{t+1}^i \leq w_t^i n_t^i + b_t^i - z_t^i + q_t h_t^i - \kappa(h_t^i) \tag{5.2}$$

where $\kappa(h_t^i)$ is a cost function that captures the degree of illiquidity of housing. In this economy, housing has a double nature. As aforementioned, h is a durable consumption good, it can be purchased at any period t for utility-maximization purposes, and 'consumed' over several periods of time. On the other hand, because it is storable, housing can be used to transfer resources from one period to another, acting as an investment good like the government bonds b. A core difference between b and h is that, while the former is purchased at a discount and sold at face value after one period of bond holding - thus providing a safe return - the latter can be sold at any point in the future that period price q_t, which may be lower than the purchasing price. Additionally, I set that the cost function κ depletes the value of the household's housing stock over time. κ can be interpreted as a periodical maintenance cost that households need to bear in order to keep their dwelling marketable, so that its value does not deteriorate with time. This implies that, even when agents are not interested in expanding the size of their housing, they need to keeping investing in h at every period. Following the standard literature, I assume that κ is linear, and proportional to the value of h:

$$\kappa(h_t^i) = \kappa q_t h_t^i \tag{5.3}$$

No further restrictions for the liquidity of the housing are imposed through the remaining part of this chapter[2]. In addition to this, I impose a 'no-rough-sleeping' constraint, i.e.

[1] See, for instance, Fernandez-Villaverde and Krueger (2011)

[2] While a few authors assume the existence of adjustment costs that individuals have to bear when purchasing illiquid assets, I choose not to include them in the model. For example, Hintermaier and Koeniger (2010), in their model with durable and non-durable consumption goods, assume a quadratic costs function for individuals who buy and sell durables. In my framework, I follow the mainstream literature concerning housing in macroeconomic models, such as Iacoviello (2005), Adam et al. (2011), and Ascari et al. (2018), who exclude the presence of adjustment costs. In such setting, the illiquidity of housing is driven by the depreciation or maintenance costs related to housing, which force agents to continuously invest in housing in order to maintain their housing at full value. What is more, from the mathematical point of view, the absence of a convex cost function helps to keep the household's problem concave.

at any period t households are required to possess a minimum, or subsistence, level of housing $\underline{h} > 0$. Formally, at any t:

$$h^i_{t+1} \geq \underline{h} > 0 \tag{5.4}$$

I will show in the next sections how the choice of $\underline{h}$ is pivotal for the computation of the equilibrium, and how different constraints lead to different outcomes in terms of policy functions and asset distributions.

As in Chapter 4, I postulate the existence of a fully competitive intermediation sector and of transaction costs ε, which is proportional to the borrowed amount of funds. To cover for these costs, intermediaries charge a mark-up on loans. Hence, the borrowers face an effective cost of debt, R^b_t at time t, which is higher than the real gross interest rate of the economy R_t. Therefore, if $b' >= 0$ we have that

$$p^*_t = \frac{1}{R_t}$$

while, if $b' < 0$:

$$p^*_t = \frac{1}{R^b_t} = \frac{1-\varepsilon}{R_t}$$

Households are subject to a borrowing constraint: individuals can borrow up to a fraction $\phi \in]0, 1[$ of the value of their investment in housing. More formally:

$$b^i_{t+1} \geq -\phi_h(1-\kappa)q_t h_{t+1} \tag{5.5}$$

In this economy, housing acts as collateral: individuals do not necessarily borrow for real estate purposes, but if they do they need to provide collateral to cover part of the loan. If borrowing is for purchasing new housing, then ϕ can be interpreted as the maximum loan-to-value ratio for the said investment.

As in Hintermaier and Koeniger (2010), I define households' net worth x_t as the sum of the net investment positions in both bonds and housing at time t. More formally:

$$x^i_t \equiv b^i_t + (1-\kappa)q_t h^i_t \tag{5.6}$$

Therefore, it is possible to re-write the budget constraint (5.2) and the borrowing constraint (5.5) in terms of wealth x, i.e.:

$$\begin{aligned} p^*_t b^i_{t+1} + c^i_t + q_t h^i_{t+1} &\leq w^i_t n^i_t - z^i_t + x^i_t \\ x^i_{t+1} &\geq (1-\phi_h)(1-\kappa)q_t h_{t+1} \end{aligned}$$

By taking all these elements into account, I can write the new household's optimization problem in the following recursive form:

$$V(w, h, x) = \max_{c, h', b', n} u(c, h, n) + \beta\mathbb{E}[V(w', h', x')|w]$$

subject to:

$$
\begin{aligned}
c &\leq wn - z + x - p^* b' - qh' \\
x' &\geq (1 - \phi_h)(1 - \kappa) q h' \\
x' &= b' + (1 - \kappa) q h' \\
c &\geq 0 \\
n &\in [0, 1] \\
h' &\geq \underline{h} > 0
\end{aligned}
$$

where the current x, h and productivity w represent the state variables of the problem, c, h', b' and n represent the choice variables, and V is the household's value function.

Finally, as in the previous chapters, I assume the presence of a public sector in the economy, whose role is to provide unemployment benefits to individuals with zero wage/productivity, and to rise funds through taxation and by issuing government bonds on the market.

5.2. Stationary Equilibrium

In this economy, the following definition of stationary equilibrium holds. Let us denote with $\mathbf{W}$, $\mathbf{X}$, $\mathbf{H}$, respectively, the sets of all the possible values for w, x, and h. Define $\mathbf{S}$ as the set of all the possible states of the world in this economy, i.e. $\mathbf{S} = \mathbf{W} \times \mathbf{X} \times \mathbf{H}$, and with $\mathcal{B}$ the associated Borel σ-algebra. As above, I denote with Λ the set of all the possible probability measures over the measurable space $(\mathbf{S}, \mathcal{B})$ and, for any subset $\mathcal{S} \in \mathcal{B}$, I denote with $\lambda(\mathcal{S})$ the measure of agents in $\mathcal{S}$. Therefore:

Definition 2 *A **stationary recursive competitive equilibrium** in a Bewley economy with consumption and housing is a value function V, a set of interest rates $\{R, R^b\}$, housing market variables $\{q, H^s\}$, a set of policy functions $\{c, b', h', n\}$, a set of government policies $\{\tau, v, B\}$, and a stationary probability measure $\lambda \in \Lambda$, such that:*

I. *Given $\{R, R^b\}$, $\{q, H^s\}$ and $\{\tau, v, B\}$, and for any triple $(w, x, h) \in \mathbf{S}$, policy functions $c : \mathbf{S} \to \mathbb{R}_+$, $b' : \mathbf{S} \to \mathbb{R}$, and $h' : \mathbf{S} \to [\underline{h}, H^S]$ solve household's problem, and $V : \mathbf{S} \to \mathbb{R}$ is the associated value function;*

II. *The financial market is in equilibrium, i.e. aggregate bond demand is equal to the exogenous supply B:*

$$\int_{\mathbf{S}} b'(w, x, h) d\lambda(w, x, h) = B$$

III. *The housing market is in equilibrium, i.e. demand must be equal to supply H^s:*

$$H^d \equiv \int_{\mathbf{S}} h'(w, x, h) d\lambda(w, x, h) = H^s$$

IV. *Government policies* $\{\tau, \nu, B\}$ *satisfy the government's budget constraint:*

$$pB + u\nu = \tau + B$$

V. *The market for goods is in equilibrium. Aggregate labour supply at equilibrium is:*

$$N = \int_{\mathbf{S}} n(w, x, h) d\lambda(w, x, h)$$

while output is:

$$Y = \int_{\mathbf{S}} y(w, x, h) d\lambda(w, x, h)$$

with $y(w, x, h) = w \times n(w, x, h)$ *being the optimal efficiency unit of labour at equilibrium, for a given initial* w, x *and* h. *Good market clears when:*

$$Y = C + \kappa q H^s + \frac{\varepsilon}{R^b} L$$

where C *is aggregate consumption,* $\kappa q H^s$ *represents aggregate maintenance costs for housing, and* L *represents the aggregate debt outstanding of the economy.*

VI. *For all subsets* $\mathcal{S} \in \mathcal{B}$, *the probability measure* λ *is such that:*

$$\lambda(\mathcal{S}) = \int_{\mathbf{S}} P((w, x, h), \mathcal{S}) d\lambda(w, x, h)$$

where $P((w, x, h), \mathcal{S})$ *is the Markov transition function associated to the household's problem, which maps the transition from the current state* (w, x, h) *to the set* $\mathcal{S}$ *in the next period.*

In the economy with housing, the equilibrium solution to the consumer's problem is characterized by the following system of equations:

$$u_c(c, h, n) \geq R\beta\mathbb{E}[u_c(c', h', n')] \tag{5.7}$$

$$qu_c(c, h) \geq \beta q(1 - \kappa)\mathbb{E}[u_c(c', h')] + \beta\mathbb{E}[u_h(c', h')] \tag{5.8}$$

$$wu_c(c, h, n) \leq -u_n(c, h, n) \tag{5.9}$$

$$c = wn - z + b - pb' \tag{5.10}$$

$$c > 0 \tag{5.11}$$

$$h \geq \underline{h} \tag{5.12}$$

where each u_k denotes the partial derivative of the utility function u with respect to variable $k = c, h, n$. Note that the Euler equation (5.7) is satisfied with equality if the borrowing constraint (5.5) is slack, the equilibrium condition on the housing market (5.8) is satisfied with equality if both (5.5) is slack and $h \geq \underline{h}$, and the intratemporal optimality condition (5.9) is satisfied with equality if $n > 0$.

Variable	Value	Target/Source
Average employment	40% of total time endowment	Nekarda and Ramey (2020)
Unemployment benefit	40% of GDP (quarterly)	Nekarda and Ramey (2020)
Total Liquid Asset	178% of GDP (annualized)	US economy in 2006
Value of Housing	238% of GDP (annualized)	US economy in 2006
House Prices	1.2	US economy in 2006
Real Interest Rate	2.5% (annualized)	
Debt-to-GDP ratio	28% (annualized)	

Table 5.1. *Targets for the Calibration of the Stationary Equilibrium - Model with Housing*

5.2.1. Calibration

As above, the model is calibrated and simulated using numerical methods. Table (5.1) sets out the long-term targets for the parametrization of the model at the stationary equilibrium. Said targets are chosen to match the US economy in 2006. Values for employment, unemployment benefits, liquid assets, and real interest rate are set to be the same as in the previous setups without housing. For what concerns the housing market, targets are also taken from the US economic aggregate data from 2006. More specifically, the value of the housing held by households in the US was equal to 238% of annual GDP, while the aggregate house price index of the US was 1.2 times bigger than the standard consumer price index. Therefore, since the price level for goods in this economy is normalized to 1, I set that $q = 1.2$ at the stationary equilibrium. To match such targets, I use the calibration laid out in Table (5.2).

For what concerns household debt, I set out a 28% target for the annual debt-to-GDP ratio, although this value does not match the 2006 data for the United States, when the household debt-to-GDP ratio was above 70%. To match such high target, I would need to impose a debt limit ϕ_h well above 100% in the model calibration, but the economy would not converge to an equilibrium in this case. Nonetheless, for my initial calibration of the model, I consider an economy with loose borrowing limits, where individuals can finance almost all their housing investment using debt. Therefore, I set 28% as the arbitrary target value for the annual debt-to-GDP ratio, which is consistent with $\phi_h = 95.5\%$.

5.2.2. Results

The four panels of Figure (5.1) display, clockwise, the policy functions for c, h', b' and n, respectively - as function of total net worth x, for a fixed value of initial housing h, and for three different levels of productivity w. Figure (5.1a) shows that policy functions for consumption are similar to those in the baseline model without housing: for low net worth, agents are more risk averse and, hence, the curves are strictly concave. However, for higher values of x, the curves of the policy function become more linearized, as consumption

Parameter	Description	Value	Target or Source
β	Rate of time preference	0.99	Int. rate = 2.5% annualized
γ	Constant Risk Aversion	4	
η	Concavity disutility from labour	1.5	Frisch elasticity = 1
Ψ	Coefficient disutility from labour	5.9	Avg. employment 40% of annual GDP
θ	Cobb-Douglas elasticity	0.8	House price target $q = 1.2$
ϕ_h	Max LTV ratio	95.5%	Implies aggregate debt equal to 29% of annual GDP
κ	Maintenance costs for housing	1.5%	
ν	Unemp. benefits	0.1675	Target = 40% of quarterly GDP
B	Bond supply	2.51	Target = 150% of annual GDP
μ	Uncond. mean wage process	0	Set mean wage $w = 1$
ρ	Persistence wage process	0.967	Flodén and Lindé (2001)
σ^2	Variance wage process	0.017	Flodén and Lindé (2001)
J	Number of wage states (unemployment excluded)	5	
π_{ue}	Prob. of finding a job	0.882	From Shimer (2005)
π_{eu}	Prob. of losing a job	0.057	From Shimer (2005)
ε	Transaction Costs	1%	Annualized Value
$\underline{h}$	Subsistence Housing	0.01	

Table 5.2. *Calibration of the Stationary Equilibrium - Model with Housing.*

choices depend on the total net worth, rather than on income levels. Also policy functions for labour supply n follow a similar pattern as in the economies without housing (5.1d). When the initial x is low, individuals with low wage provide supply higher labour to the economy, when compared to individuals with higher w. Once again, the income effect is stronger when starting net worth is low. As x grows, high-wage individuals supply more labour then their low-wage counterparties, because substitution effect dominates over income effect in this case.

For what concerns investments in housing and bonds, optimal decisions strictly depend on initial conditions in terms of housing, net worth and labour productivity, as well as on whether individuals purchase housing through leverage or solely using their own funds. For convenience, I segment the population on this economy based on their investment decisions, into three categories: the borrowers, the lenders, and the 'strictly hand-to-mouth' agents. In the equilibrium plots, borrowers are located at the very left-hand side of the grid for x in Figures (5.1b) and (5.1c). The savings curve for borrowers is u-shaped. Under the current calibration, when the net worth is low enough (e.g. circa $x < 1$ in the graph), individuals are borrowing constrained, i.e. they borrow as much as possible to purchase their housing stock: the picture shows that the size of their debt $|b'|$ increases linearly with their optimal housing decisions h'. Those individuals, are

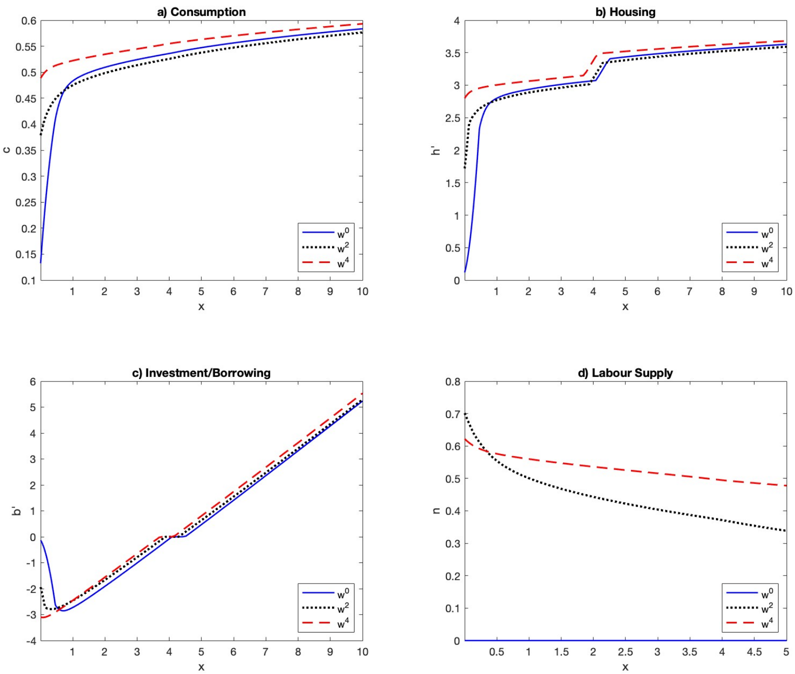

Figure 5.1. *Policy functions at the Stationary Equilibrium, as function of total wealth h, for three different levels of productivity w, and for a fixed initial level of housing $h = 0.2790$ - Model with Housing*

the most affected by movements in ϕ_h. As x increases, individuals borrow less than the maximum allowed amount, and use an increasing share of their total resources to buy their dwelling, therefore the savings curve is upwards shaped when $x \in]1, 3.5[$ in the figure. Lenders are located on the right-hand side of the figure. Both optimal investment in bonds and housing purchases are now increasing functions of their starting net worth, and differences among income levels tend to disappear, in fact as x grows, $n \longrightarrow 0$.

The 'jump' in Figure (5.1b), together with the flattened part of the savings curves in (5.1c), represent the optimal decisions in terms of h' and b', respectively, of strictly hand-to-mouth households. Their policy function for housing is proportional to their initial level of wealth x. Such individuals do not participate in the credit market, not because they are cut off by external forces, but because it is sub-optimal for them to either borrow or invest in bonds: the rationale is that, according to their preference structure, R is too low to invest and R^b is too high to borrow. Optimal consumption decisions at equilibrium must satisfy the Euler condition, which is the following, provided that the

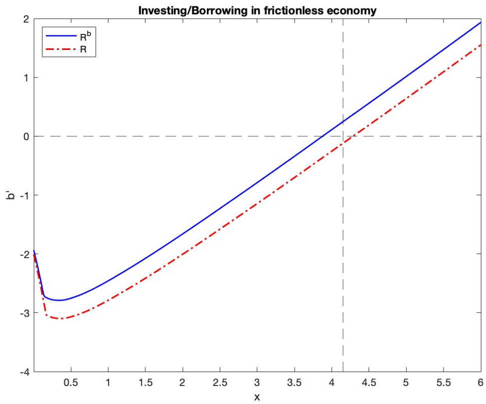

Figure 5.2. *Policy functions for b', assuming that the economy is frictionless and for two different interest rates R and R^b. The two policies are displayed as function of x and for given levels of $w = w^2$ and $h = 0.2790$.*

individual is not constrained:

$$MUC = \beta R^* \mathbb{E}[MUC']$$

where $R^* = R$ or R^b, depending on the agent's net financial position b', and with MUC denoting the marginal utility of consumption. Suppose for now that $\varepsilon = 0$ and that R is the interest rate for both borrowing and lending. The policy function for b' (for given h and w) is represented by the dashed and dotted line in Figure (5.2). If we consider, for example, an individual with initial net worth $x = 4$, their optimal financial decision would be to borrow now to consume more tomorrow, hence $b' < 0$. If, for any given reason, the interest rate of the economy suddenly jumps to R^b, the RHS of the Euler equation would, *ceteris paribus*, exceed the LHS. For the equilibrium to be restored, the agent would need to reduce their present day consumption, so that the MUC is higher and equal to the new RHS. The equilibrium policy function at this rate is given by the solid line in Figure (5.2). If, once again, we consider an individual with $x = 4$, when the interest rate is R^b their optimal financial decision would be to invest, i.e. $b' > 0$. Therefore, in an economy with transaction costs, borrowing at R^b and investing at R would both be sub-optimal decisions for an agent with starting net worth equal to 4, therefore such individual would be better off with $b' = 0$[3].

Figure (5.3) displays stationary density functions for housing h and net worth x at equilibrium. As regards housing (Figure 5.3a), the distribution exhibits bimodality, due to the segmentation of household into borrowers, lenders, and strictly hand-to-mouth, and

[3]In the following sections, I will show that the number of strictly hand-to-mouth agents increases with ε.

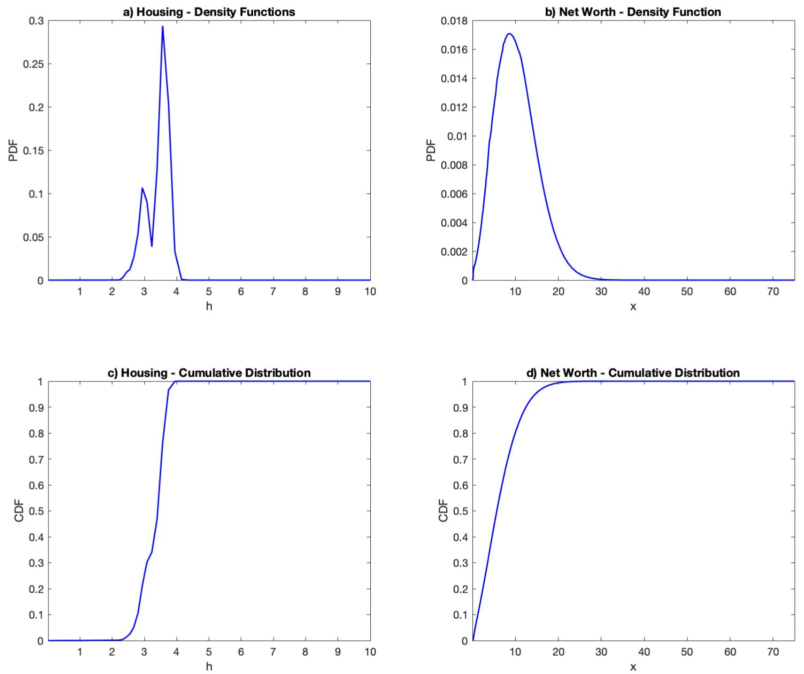

Figure 5.3. *Stationary Distributions at the Stationary Equilibrium - Model with Housing*

because of the impact of intermediation costs. The spike on the left-hand side (right-hand side) of the distribution represents total housing owned by the borrowers (lenders); strictly hand-to-mouth agents are located in the saddle point between the two concavities. The sizeable difference in height between the two maxima points denotes large inequality in the distribution of illiquid wealth across the different classes of households, with the majority of the housing stock H^s in the economy in the hands of the borrowers. Figure (5.4a) plots the average housing owned per level of wealth and it shows that housing capital accumulation, on average, decreases with x. Individuals with initial low net worth tend to invest more in housing, while investment of agents who are already wealthy is proportional to their existing net worth x. Furthermore, the extremely thin tails at the ends of the distribution in Figure (5.3a), as opposed to the tall spikes at the center, together with the steep slope of the cumulative distribution for h, may suggest that housing is highly concentrated in the hands of a few individuals in the economy. Figure (5.4a), which plots the distribution of housing across different levels of x, suggests that households with $x < 21$, i.e. the bottom 28% of individuals in terms of net worth, hold 99.4% of the total housing stock of the economy.

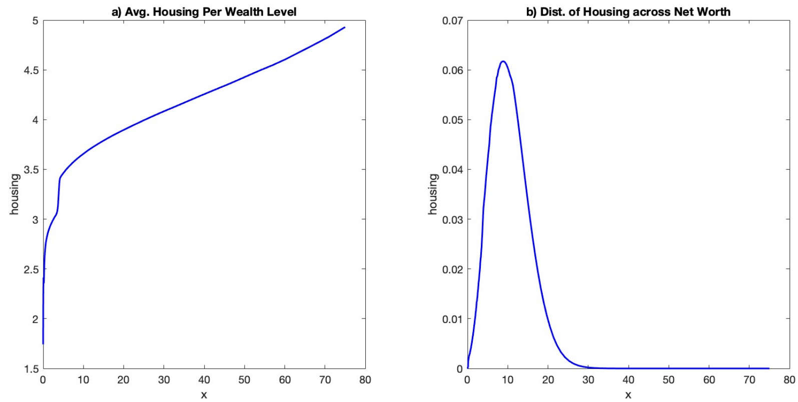

Figure 5.4. *The graph on the left displays the average level of housing owned at different level of wealth x. The graph on the right shows the distribution of housing owned across net worth levels x.*

5.2.3. Subsistence Housing and Equilibrium Results

The no-rough-sleeping constraint (5.4) is a pivotal determinant of the equilibrium results we observed in Section 5.2.2. Imposing a minimum value for h is necessary to model a realistic economy. Firstly, $\underline{h}$ acts as a physical constraint, i.e. a dwelling needs to be 'big enough' to allow the individual to live in it, and present some basic features (e.g. a bedroom, a kitchen and a bathroom). In this sense, $\underline{h}$ can be thought as the minimum number of square meters that a house or apartment needs to have to include such basic features. Secondly, each individual's equilibrium choice of consumption and housing needs to be plausible, so that for example a household would not select an extremely large quantity of consumption good, and very little housing (or vice versa). Nonetheless such eventuality is also ruled out by the shape of the utility function: optimal c and h are primarily determined by the elasticity θ, which represents the household's 'degree' of preference of standard consumption with respect to housing, e.g. a higher θ implies a higher preference for consumption. Thirdly, and most importantly, imposing a minimum dimension for h also responds to the economic necessity of providing the households in the economy with a substantial level of housing, so that each individual has the necessary economic resources to satisfy their basic needs. In this optic, it would be interesting to analyse how variations in the levels of subsistence housing contribute to shape the individual's optimal decision rules.

Figure (5.5) displays individual's policy functions at the stationary equilibrium for three different levels of subsistence housing $\underline{h}$. The solid line represent the standard case, explored in the previous section, in which $\underline{h} = 0.01$, while the dashed and the dotted

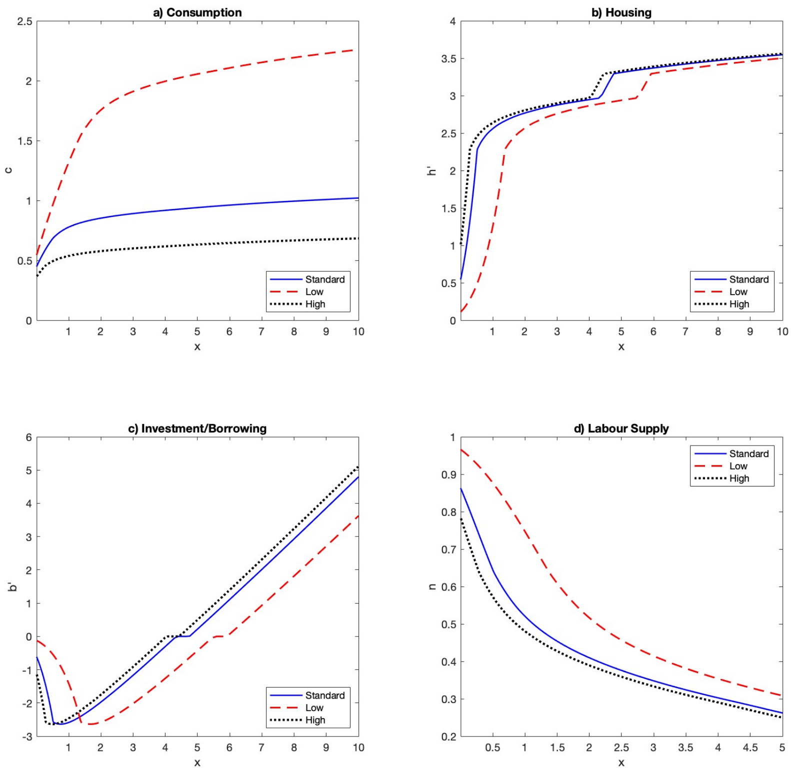

Figure 5.5. *Changes in equilibrium results for different values of* $\underline{h}$. *The solid line displays the standard case with* $\underline{h} = 10^{-2}$. *The dashed line displays the case in which* $\underline{h} = 10^{-4}$. *The dotted line displays the case in which* $\underline{h} = 10^{-1}$. *Wage* w *and initial housing* h *are fixed to, respectively,* $w = w^2$ *and* $h = 0.2790$.

lines represent, respectively, economies in which $\underline{h} = 0.001$ and $\underline{h} = 0.1$[4]. Once again, the graph display optimal choices as function of the net worth x, for a fixed productivity level $w = w^2$ and for a fixed initial level of housing $h = \underline{h}$. The figure shows that a higher level of $\underline{h}$ yields a positive wealth effect for individuals. Since they are endowed with a higher level of housing, agents are prompted to increase their investment in housing. Furthermore, a large house endowment allows agents to potentially borrow more, since they now can provide additional collateral. In fact, the policy function for b' shows that borrowing constrained agents (for $x < 1$) take additional loans when $\underline{h} = 0.1$, than in

[4]In these two alternative economies, the other calibrated parameters remain unchanged, with respect to the standard equilibrium case.

the standard case. However, unconstrained borrowers tend to borrow less. Given the increased availability of resources for households, they can reduce their labour supply, when the subsistence level of housing is higher (Figure 5.5d). However, the effect on consumption is negative.

5.3. Credit Crunch

In this section, I assume that the economy described above is hit by a one-time and permanent credit crunch shock, which entails a sharp decline in the borrowing limit ϕ_h. More specifically I assume that the drop in ϕ_h is consistent with a long-term decline of the annual debt-to-GDP ratio of 5% - dropping from 29% in the original equilibrium to 24% in the new stationary equilibrium, at which the economy eventually converges after the shock. The final new value for ϕ_h is 71.45%. As in the previous setups, I assume that the adjustment of ϕ_h to its new value takes place gradually, over a period of $s = 9$ quarters. As above, the rationale behind this latter assumption is to avoid a situation in which constrained agents are forced to make repayments that are so big so that the borrowing and no-rough-sleeping constraints of this economy are violated[5].

5.3.1. New Stationary Equilibrium

Before analysing the consequence of the shock on the aggregate economy and on the transitional dynamics, it is interesting to study how the credit crunch affects households at the micro level, and how the policy functions adjust to the new financial regime. To do so, I compare the policy functions of the new stationary equilibrium at which the economy converges after the shock, with those at the starting equilibrium. Such comparison is displayed in Figure (5.6). The shock yields a heterogeneous effect on households, depending on their initial conditions in terms of net worth, housing and wage. Individuals with low x - especially those for which the borrowing constraint (5.5) is binding - are the most negatively affected: since their borrowing possibilities are reduced and, hence, their available resources are cut, low-wealth individuals are forced to consume less than in the original equilibrium, and provide additional labour (as shown by Figures 5.6a and 5.6d). Borrowing constrained individuals are forced to sell part of their housing to cope with consumption expenditure, while those who are double constrained - i.e. for which both equations (5.5) and (5.4) are binding - and who cannot further downsize their housing,

[5]One can argue that when ϕ_h decreases, borrowing constrained agents can simply repay the excessive part of their debt by downsizing their housing stock h' thus avoiding any possibility of default. However, if an agent is double constrained, i.e. both conditions (5.5) and $h' \geq \underline{h}$ are binding, she cannot repay the excessive part of her debt by simply reducing h'. Hence in such situations, if the repayment is too high to satisfy the budget constraint without reducing h', and if $h' = \underline{h}$ already, then individuals cannot cope with the repayment and they are forced to default.

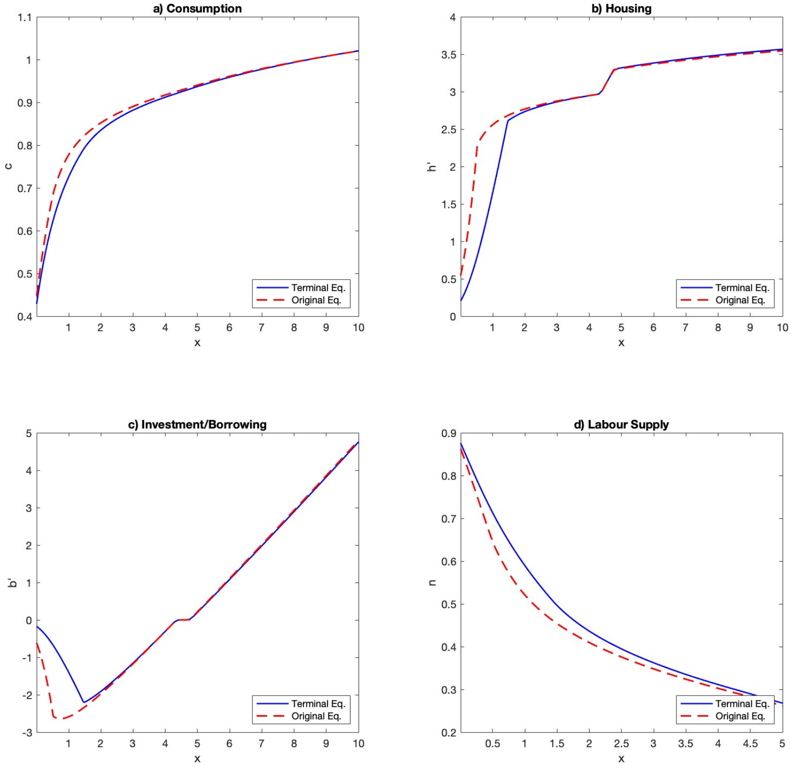

Figure 5.6. *Policy Functions at the New Stationary Equilibrium, for a given initial value of h and $w = w^2$ - Comparison with Original Equilibrium (dashed lines).*

are forced to further reduce their consumption. When the net worth increases (e.g. $x > 5$ in the graph), the differences in terms of c and n between the two equilibria are reduced.

For what concerns investments and house purchase, the shock generates a portfolio re-allocation effect. On the one hand, (part of) the borrowers are forced to repay some of their debt, and downsize their housing. On the other hand, the lenders face a reduced demand for credit, and an increased availability of housing, therefore they reduce their investments in bonds and purchase more dwellings. Such market dynamics make the interest rates fall at the new equilibrium and the house price q rise. More specifically, R and R^b drop, respectively, from 2.5% to 2.44%, and from 3.57% to 3.48%, while the house price jumps to 1.24. Borrowers who were closer to the original debt limit now are forced to reduce the size of their debt, and to downsize their housing (if they can). The drop in individual debt levels can be seen by the difference in policy functions for b' between the

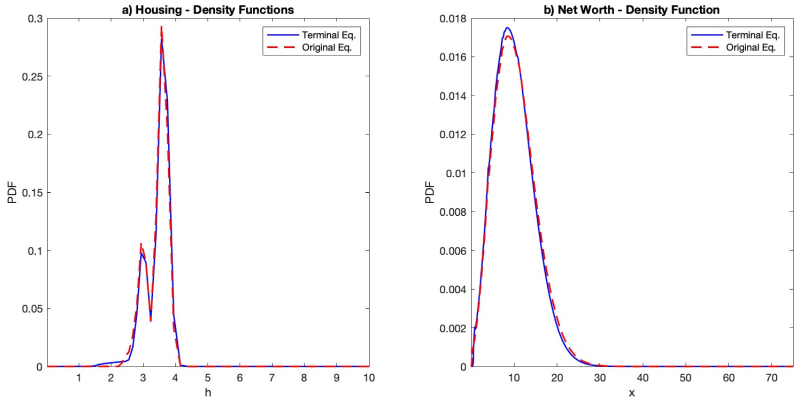

Figure 5.7. *Stationary Distributions at the New Stationary Equilibrium - Comparison with Original Equilibrium (dashed lines).*

new (solid line) and the original equilibrium (dashed line), which are shown in Figure (5.6c) for $x < 1$. Such agents also greatly reduce the size of their housing, as they need to repay the excessive part of their debt (Figure 5.6b). At the same time, those borrowers who, in the original steady state, had a loan worth less than 68.6% of their net housing investment, take advantage of the lower rates, and increase the size of their debt in the new equilibrium. In fact, Figure (5.6c) shows that the policy function has moved towards right in the new equilibrium. Additionally, some individuals who were strictly hand-to-mouth in the original equilibrium, now prefer to borrow. As a result, differences in housing choices across the two equilibria are relatively small (Figure 5.6b). At the new equilibrium, lenders reduce their investment in bonds, given the reduced demand and the consequent fall in interest rates, and increase their demand for housing.

The new equilibrium displays increased wealth inequality across the households, in particular for what concerns housing. Figure (5.7a) shows that at the new equilibrium, borrowers hold a smaller share of total housing in the economy, as the tail on the left-hand side is now smaller than in the original stationary equilibrium, and the smaller peak on the left is slightly smaller. At the same time, the size of housing in the hands of the lenders has increased compared to the original equilibrium. The share of housing in the hands of the strictly hand-to-mouth - who are represented by the saddle point in between the two peaks - does not change. Such dynamics are also reflected in the distribution of net worth x (Figure 5.7b): the central part of the distribution is higher, while the tails are slimmer, which entails a higher concentration of assets in the hands of the lenders.

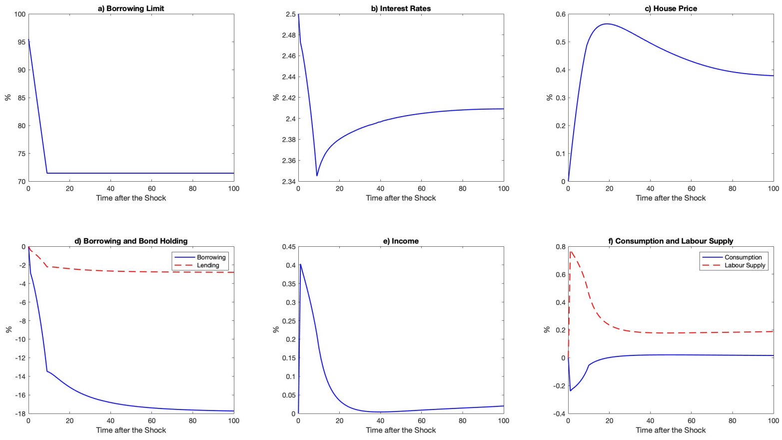

Figure 5.8. *Transition Dynamics after an MIT shock to ϕ_h - Model with Housing*

5.3.2. Transitional Dynamics

The transitional dynamics after the shock are shown in Figure (5.8). The drop in ϕ_h yields an heterogeneous effect on agents, depending on whether they buy housing with borrowing or with solely their own funds. As aforementioned, the demand for loans is reduced after the shock, therefore, following the initial drop in ϕ_h, the real interest rate has to decrease to dissuade lenders from putting unnecessary resources in the credit market. R_t falls down to 2.34% in the 9 quarters after the shock, but this drop is then followed by a rise, due to the increased demand for loans stemming by non-constrained agents, i.e. those who borrow less than the limit, who are attracted by the lower cost of debt. The rate then converges to its new equilibrium value of circa 2.44% per annum. The lending rate R_t^b follows the same path, since the intermediation cost does not change. For what concerns the housing market, there are two opposite forces at play. On the one hand, the constrained borrowers sell the size of housing that they can no longer afford - hence their demand for h is lower. On the other hand, the demand for housing from lenders and unconstrained agents is still high, as they are not affected (or less affected) by the drop in ϕ, and the interest rates are now lower. Since constrained agents owned a relatively small share of total housing in the economy, the increased availability of dwellings is not enough to satisfy the increased demand stemming from lenders and unconstrained borrowers. Therefore, the price of housing increases over time, until convergence to its new equilibrium value (Figure 5.8c).

For what concerns the real economy, the shock yields a positive effect on aggregate income, both in the short and in the long run. Figure (5.8e) shows that, as the shock hits, output increases by 1.2% in just one quarter, and then it slowly converges to its new

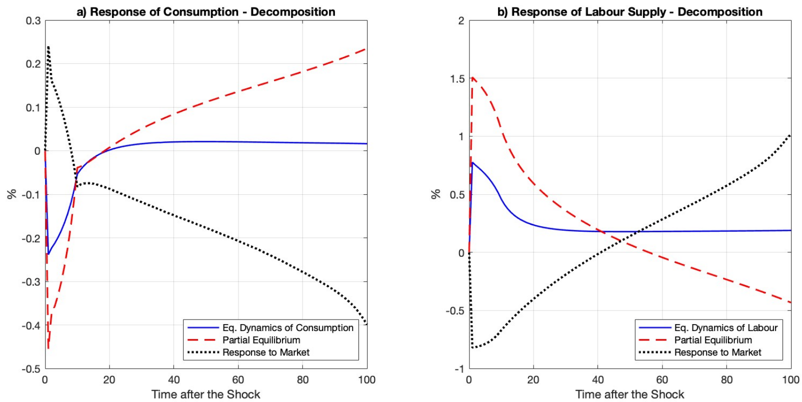

Figure 5.9. *Decomposition of the transitional dynamics of aggregate consumption (left) and labour supply (right) into the partial equilibrium channel (dashed line) and the response-to-market-variables channel (dotted line). The equilibrium transition path is represented by the solid line.*

equilibrium value, which is 0.4% higher than the original one. Such dynamics of Y depend on how the crunch affects aggregate consumption and labour supply: according to Figure (5.8f), consumption responds negatively to the drop in ϕ_b, while the overall effect on N is positive. As in the baseline setup, the crunch reduces available resources for households, especially for borrowers. To compensate for such loss, agents affected by the crunch supply additional hours of work to the economy: as a result, labour supply increases by 1.5% following the shock, while its new long-term value is 0.55% higher than in the initial equilibrium. At the same time, since resources are scarcer, standard consumption drops immediately after the shock (by circa 0.5%), and it converges to a new value which is slightly lower than the one in the original steady state. Figure (5.9) disentangle the partial equilibrium and the market channels of propagation of the shock: as in the previous setups, the response of aggregate consumption and labour supply is primarily determined by the partial equilibrium channel, while the movements in interest rates help mitigate it. In contrast with the previous setups, the positive effect of the drop in ϕ_b is stronger on labour than the contemporaneous negative effect on consumption, thus explaining why output increases after the shock.

5.4. Intermediation Shock

In this section, I consider that the economy, at the original stationary equilibrium of Section 5.2, is hit by an MIT shock to the transaction costs ε. More specifically, I assume that transaction costs rise up to 1.65%, on an annual basis, a value that is consistent with

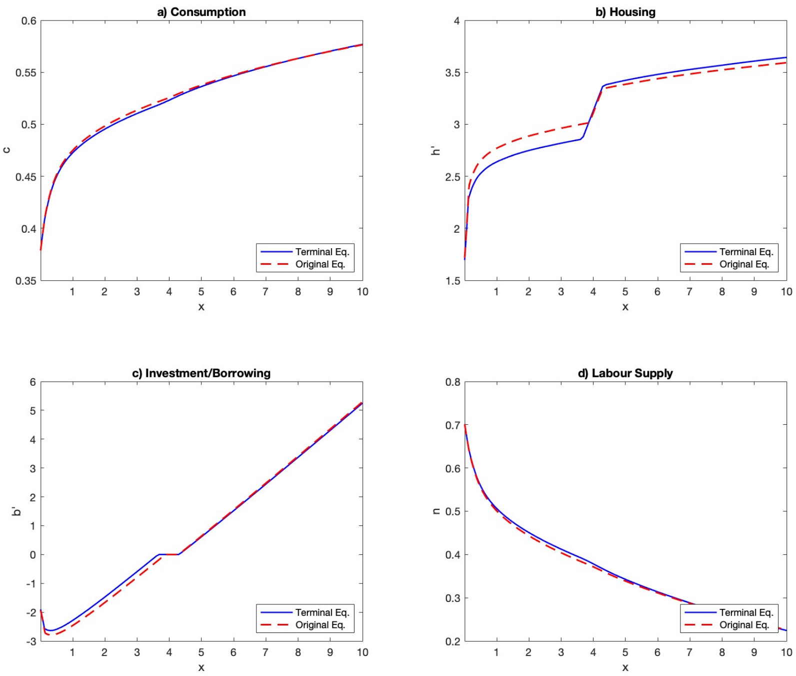

Figure 5.10. *Equilibrium policy functions after (solid lines) and before (dashed lines) an MIT shock to the intermediation costs ε, for $w = w^3$ - Model with Housing.*

a decrease of 5% in the annual debt-to-GDP ratio, which, once again, drops to 23% at the new equilibrium. Unlike the previous experiment, I assume that ε adjusts to its new value immediately, and not after s quarters. In fact, provided that the new final value for the transaction cost is not too big, under the current calibration, agents are not forced to default if ε adjusts to its new value only within one period. I consider the shock to be permanent.

5.4.1. New Stationary Equilibrium

Figure (5.10) compares the household's policy functions at the new equilibrium with those at the original equilibrium, for a given productivity level and a given initial level of housing. The difference in consumption and labour supply is minimal, across the two equilibria: borrower's consumption and labour supply seem to remain very close to the original equilibrium, while, for what concerns the lenders, they seem to increase standard consumption but reduce labour supply. Unlike the previous case, the intermediation

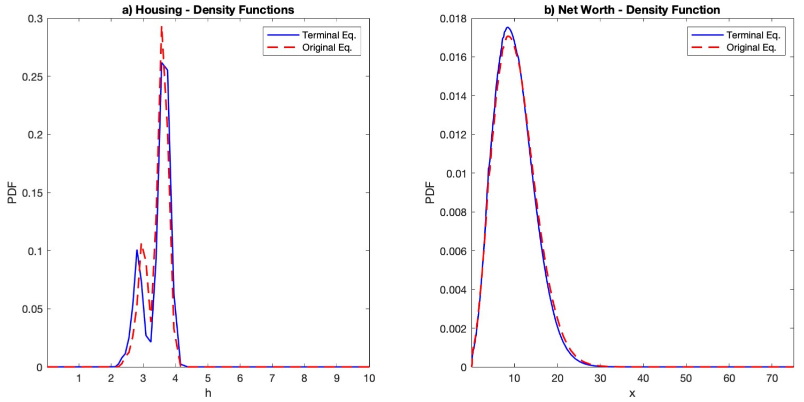

Figure 5.11. *Equilibrium housing and net worth distributions after (solid lines) and before (dashed lines) an MIT shock to the intermediation costs ε, for $w = w^3$ - Model with Housing.*

shock seem to affect all the borrowers at the same level. The policy function for b' sensibly moves towards the left when $b' < 0$: even those agents that are not close to the borrowing limit need to reduce their size of debt following the increased cost of debt. As a matter of fact, at the new equilibrium the net borrowing rate rises from 3.5% to 4.1%. Furthermore, The interval of net worth x for which agents prefer to be strictly hand-to-mouth than borrower increases. Furthermore, Figure (5.11b) shows that all the borrowers sensibly reduce their investment in housing, and not those who are closer to the borrowing limit as in Section 5.3. For what concerns lenders, we no longer observe the portfolio reallocation effect that characterized the credit crunch. At the new equilibrium lenders increase both their investments in housing and bonds, despite the net real interest rate of the economy is now lower and equal to 2.4% per annum. The reason lies in the fact that the house price has dropped as well, by 0.4% compared to the original equilibrium. Therefore, lenders still find it profitable to use bonds as a precautionary saving tool, instead of housing. In terms of asset distributions, the increased transaction costs generate further inequality across households, similarly to credit crunch. The higher ε widens the gap between borrowers and lenders in terms of housing owning (Figure 5.11). However, the difference in total wealth, across the two equilibria, is very small.

5.4.2. Transitional Dynamics

Figure (5.12) displays the transitional dynamics of the aggregate economy after the shock. The rise in ε widens the wedge between borrowing and lending rate, although it does not necessarily translate in a mere increase of R^b, while R remains unchanged. As in the credit crunch case, the increase in ε drives the demand for loans down, only this time evenly across

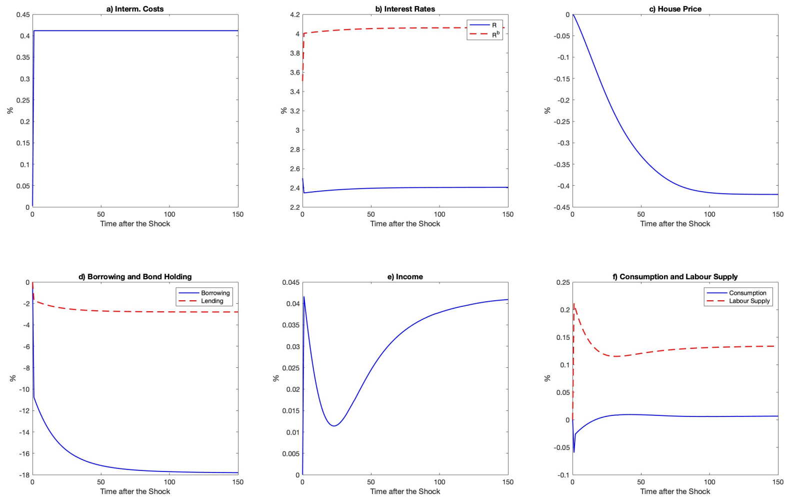

Figure 5.12. *Transition Dynamics after an MIT shock to the intermediation costs ε - Model with Housing.*

the borrowers. Additionally lenders are more willing to invest in bonds. The deleveraging of borrowers and the higher demand for bonds generate an immediate drop in the interest rate, and its consequent adjust at a lower equilibrium level. Similarly, the rise in costs reduce the demand for housing from the borrowers, although lenders are now more willing to purchase. However, the combined effect on the housing market is negative, so the drop in demand makes the house price q_t reduce over the quarters following the shock, until convergence to the new lower level of 1.195 at the terminal equilibrium. Unsurprisingly, aggregate borrowing and lending suffer a consistent drop after the shock too. Similarly to the credit crunch case, the rise in transaction costs yields a negative effect on consumption and a positive effect on labour supply, albeit the final effect on output is negligible. On the one hand, a higher ε reduces available resources for borrowing households, forcing them to work more and reduce their standard consumption and investment in housing. On the other hand, the lower real interest rate mitigate the impact of a higher ε on borrower's expenses: R^b does not increase proportionally with the intermediation costs, as credit market equilibrium dynamics make R decrease, as the shock hits. Therefore, households do not bear the full impact of the higher borrowing costs. Lower house prices also facilitate consumption. As a result, aggregate consumption slightly decreases in the short run, but in the long term it grows back to its pre-shock level.

Conclusions

In this dissertation, I have analyzed how financial frictions and credit constraints affect the capacity of households to invest and borrow on asset markets. In Chapter 1, I described and analyzed in detail the characteristics of a baseline one-asset model, developed in the well-known investigation of Guerrieri and Lorenzoni (2017), and I explained the role of the precautionary motive behind household financial decisions. In Chapter 2, I replicated the numerical experiment in Guerrieri and Lorenzoni (2017) by considering the effect of a credit crunch, i.e. a reduction to the exogeneous borrowing limit, on the economy. The results show that the reduced availability of resources pushed constrained agents to save even more against an adverse income shock, while wealthy agents reduce their asset holdings due to the drop in interest rates following the shock, and prefer to consume more. However, the overall effect of the shock is a systematic reduction of aggregate consumption, and an increase in labour supply, due to the adverse effect of the shock on low-wealth households' available resources. In Chapter 3, I also analysed how the aforementioned results change when the income process is calibrated using different parametrization: a higher (lower) persistence or innovation variance for the wage process exacerbates (alleviates) the effect of the credit crunch shock both in the short and long term. In Chapter 4, I extended the baseline framework to an economy characterized by intermediation costs, which raise the cost of debt. I further show that, under the same calibration as above, a credit crunch yields a worse effect on aggregate variables and cross-sectional distribution, because the presence of transaction costs further reduce borrowing possibilities and the availability of resources. However an increase in transaction costs may have a slightly positive effect on aggregate output (albeit it still yields a negative effect on consumption. On the one hand, the shock pushes agents to supply additional labour to make up for the drop in financial resources. On the other hand, the increased income is wasted in order to cover the increased transaction costs. In Chapter 5, I present a two-asset economy, with liquid bonds and an illiquid asset (housing), both supplied in a fixed amount, and I analyse households portfolio dynamics following a tightening of financial conditions. Agents are subject to a collateral constraint if they want to borrow on the market. At equilibrium, agents are segmented into borrowers, who purchase housing with debt, lenders, who invest both in housing and bonds, and strictly hand-to-mouth individuals, which buy housing and consumption goods with their own resources but have zero bond holdings ($b' = 0$). At equilibrium, housing capital is concentrated in the hands of the lenders, while, due to the presence of intermediation costs, borrowers own a smaller fraction. In terms of net-worth levels, results show that individuals with lower initial wealth, on average, tend to invest more in housing than their wealthy counterparties, and, hence, the majority of housing capital in this economy is concentrated among the bottom 28% of households in terms of net worth. The tightening of financial markets, in terms of a credit crunch or a rise in transaction costs, induces a portfolio reallocation across households: low-wealth agents sell housing and purchase liquid bonds in an attempt to increase their precautionary savings against future adverse income shocks, while, following a drop in the interest rates, high-wealth agents sell bonds and buy more housing, to look for more profitable investment opportunities. Thus, worse financial conditions increase wealth inequality across households, as housing becomes more concentrated in the hands of 'richer' agents. While the analysis provides a clear picture of how financial shocks affect the aggregate economy and individuals households, it ignores some pivotal dimensions connected to loans and housing. As argued above, the possibility of defaults is ignored by the model and hence the investigation disregards the effect of credit risk on individual access to credit and financial dynamics. A promising area for future research would be to include the risk dimension in the analysis, and to study how investment and borrowing decisions are formed when agents face financial uncertainty other than income risk. Additionally, the housing model could be generalized to a version in which agents can either buy houses or rent them, to understand how market frictions and financial risks may prevent agents from becoming homeowners.

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Appendix A

Mathematical Solutions

A.1. Solution of the Baseline Model

Using recursive notation, the household's problem can be written in the following way:

$$V(w, b) = \max_{b', b', n} u\left(wn - z + b - pb', n\right) + \beta \mathbb{E}\left[V(w', b')|w\right] \tag{A.1}$$

s.t.

$$b' \geq -\phi \tag{A.2}$$

$$c \geq 0 \tag{A.3}$$

$$n \geq 0 \tag{A.4}$$

$$n \leq 1 \tag{A.5}$$

In equation (A.1) I substitute c with the right-hand side of the budget constraint (1.3). Because of resource optimization, at any stationary equilibrium, household's budget constraint is satisfied with equality. Therefore, it is possible to operate the substitution above without loss of generality. Note that, under Assumption (1), there cannot be an equilibrium with $c = 0$ and $n = 1$, as any other feasible value for c and n would yield higher utility. Therefore, at equilibrium, constraints (A.3) and (A.5) are satisfied with the inequality sign and the corresponding Lagrange multipliers will be equal to zero. Therefore, let us denote with λ and μ, respectively, the Lagrange multipliers associated with the borrowing constraint (A.2) and the constraint (A.4), which are only occasionally binding. The necessary first order conditions are therefore given by:

$$[b']: \qquad \frac{\partial}{\partial c}u(c, n) \cdot \frac{\partial}{\partial b'}c + \beta \mathbb{E}\left[\frac{\partial}{\partial b'}V(w', b')\right] + \lambda = 0$$

$$[n]: \qquad \frac{\partial}{\partial c}u(c, n) \cdot \frac{\partial}{\partial n}c + \frac{\partial}{\partial n}u(c, n) + \mu = 0$$

Because of the Envelope Theorem, the following envelope condition holds for b:

$$\frac{\partial}{\partial b}V(w,b) = \frac{\partial}{\partial c}u(c,n) \tag{A.6}$$

Given equation (A.6), and given that:

$$\frac{\partial}{\partial b}c = 1, \qquad \frac{\partial}{\partial b'}c = -p, \qquad \frac{\partial}{\partial n}c = w$$

from the budget constraint, it is possible to re-write the first order conditions in the following way:

$$[b']: \qquad \frac{\partial}{\partial c}u(c,n) = \frac{1}{p}\left\{\beta\mathbb{E}\left[\frac{\partial}{\partial c'}u(c',n')\right] + \lambda\right\} \tag{A.7}$$

$$[n]: \qquad w\frac{\partial}{\partial c}u(c,n) = -\frac{\partial}{\partial n}u(c,n) - \mu \tag{A.8}$$

which, together with the budget constraint, satisfied with equality, complete the equilibrium condition in the baseline economy:

$$c = wn - z + b - pb' \tag{A.9}$$

Since the system of constraints in the household problem describe a compact set, the first order conditions are necessary and sufficient to identify a stationary recursive equilibrium.

A.2. Solution of the Model with Housing

Using the definition of net worth in equation (5.6) and recursive notation, we can write the household's problem in the following way:

$$V(w,h,x) = \max_{h',b',n} u\left(wn - z + x - p^* b' - qh', h, n\right) + \beta\mathbb{E}\left[V(w',h',x')|w\right] \tag{A.10}$$

s.t.

$$x' \geq (1-\phi_h)(1-\kappa)qh' \tag{A.11}$$

$$x' = b' + (1-\kappa)qh' \tag{A.12}$$

$$h' \geq \underline{h} \tag{A.13}$$

$$c \geq 0 \tag{A.14}$$

$$n \geq 0 \tag{A.15}$$

$$n \leq 1 \tag{A.16}$$

where p^* could be the inverse of R or R^b, depending on whether the individual is a lender or a borrower. As for the baseline models constraints (A.14) and (A.16) are never binding, while constraints (A.11), (A.13) and (A.15) are occasionally binding. Let us denote with λ, μ_h and μ_n, respectively, the Lagrange multipliers associated to the three occasionally binding constraints. The necessary first order conditions are the following:

$$[b'] : \quad \frac{\partial}{\partial c}u(c,h,n)\cdot\frac{\partial}{\partial b'}c+\beta\mathbb{E}\left[\frac{\partial}{\partial x'}V(w',h',x')\cdot\frac{\partial}{\partial b'}x'\right]+\lambda\frac{\partial}{\partial b'}x'=0$$

$$[h'] : \quad \frac{\partial}{\partial c}u(c,h,n)\cdot\frac{\partial}{\partial h'}c+\beta\mathbb{E}\left[\frac{\partial}{\partial x'}V(w',h',x')\cdot\frac{\partial}{\partial h'}x'+\frac{\partial}{\partial h'}V(w',h',x')\right]$$
$$+\lambda\left[\frac{\partial}{\partial b'}x'-(1-\phi_h)(1-\kappa)q\right]+\mu_h=0$$

$$[n] : \quad \frac{\partial}{\partial c}u(c,h,n)\cdot\frac{\partial}{\partial n}c+\frac{\partial}{\partial n}u(c,h,n)+\mu_n=0$$

The following envelope conditions hold, respectively, for x and h:

$$\frac{\partial}{\partial x}V(w,h,x)=\frac{\partial}{\partial c}u(c,h,n)$$
$$\frac{\partial}{\partial h}V(w,h,x)=\frac{\partial}{\partial h}u(c,h,n)$$

Since:

$$\frac{\partial}{\partial x}c=1, \qquad \frac{\partial}{\partial b'}c=-p^*, \qquad \frac{\partial}{\partial n}c=w$$
$$\frac{\partial}{\partial b'}x'=1, \qquad \frac{\partial}{\partial h'}c=-q, \qquad \frac{\partial}{\partial h'}x'=(1-\kappa)q$$

we can re-write the first order conditions in the following way:

$$[b'] : \quad \frac{\partial}{\partial c}u(c,h,n)=\frac{1}{p^*}\left\{\beta\mathbb{E}\left[\frac{\partial}{\partial c'}u(c',h',n')\right]+\lambda x'\right\} \tag{A.17}$$

$$[h'] : \quad q\frac{\partial}{\partial c}u(c,h,n)=\beta\mathbb{E}\left[q(1-\kappa)\frac{\partial}{\partial c'}u(c',h',n')+\frac{\partial}{\partial h'}u(c',h',n')\right] \tag{A.18}$$
$$+\lambda\left[1-(1-\phi_h)(1-\kappa)q\right]+\mu_h \tag{A.19}$$

$$[n] : \quad w\frac{\partial}{\partial c}u(c,h,n)=-\frac{\partial}{\partial n}u(c,h,n)-\mu_n \tag{A.20}$$

The first order conditions above, together with:

$$c=wn-z+x-p^*b-qh'$$
$$x'=b'+(1-\kappa)qh'$$

describe the stationary equilibrium in this economy, since the set describe by the constraints (A.11)-(A.16) is compact.

Appendix B

Alternative Calibrations

B.1. Alternative Income Processes for the Baseline Model

In this section, I explore how the results analysed in Chapters 1 and 2 change when using different underlying income processes, their parametrization, and the chosen method for the discretization into finite states. Following the original paper of Guerrieri and Lorenzoni (2017), I postulated a Gaussian AR(1) process for productivity (3) Assumption (2). Then, as in the paper, I set $\mu = 0, \rho = 0.967$ and $\sigma^2 = 0.017$ as, respectively, the unconditional mean, persistence and error variance of the process. For the discretization, I employed Tauchen's (1986) method, which is used by the original paper, and widely adopted by the standard macroeconomic literature. However, more recent analyses shed some doubt on the reliability of Gaussuian AR(1) processes and Tauchen's method for the modelling realistic income and wage processes. For example, Kopecky and Suen (2010) show that, Tauchen's method performs poorly for the approximation of highly persistent stationary AR(1) processes (i.e. with $|\rho| \to 1$, like the one in Guerrieri and Lorenzoni, 2017), when compared to alternative procedures. Kopecky and Suen (2010) suggest that the best performing approximation method in such cases is the one formulated by Rouwenhorst (1995). Such claims is also corroborated by the numerical simulations run by Galindev and Lkhagvasuren (2010).

Furthermore, Guvenen et al. (2021), using using panel data on income and labour-market-related information of U.S. workers[1] show that (log) income distribution across individuals exhibits consistent non-normalities, such as negative skewness, and positive and high excess kurtosis. Additionally, they show that the moments of the distribution and the persistence of the process vary sensibly, depending on the worker's age and earning level, and that income is affected by both persistent and transitory shocks. Therefore, the

[1] More specifically, the authors use U.S. Social Security Administration (SSA) and Panel Study of Income Dynamics (PSID) data

authors employ a AR(1) process with normal mixture innovations (with both persistent and transitory components) to calibrate an income process that fits the observed empirical evidence[2]. Such findings are also confirmed by Arellano et al. (2017), who find that income shock persistence evolves nonlinearly with workers' age, and it has a pivotal rule in shaping agent's consumption decisions. The growing body of research over life-cycle macroeconomic models has incorporated the age and earning dependence of income processes when

B.1.1. Tauchen (1986) vs. Rouwenhorst (1995)

Several authors, including Galindev and Lkhagvasuren (2010) and Kopecky and Suen (2010), have showed that Rouwenhorst's (1995) method produces better approximations of stationary Gaussian AR(1) processes than Tauchen's (1986) one, when $|\rho| \rightarrow 1$. Since I set $\rho = 0.967$ in the model calibration, I will now replicate the analysis above by discretizing the productivity process for w using Rouwenhorst's (1995) method. I will first briefly explain the main differences between the two approaches. Let us consider the (covariance-stationary) AR(1) process

$$y_t = \rho y_{t-1} + \varepsilon_t \tag{B.1}$$

with $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$, and unconditional mean $\mu = 0$. The variance of the process σ_y^2 is given by $\sigma_\varepsilon^2 (1-\rho)^{-2}$.

According to Tauchen's (1986) method, the state space is discretized in $n \geq 2$ evenly-spaced states $y_1, ..., y_n$, so that $y_n = -y_1 = m\sigma_y$, where m is a positive real number, arbitrarily set by the researcher. For any given two states $i, j \in 1, ..., n$, the transition probability from i to j in the Markov chain is given by:

$$\pi_{i,j} = \begin{cases} \Phi\left(\frac{y_j - \rho y_i + 0.5\iota}{\sigma_\varepsilon}\right) & \text{if } j = 1 \\ \Phi\left(\frac{y_j - \rho y_i + 0.5\iota}{\sigma_\varepsilon}\right) - \Phi\left(\frac{y_j - \rho y_i - 0.5\iota}{\sigma_\varepsilon}\right) & \text{if } j = 2, ..., n-1 \\ 1 - \Phi\left(\frac{y_j - \rho y_i - 0.5\iota}{\sigma_\varepsilon}\right) & \text{if } j = n \end{cases} \tag{B.2}$$

where Φ represents the standard normal cumulative distribution function and ι is the constant distance between two consecutive states. In order to obtain the best possible approximation using this method, one should select m so that the variance of the discretized process closely matches the variance of the original AR(1) process. Kopecky and Suen (2010) state that such arbitrary choice is one of the biggest drawbacks of this approach. In Guerrieri and Lorenzoni (2017), the authors set $m \simeq 2.07$.

Like the previous one, the method of Rouwenhorst (1995) requires that the state space is discretized in $n \geq 2$ points that are equally distanced on a real-valued grid: $y_1, ..., y_n$,

[2] For a thorough description and analysis of normal mixture innovations autoregressive processes (NMAR) see Civale et al. (2017).

Method	w^1	w^3	w^5	w^8	w^{10}	w^{12}
Tauchen (1986)	0.3463	0.5093	0.7489	1.3353	1.9635	2.8873
Rouwenhorst (1995)	0.1832	0.3396	0.6295	1.5887	2.9450	5.4593

Table B.1. *Discretized state-space for productivity levels w using different approximation methods.*

so that $y_n = -y_1 = \sqrt{(n-1)\sigma_y^2}$. Then the researcher should select two probabilities $p, q \in]0,1[$, more specifically:

$$p = q = \frac{1-\rho}{2}$$

so that the transition matrix, when $n = 2$, could be written as:

$$\Pi_2 = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix} \tag{B.3}$$

Then,for any other state space size $n \geq 3$, one could compute the transition matrix using the following formula:

$$\Pi_n = p\begin{bmatrix} \Pi_{n-1} & \mathbf{0} \\ \mathbf{0}' & 0 \end{bmatrix} + (1-p)\begin{bmatrix} \mathbf{0} & \Pi_{n-1} \\ 0 & \mathbf{0}' \end{bmatrix} + (1-q)\begin{bmatrix} \mathbf{0}' & 0 \\ \Pi_{n-1} & \mathbf{0} \end{bmatrix} + q\begin{bmatrix} 0 & \mathbf{0}' \\ \mathbf{0} & \Pi_{n-1} \end{bmatrix} \tag{B.4}$$

Kopecky and Suen (2010) show that the invariant distribution associated with the Markov chain thus discretized is binomial, with parameters $n-1$ and $1-s$, with $s \equiv (1-q)[2-(p+q)]^{-1}$.

The discretization of the wage process in equation in (1.4), under the calibration set out in Table (1.2), using Tauchen's method with $m = 2.07$ and with $n = 12$ gridpoints leads to the interval of productivity values set out in the first row of Table (B.1). The discretization of the same process with Rouwenhorst's method leads to the values in the second row. Note that the amplitude of the discretized space under Rouwenhorst's method is higher than the corresponding one obtained with Tauchen's one. In fact, if σ_w is the variance of the process for $\ln(w_t)$ and w^1 and w^n are, respectively, the first and the last state on the grid, we have that:

$$\ln(w_{\mathrm{R}}^n) = -\ln(w_{\mathrm{R}}^1) = \sqrt{(n-1)\sigma_w^2} \simeq 3.32\sigma_w > 2.07\sigma_w = \ln(w_{\mathrm{T}}^n) = -\ln(w_{\mathrm{T}}^1)$$

Therefore, productivity states that are below (above) the average are smaller (bigger) when Rouwenhorst's method is used. This generates several implications for both the equilibrium results and the post-credit-shock transitional dynamics. Figure (B.1) compares stationary equilibrium results, in terms of policy functions and stationary asset distributions, across the two discretization methods. For $w < 1$ ($w > 1$) policy functions

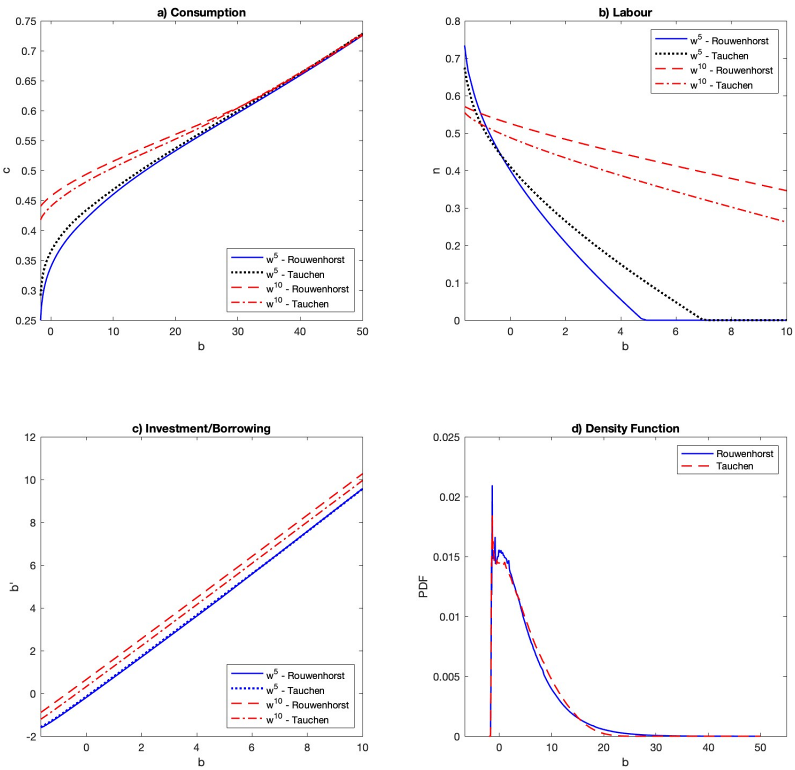

Figure B.1. *Comparison among equilibrium results in terms of policy functions and stationary asset distributions, when the wage process is discretized following Rouwenhorst's method and Tauchen's method*

for consumption and b' are strictly lower (greater) than those computed with Tauchen-discretized productivity levels. Due to the wage states being smaller under Rouwenhorst, the income effect is now stronger for low productivity states, when net worth b is small enough: agents prefer to supply additional labour to enjoy more consumption today. At the same time, substitution effect is also larger under Rouwenhorst for large values of b. In terms of asset distribution, the 'wider' grid for productivity generates additional inequality in terms of asset holdings: the share of individuals who borrow, and those who are borrowing constrained is greater than in the Tauchen's case, while larger wage states (when $w > 1$) lead to an increased bond holding for high-productivity agents.

Figure (B.2) displays the transition dynamics of the aggregate economy, following the credit crunch shock described in Chapter 2, when the wage process is discretized using

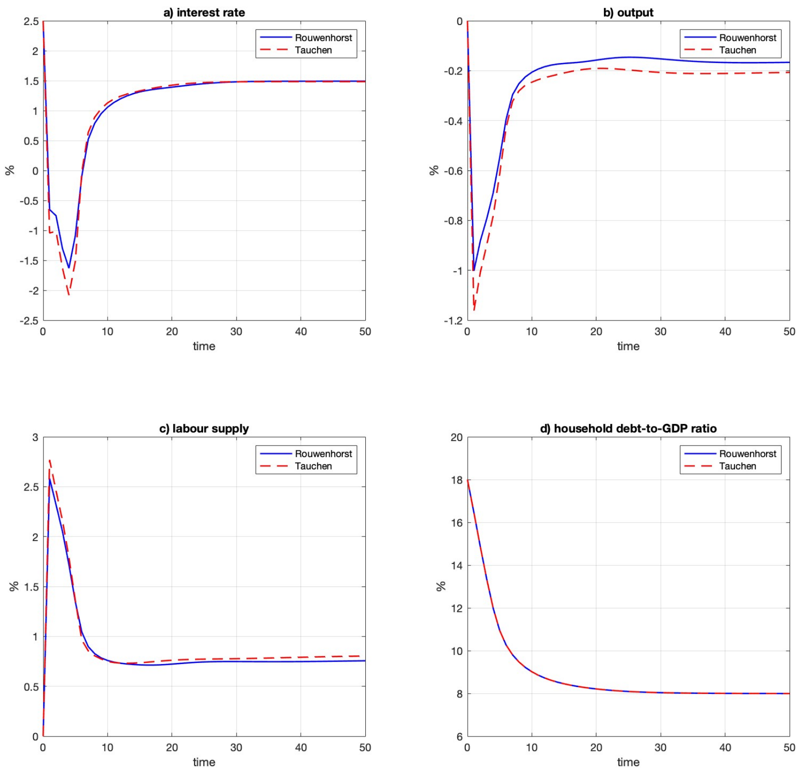

Figure B.2. *Transitional dynamics following the credit crunch shock analysed in Chapter 2: comparison between the two cases when the wage process is discretized accorideing to Rouwenhorst method (solid line) and via Tauchen's method (dashed line).*

Rouwenhorst's method (solid line) and Tauchen's method (dashed line). Although the dynamics of the economy do not differ greatly between the two cases under scrutiny, and that the new long-term equilibrium values are very close, it can be noticed that, in the Rouwenhorst case, the short-term effect of the shock is slightly smaller. This is due to the lower productivity levels in the Rouwenhorst economy, when $w < 1$, which make individuals more dependent on credit - so that the share of agents who are borrowing constrained is still greater than in the Tauchen's case.

B.1.2. Age-dependent processes

Recent empirical studies have shown that household income variability depend on the individuals' age. Among these, the analyses in Arellano et al. (2017), De Nardi et al. (2020) and Guvenen et al. (2021) all support the concept that the persistence and the distribution of income shocks evolve non-linearly with worker's age. De Nardi et al. (2020) then construct a life-cycle macroeconomic setup and show that introducing age-dependent income processes allows for a better modelling of consumption paths over the life cycle,cross-sectional consumption and wealth inequality, and how individuals self-insure against income risk. Unfortunately, the Bewley-type model used in this analysis does not support any life-cycle component, as it assumes that individuals live for an infinite amount of time, and that the size of the economy does not change (e.g. there are no overlapping generations of individuals). However, to internalise some of the age-specific features advanced by the aforementioned papers, I will re-run my analysis performing different numerical experiment, and in each of them I will assume that all the infinitely-lived agents in the economy find themselves at a different moments of their life cycle, and face moment-specific income processes.

In their empirical analysis, Karahan and Ozkan (2013) estimate how the persistence and the variance of both permanent and transitory shocks varies with age. They divide their sample of workers in three age cohorts: the first one groups people aged between 24 and 33, the second individuals aged between 34 and 52, the third one individuals those who are close to the retirement age, between 53 and 60 years old. Estimates show that 'young' workers' permanent income process displays the lowest persistence and the highest variance, among the three groups considered. The 'middle age' one, on the other hand, shows the highest persistence and the smallest variance. The cohort of the 'old' individuals lies in the middle, with a persistence value that is close to the one of the middle aged individuals, but high variance as for the young workers. For the following numerical exercise, I replicate the analyses in Chapters 1 and 2, by assuming that all the agents are 'young', i.e. characterized by a permanent income process with lower persistence and sensibly higher variance, with respect to the ones calibrated in Section 1.2.2[3]. In the second numerical experiment, I will postulate that the agents are 'old', with slightly lower income persistence compared to the standard calibration, but higher variance.

Results of these experiments are reported in Figures (B.3) and (B.4). To perform such analysis I do not merely use the estimated values in Karahan and Ozkan (2013) because they refer to income processes rather than a wage process, such as the one considered here, and persistence values lower than 0.9 may clash with the remaining calibration. For the 'young' case I impose that $\rho = 0.936$ and $\sigma_\varepsilon^2 = 0.029$. For the 'old' case, I set $\rho = 0.955$ and $\sigma_\varepsilon^2 = 0.024$. The 'middle-aged' case corresponds to the standard calibration targets

[3]Since this is not a life-cycle set-up, I ignore any transitory income effect identified by Karahan and Ozkan (2013), and, as in the previous section, focus on the permanent income process.

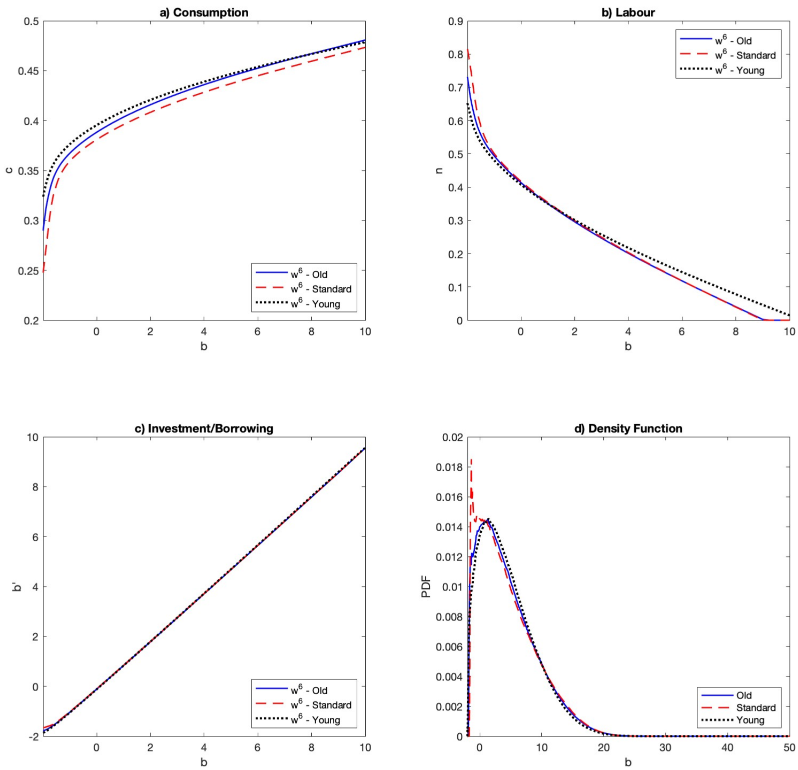

Figure B.3. *Policy functions and stationary distributions at the stationary equilibrium (for a given level of wages $w = w^6$) for the three cases in which the economy follows the standard calibration (dashed line) of the wage process, the 'young' calibration (dotted line), and the 'old' calibration (solid line).*

of Section 1.2.2. To ensure comparability with the standard calibration, the processed are discretized with Tauchen's (1986) method. Despite the higher variance, the lower persistence contributes to the discretization of wage states that are less dispersed around the unitary mean. Therefore all the wage states that are less than 1, in the young and old calibrations, are higher than the corresponding ones in the standard calibration. Therefore, policy functions show higher consumption levels for the 'young' and the 'old', and lower labour supply when the initial b is low. At the same time, the share of individuals that are close to the borrowing constraint is smaller both in the 'young' and in the 'old' case. For what concerns the response of the economy to a credit crunch, when the persistence of the wage process is lower, the short-term effect of the shock is reduced when the persistence is

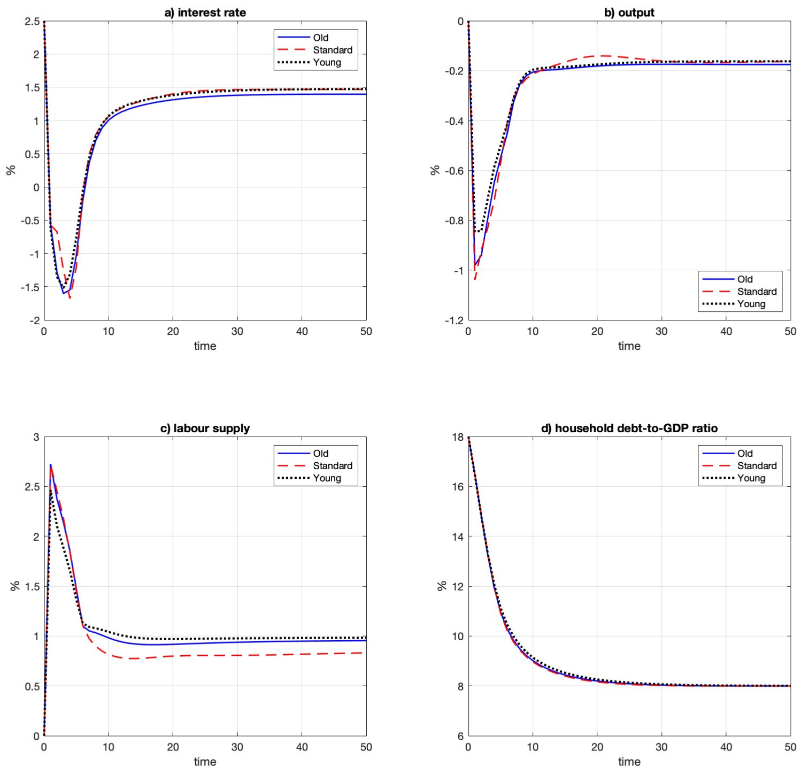

Figure B.4. *Transitional dynamics to a credit crunch shock, for the three cases in which the economy follows the standard calibration (dashed line) of the wage process, the 'young' calibration (dotted line), and the 'old' calibration (solid line).*

lower. For example, Figure (B.4) shows that the short-term drop in Y for the 'young' case is circa 2/3 the size of the corresponding one in the standard case. The new long-term values for the aggregate economy, however, are very close to the standard case.

B.1.3. Non-normalities

Although the mainstream macroeconomic literature treats income processes as perfectly Gaussian and focus on calibrating only the first two moments of the distributions, recent analyses from Gospodinov and Lkhagvasuren (2014), Civale et al. (2017) and Guvenen et al. (2021) have shown that such processes are usually marked by negative skewness and positive kurtosis. Therefore, In this section, I will examine how the previous result change

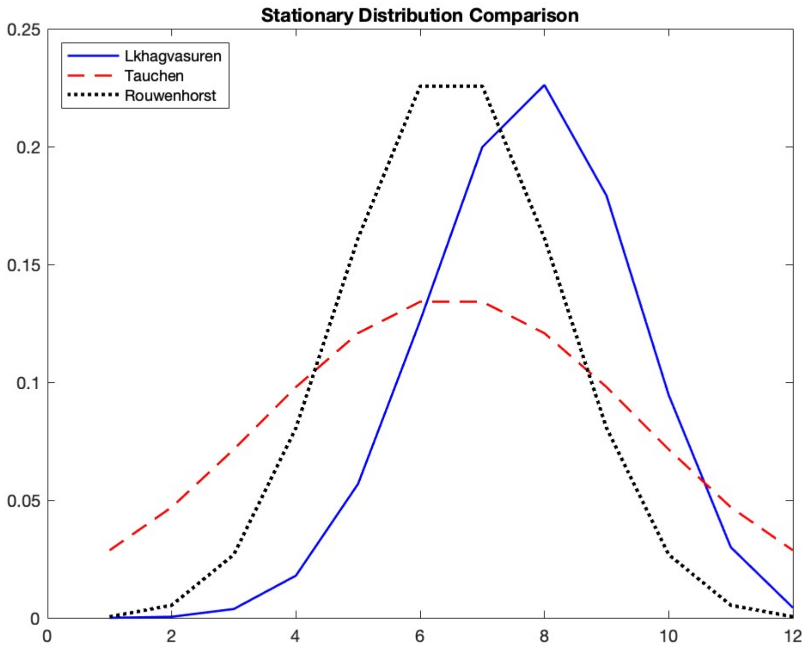

Figure B.5. *Comparison among the invariant wage distributions under the three different modelling methods: Tauchen (dashed line), Rouwenhorst (dotted line), Lkhagvasuren (solid line).*

when the assumption of perfect normality is relaxed. To model and discretize a non-normal processes I use the procedure developed by Lkhagvasuren (2023), which consists of a generalization of the method of Rouwenhorst (1995). The researcher constructs a binomial Markov chain by targeting the higher moments of the distribution of the process, other than the mean and variance, as in the previous sections. However, the main drawback of this method is that you can only perfectly target the skewness or the kurtosis of the process, but not both at the same time. For the numerical exercise in this section, I take into account and income process with $\rho = 0.967$ and $\sigma_{\varepsilon}^2 = 0.017$ as above, and, using the estimated values in Guvenen et al. (2021), I calibrate the skewness of the wage process to be equal to -0.14[4]. Figure (B.5) compares the distribution obtained using the new method with the ones calculated via Tauchen (1986) and Rouwenhorst (1995): even if the calibrated skewness is relatively small, and the remaining parametrization is unchanged, the difference with the normal case is sizeable.

[4]For the moment, I still consider the distribution as mesokurtic

The discretization done through the method of Lkhagvasuren (2023) produces smaller states for w, when compared to the traditional methods. The reduction is global across all the states. However, those that are lower than 1 are the ones that are most affected, while the differences in productivity levels between Tauchen's and Lkhagvasuren's methods are generally small. Therefore, at the stationary equilibrium, consumption is reduced for any value of w, although the drop for individuals with $w < 1$ is more accentuated (Figure B.6). For what concerns labour supply, under the new method, income effect is stronger than substitution effect when b is small, while the opposite holds true for greater values of b. In terms of asset distribution, the proportion of constrained agents is slightly smaller, although we can observe a higher concentration of bonds in the hands of the wealthy agents.

I consider now the case of a credit crunch shock, whose size is the same as in Chapter 2. To prevent defaults, I set that the limit ϕ adjusts to its new value after 9 quarters instead of 6 as in the standard Tauchen's case. Under the new process, wage states are smaller than in the Tauchen's case, when $w < 1$. Therefore, agents with low productivity are more affected by the shock: when the same amount of labour is provided, their income is lower under the new process than in the Tauchen's economy. Hence, for an adjustment horizon smaller than 9 quarters, defaults take place. Figure (B.7) displays the transitional dynamics following the shock, and compares them to the standard Tauchen's case[5]. When the methodology of Lkhagvasuren (2023) is applied, the effect on r_t is slightly bigger than in the standard case. Under the new framework, constrained individuals deleverage faster and accumulate additional bonds, as they are more affected by the crunch, than their counterparties in the standard framework. The effect of the shock on Y_t and N_t is not dissimilar from the standard case.

[5]To ensure the comparability of the two cases, I also solved the standard case considering an adjustment horizon of 9 quarters. As showed in the previous sections, the length of the adjustment horizon affects the short-term impact of the shock and how fast the economy converges to its new equilibrium.

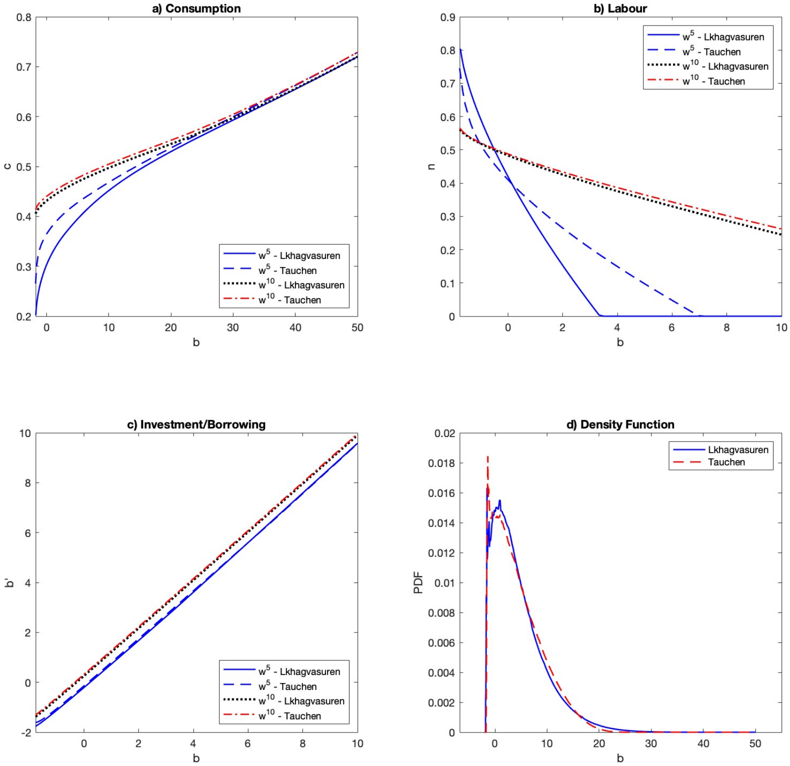

Figure B.6. *Comparison among equilibrium results in terms of policy functions and stationary asset distributions, when the wage process is discretized following Lkhagvasuren's method and Tauchen's method.*

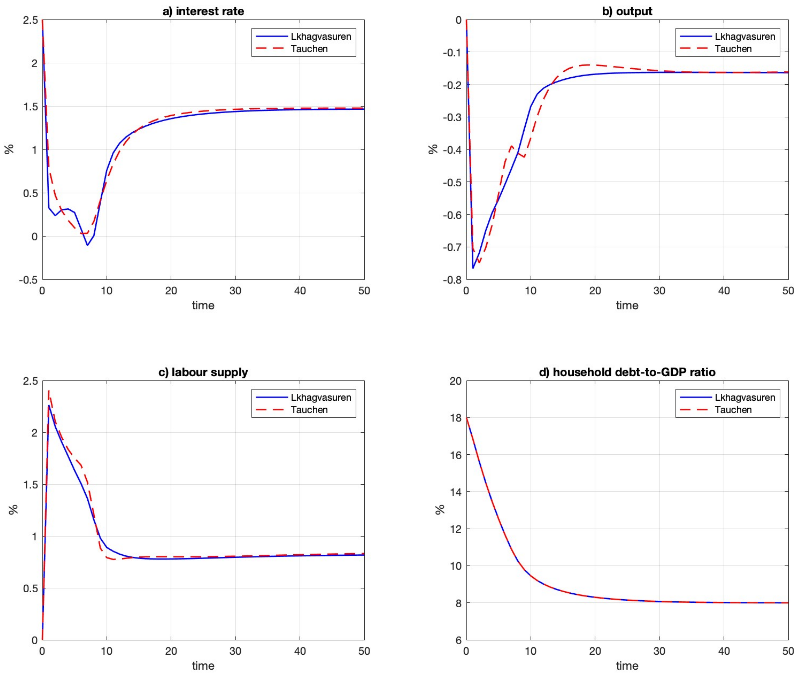

Figure B.7. *Transitional dynamics to a credit crunch shock, when the wage process is discretized following Lkhagvasuren's method (solid lines) and Tauchen's method (dashed lines)*

B.2. Alternative Calibrations of the Model with Housing

In this section, I consider variations of the housing model described in Chapter 5, by using alternative calibrations. The purpose is to study how the observed results, both at equilibrium and after the shocks, change with the selected parametrization, and what the relevant implications are.

B.2.1. Higher Disutility from Work

In Chapter 5, I specify a 40% long-term target for average employment (with respect to the total time endowment). Such target was consistent with a $\Psi = 5.9$ in the model(see Tables 5.1 and 5.2). In this section, instead, I lower the employment target to 35% of the total time endowment, which, in terms of parametrization, corresponds an almost double value for Ψ, i.e. equal to 11.32. The remaining targets imposed for this exercise remain unchanged from the ones listed in Table (5.1). The calibration that is consistent with these targets is detailed in Table(B.2). It can be seen that the optimal supply of bonds and housing in the economy, namely B and H in the table, are lower than under the standard calibration. Such difference can be explained by the fact that, under the new parametrization, individuals invest less resources in both housing and bonds, and both government and individual agents borrow less. Because of this, the equilibrium value for ϕ_h is also lower and equal to 83.64%, despite the debt-to-GDP target remains the same.

Figure (B.8) displays policy functions at the stationary equilibrium both under the new and the standard calibration - as function of net worth x and for given values of h and w. It is unsurprising that, given the higher Ψ, households supply less labour than in the standard exercise - for any initial triple (x, h, w). For a given productivity level w, a lower level of labour supply n implies lower income and, thus, less resources to cope with the expenses. Therefore, standard consumption and investment in housing are also reduced. The policy function for b' translates towards the left under the new calibration. Due to the lower investment in housing, borrowers can provide less collateral to back their loans, and borrowing possibilities are increasingly limited. At the same time, since labour income is reduced, lenders prefer to purchase additional bonds to increase their precautionary savings. In terms of distributions, Figure (B.8) shows that the distribution for housing translates towards the left, due to the lower supply of houses in the economy, hence on average households own less housing than in the standard exercise. However, the figure does not show any increased inequality across households, as the rise in Ψ seems affects all the households equally - even those who provide zero labour to the economy. Since bonds and houses are scarcer in this economy, the net worth distribution is now smaller.

Parameter	Description	Value	Target or Source
β	Rate of time preference	0.99	Int. rate = 2.5% annualized
γ	Constant Risk Aversion	4	
η	Concavity disutility from labour	1.5	Frisch elasticity = 1
Ψ	Coefficient disutility from labour	11.32	Avg. employment 35% of annual GDP
θ	Cobb-Douglas elasticity	0.79	House price target $q = 1.2$
ϕ_h	Max LTV ratio	83.64%	Target Debt-to-GDP ratio = 28%
κ	Maintenance costs for housing	1.5%	
ν	Unemp. benefits	0.147	Target = 35% of quarterly GDP
B	Bond supply	2.20	Target = 150% of annual GDP
μ	Uncond. mean wage process	0	Set mean wage $w = 1$
ρ	Persistence wage process	0.967	Flodén and Lindé (2001)
σ^2	Variance wage process	0.017	Flodén and Lindé (2001)
J	Number of wage states (unemployment excluded)	5	
π_{ue}	Prob. of finding a job	0.882	From Shimer (2005)
π_{eu}	Prob. of losing a job	0.057	From Shimer (2005)
ε	Transaction Costs	1%	Annualized Value
$\underline{h}$	Subsistence Housing	0.01	

Table B.2. *Alternative Calibration of the Stationary Equilibrium for the model with housing: employment target equal to 35% of the total time endowment.*

As in Chapter 5, I now consider a shock to the borrowing limit ϕ_h so that, at the new equilibrium, the debt-to-GDP ratio drops from 28% to 23%. The new value for ϕ_h is 65.76% . As in the previous case, I assume that the drop takes place linearly over a period of $s = 9$ quarters, to avoid a situation in which the repayment is too big for constrained households. Figure (B.10) displays the equilibrium response of the market and aggregate variables to the shock, and illustrates the comparison with the standard case. Under the alternative calibration the effect of the shock seems to be amplified, when compared to the standard case. The reduced supply of housing and bonds to the economy, together with the lower starting value for ϕ_h, make individuals more sensitive to changes to the borrowing limit. The final value for the borrowing limit is smaller (65.76% rather than 71.45%): as a result, individuals are allowed to borrow less than in the standard calibration. Therefore housing divestment and deleveraging of the constrained borrowers take place at a faster pace than in the standard calibration. The effect on the interest rate is amplified, and it falls (both in the short and long run) to a slightly lower value (Figure B.10b). At the same time, since the housing supply is lower, the the rise in housing demand stemming from the lenders makes the price increase to a higher level, compared to the standard case (Figure B.10c). Since the new value for ϕ_h is lower than under the standard calibration, he

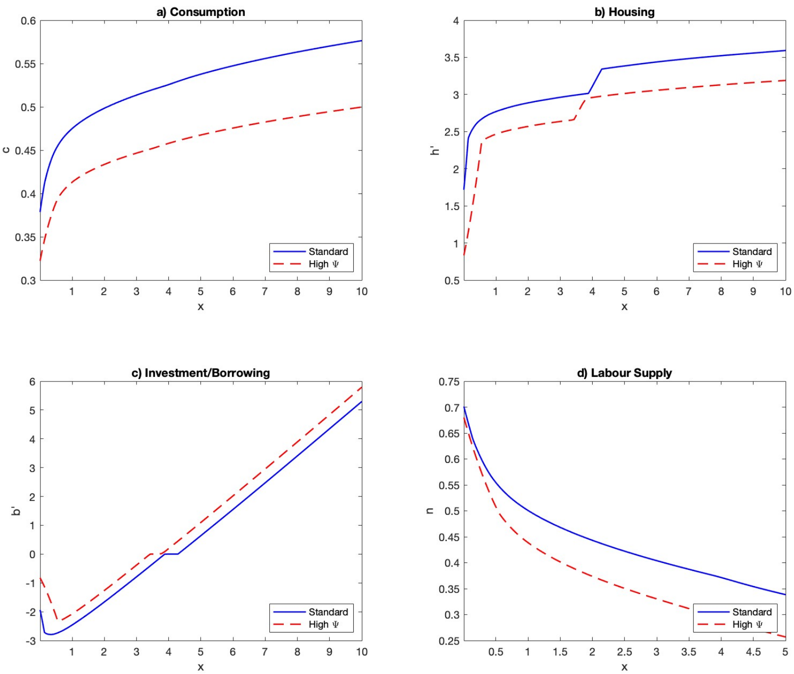

Figure B.8. *Equilibrium policy functions under the standard calibration (solid lines) and under the calibration with high Ψ (dashed lines) - for a given initial value of housing $h = 0.2790$ and $w = w^2$.*

effect of the shock on output and labour supplied is amplified, as the partial equilibrium effect keeps being stronger than the market effect. At the same time, since the fall in R is deeper, the negative impact of the shock on consumption is mitigated (Figure B.10e).

B.2.2. Response to a Credit Crunch with Low Subsistence Housing

Figure (B.11) shows the transitional dynamics of the aggregate economy to a credit crunch shock (i.e. a drop in ϕ_h), and compares two economies: the standard one with $\underline{h} = 10^{-2}$, and the one with low subsistence housing, i.e. with $\underline{h} = 10^{-46}$. While the impulse responses look similar across the two cases, it can be noticed that the effect of the shock on the market variables (q_t and r_t) is slightly reduced in the short run. As a matter of fact, Figure (5.5) shows that, in the low-$\underline{h}$ setting, individuals who are closer to the borrowing

[6]The equilibrium calibration and the size of the shock are the same used in Chapter 5.

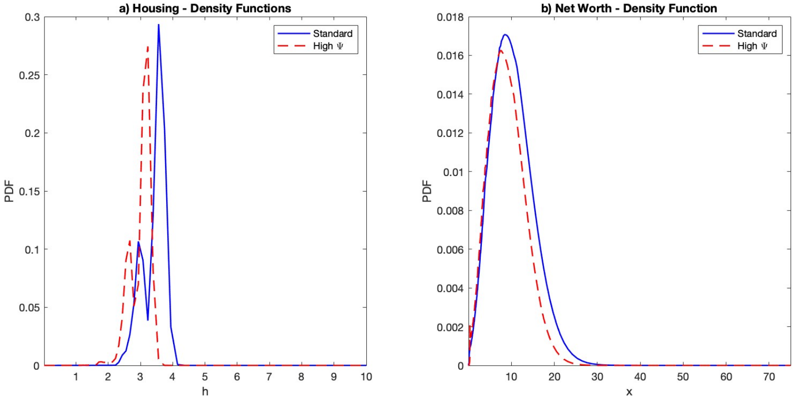

Figure B.9. *Equilibrium stationary distributions for housing and net worth under the standard calibration (solid lines) and under the calibration with high* Ψ *(dashed lines).*

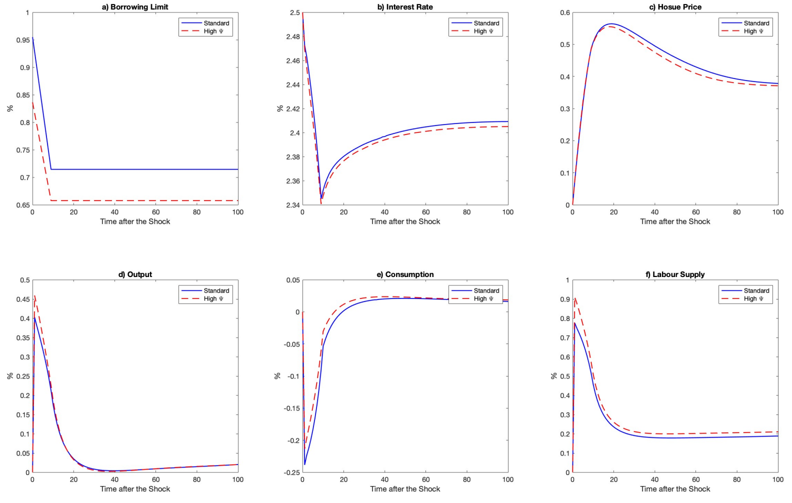

Figure B.10. *Shock to* ϕ_h*: after-shock transitional dynamics under the standard calibration (solid lines) and under the calibration with high* Ψ *(dashed lines) - Values for house prices, consumption, output and labour supply are reported as* % *deviations from the initial stationary equilibrium.*

constraint borrow less and purchase less housing (for the same initial triple (w, x, h)), when compared to their counterparties in the standard economy. Therefore, the short-term deleveraging of borrowers, and the increase in house ownership of the lenders are not as sharp as in the standard case. In terms of real aggregate variables, the effect of the

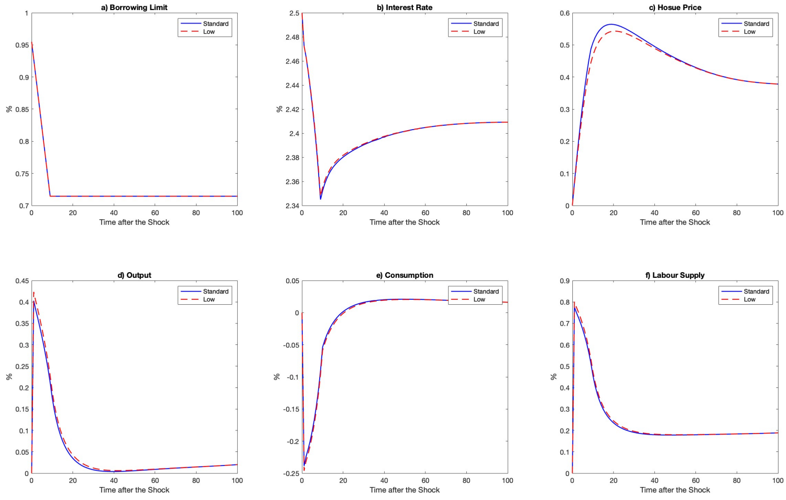

Figure B.11. *Shock to ϕ_h: after-shock transitional dynamics under the standard calibration (solid lines) and under the calibration with low $\underline{h} = 10^{-4}$ (dashed lines) - Values for house prices, consumption, output and labour supply are reported as % deviations from the initial stationary equilibrium.*

shock to consumption, labour supply and output is slightly amplified in the short term.

Appendix C

Computational Solutions

In this appendix, I describe the algorithm that I have implemented in MATLAB to solve and simulate the models in the paper. First, I am going to describe the solution algorithm for the baseline model in the paper from Guerrieri and Lorenzoni (2017), then I will describe the solution method for the model with housing.

C.1. The Baseline Model

To solve for the initial stationary equilibrium, I first set the long-term targets summarized in Table (1.1). Given the targets, I compute the steady state by iterating the policy functions, the stationary distribution for b, and the calibrated parameters until convergence is reached and the long-term targets are attained. The numerical computation consists of the following steps:

1. Given an initial guess for the calibrated parameters, I solve the consumer's problem by iterating the policy functions via the endogenous gridpoint method (EGM) of Carroll (2006).

2. Given the optimal policy functions obtained at the previous step, I compute the stationary asset distribution using the method of Young (2010).

3. Given the policy functions and the distribution obtained at the previous steps, I compute the aggregate variables and I match these values against the pre-set targets, and compute the difference between the current values and the targets. If such difference (in absolute value) is greater than some fixed tolerance level, I update the calibrated parameters accordingly and I repeat the previous steps until convergence is attained.

The EGM used to solve the consumer's problem consists of the following steps:

I. Using the method in Tauchen (1986), I discretize the stochastic process for productivity w into P nodes, and the relative Markov transition matrix.

II. I generate a discrete non-linear grid for the asset b. I firstly set a maximum and a minimum value, respectively b^{max} and b^{min}, and a number N^b of gridpoints. Then I generate a quadratic grid, so that more gridpoints are assigned to the lower end of the grid, with respect to the upper end. This way one can take advantage of the concave-shaped policy function and identify optimal values for constrained individuals, without the need to perform time-consuming maximization operations (like in the value function iteration).

III. While I let b and w vary on the respective discretized grids, I guess an initial value for the consumption policy function $c(w, b)$.

IV. Using the equilibrium conditions that were derived by solving the consumer's problem, I compute the optimal consumption, investment, and labour supply policies.

V. I check the convergence of the policy functions by computing the difference between the optimal consumption policy computed through the EGM and the initial guess.

VI. If the difference (in absolute value) is greater than some fixed tolerance value, I set the computed optimal consumption function as the new guess, and I replicate the previous steps until convergence is attained.

To solve for the final stationary equilibrium, I keep the original calibration, minus the parameter ϕ, which I set to adjust at each iteration according to the new target for the debt-to-GDP ratio after the shock. Steps 1 and 2 in the computational process remain the same, while for the third step, I set that the borrowing limit and the equilibrium interest rate adjust until convergence, i.e. until the new target is reached. To compute the transitional dynamics, the steps for the computations are similar to the ones above:

1. I set a number of period T after which I assume the economy converge to the new equilibrium. I solve the consumer's problem at every period $t = 0, ..., T$ using the EGM and backwards induction. I set that the initial guess at the end period T is the consumption policy function c at the new stationary equilibrium, and then I compute the full series of optimal policies for c, n and b' for all the periods $t = T-1, T-2, ..., 0$.

2. Once I have the full series of policy functions, I compute the stationary distribution at every $t = 0, ..., T$. I start from the initial period $t = 0$ and I continue until T. The method I use to find the stationary distribution is still the one in Young (2010).

3. I compute aggregate variables and for every $t = 0, ..., T$ and I check that markets clear at any period. If at a given point t financial or goods do not clear, I adjust period-t interest rate so that clearance is ensured.

C.2. The Model with Housing

The numerical simulation method used to solve the model with housing entails the same procedural steps as for the baseline model. The main difference consists in considering a second asset (housing), when computing the solution for the consumer's problem via EGM and computing the stationary distribution of the two assets. The method I followed to solve the consumer's problem builds upon the solution framework laid out by Hintermaier and Koeniger (2010) and it is summarized below.

For a given set of parameters, a house price q and interest rates R and R^b:

I. I discretize the stochastic process for w into a finite set of gridpoints, of size J, and I compute the Markov transition matrix using the method in Tauchen (1986).

II. I set two non-linear discrete grids, for h and x, each of them with its own dimensionality (n_h and n_x) and its own maximum and minimum value. More specifically I set $\underline{h}$ as minimum value for the grid for h, and $(1-\phi_h)(1-\kappa)\underline{h}$ as minimum value for the grid for x. Additionally, following the example in Hintermaier and Koeniger (2010), I set both grids to be triple exponential.

III. By letting w, h, and x' vary independently on their respective grids, I specify an initial guess for the consumption policy function $c(w,h,x)$.

IV. Given the initial guess for consumption, I solve the consumer's problem by iterating the optimal policy function of c, until convergence, so that at each iteration i the guess for the policy function of consumption is equal to the policy function optimally computed at the previous iteration $i-1$. At each iteration i, given the guess $c^{i-1}(w,h,x)$:

   1. I solve the consumer's problem for an economy in which the real interest rate is R^b, to compute the candidate optimal policy functions of borrowers - denoted as c_B, n_B, h'_B, b'_B;
   2. I solve the consumer's problem for an economy in which the real interest rate is R, to compute the candidate optimal policy functions of the lenders - denoted as c_L, n_L, h'_L, b'_L;
   3. I solve the consumer's problem for an economy in which the real interest rate is R and borrowing is not allowed, to compute the candidate optimal policy functions of the strictly hand-to-mouth individuals - denoted as c_S, n_S, h'_S, b'_S;
   4. For any triple (w,h,x) on the respective grid:
      - If $b'_B(w,h,x) < 0$ and $b'_L(w,h,x) < 0$, I set that the optimal policy functions are: $b'(w,h,x) = b'_B(w,h,x)$, $h'(w,h,x) = h'_B(w,h,x)$, $c(w,h,x) = c_B(w,h,x)$ and $n(w,h,x) = n_B(w,h,x)$.

- If $b'_B(w, h, x) > 0$ and $b'_L(w, h, x) > 0$, I set that the optimal policy function is $b'(w, h, x) = b'_L(w, h, x)$, $h'(w, h, x) = h'_L(w, h, x)$, $c(w, h, x) = c_L(w, h, x)$ and $n(w, h, x) = n_L(w, h, x)$.
- For any other combination, I set $b'(w, h, x) = b'_S(w, h, x)$, $h'(w, h, x) = h'_S(w, h, x)$, $c(w, h, x) = c_S(w, h, x)$ and $n(w, h, x) = n_S(w, h, x)$ - in which case individuals are better off by neither borrowing nor lending.

5. I check the convergence to the equilibrium values by computing the absolute value of the difference between new policy function $c^i(w, h, x)$, found at the current iteration i, and the guess $c^{i-1}(w, h, x)$, which is the policy function computed at the previous iteration $i-1$.
6. If the maximum value of $|c^i(w, h, x) - c^{i-1}(w, h, x)|$, for any triple (w, h, x), is smaller than a fixed tolerance level ε, then there exist a solution for the consumer's problem , under the given parameters, interest rates, and house price.
7. If $\max\{|c^i(w, h, x) - c^{i-1}(w, h, x)|\} \geq \varepsilon$ then I set $c^i(w, h, x)$ as guess for the next iteration and I repeat the steps above.

To find the solution of the consumer's problem, in each of the three cases listed above, I use the steps of Hintermaier and Koeniger (2010). At any iteration i, for a given set of parameters, a house price q, an interest rate R, grids for x and h, and for a policy function guess $c^i(w, h, x)$:

1. Using the first order conditions of the consumer's problem and the envelope conditions of the value function V - computed in Appendix A.2 - I compute the expected value of the optimal derivatives for the value function, w.r.t x and h, and denoted as V_x and V_h respectively.

2. I look for values of h' that are consistent with an interior solution to the consumer's problem, i.e. for cases in which $\lambda = \mu_h = 0$. To do so, I fix values for w and h on the respective grids, then I impose that $\lambda = \mu_h = 0$, and I look for which values of x Equations (A.17) and (A.18) hold, by using the values for V_x and V_h that I computed at the previous step. I repeat this step for any value of w and h.

3. For those values of the x that do not satisfy the conditions of the previous step, I check if either the no-rough-sleeping constraint is satisfied with equality, or if the borrowing constraint holds with equality or both. In such cases I impose $h'(w, h, x) = \underline{(h)}$ or $h'(w, h, x) = x[(1-\phi_h)(1-\kappa)]^{-1}$ or both, and I compute the relevant, positive, Lagrange multipliers.

4. Once I have the candidates for h', I compute potential optimal values for c, n and b' via the EGM method, by using the optimal first order conditions of the consumer's problem.

5. I interpolate the computed values for consumption on the grid for x, and then, through the first order conditions of the consumer's problem, and I find policy functions for c, n, h' and b'.

The computation of the transitional dynamics follows the same steps as for the model without housing.