

From BTYD to CBCV: Praising a Remarkable Model of Repeat Purchasing

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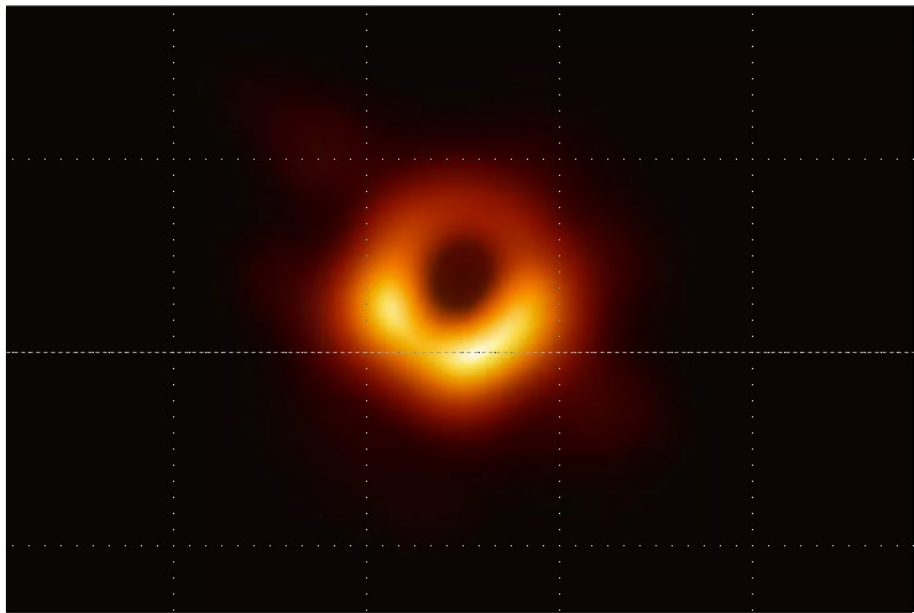
13 March 2023

Historic 1st Photo of a Black Hole Named Science Breakthrough of 2019

By Charles Q. Choi December 19, 2019

It was once thought impossible. Then it happened.

 Comments (0)



The Event Horizon Telescope, a planet-scale array of eight ground-based radio telescopes forged through international collaboration, captured this image of the supermassive black hole in the center of the galaxy M87 and its shadow. (Image: © EHT Collaboration)

The [first image of a black hole](#), previously thought nigh impossible to capture, was named the top scientific breakthrough of 2019 by the journal Science.

Press Release: The Nobel Prize in Physics 2017

3 October 2017

The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics 2017 with one half to

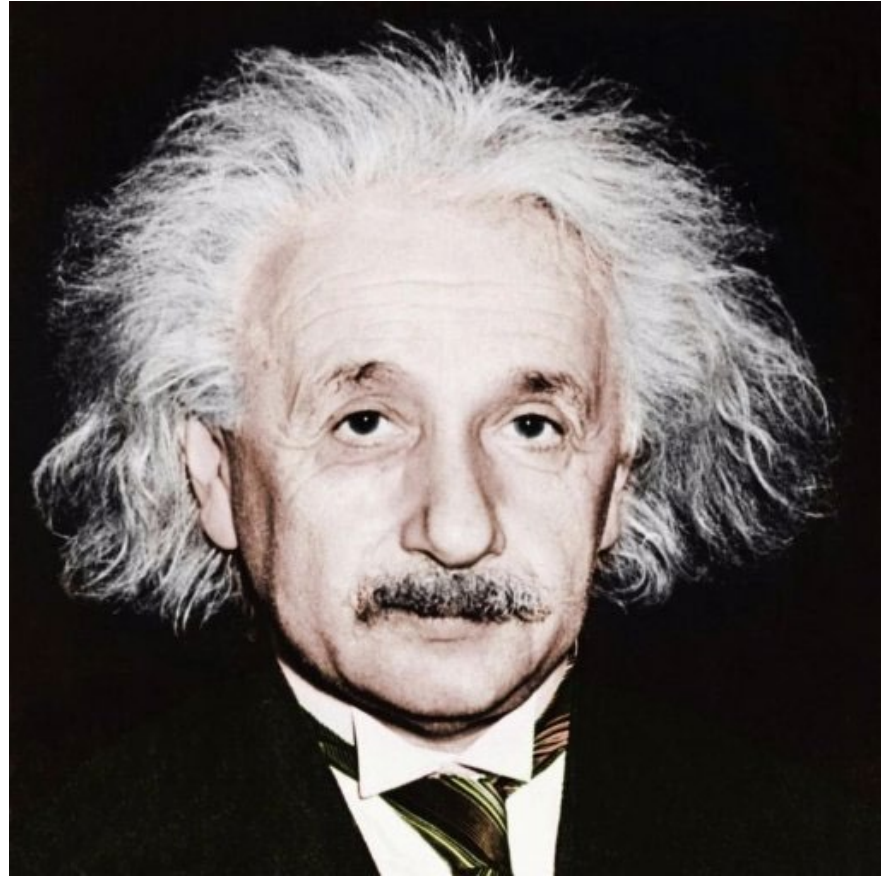
Rainer Weiss
LIGO/VIRGO Collaboration

and the other half jointly to

Barry C. Barish
LIGO/VIRGO Collaboration

and

Kip S. Thorne
LIGO/VIRGO Collaboration





Is the Customer Still Active?

DAVID C. SCHMITTLEIN and DONALD G. MORRISON*

A customer who makes a large number of purchases early on and then makes no purchases for a long time afterward is likely to be an inactive customer. Letting the observation period be one unit of time and assuming Poisson-process purchasing during the customer's active phase, the number of purchases n and the time of the last purchase t contain all of the information. The p level for testing the hypothesis that the customer is still active at the end of the observation period is found to be simply t^n . The p level is also derived for the more general case in which interpurchase times follow a gamma or Erlang distribution.

KEY WORDS: Random purchases; Beta order statistics; One-tailed tests.

1. INTRODUCTION

Consider the four customers in Figure 1 whose purchasing histories have been observed over one unit of time. An X indicates a purchase (e.g., buying a stock from a broker, purchasing a new brand of coffee). Looking at these four event histories, we wish to test the hypothesis that each of these customers is still an active customer. The purpose of this article is to develop an easy, insightful method for doing this hypothesis testing.

Intuitively, A is more likely than B to be still an active customer. Both have made four purchases, but B's purchases are clustered in the early period with a long lapse of no purchases. Customer B likely has lost interest in the product or brand. A's recent purchase is indicative of a customer who is still active. Customers C and D both have the same elapsed time since the last purchase, but C was a much more frequent purchaser than D. Customer C's long lapse at the end seems inconsistent with the large number of early purchases. Customer D's lapse in purchasing seems natural for an infrequent but still active customer. Clearly D is more likely to be active than is C. Now we will quantify these qualitative impressions.

2. THE MODEL FOR POISSON PURCHASES

For much of the discussion in this article, purchases will be assumed to be generated by a Poisson process while the customer is active. This assumption is appealing for products not bought too regularly, and it has extensive empirical support for the purchasing of frequently purchased consumer packaged goods. With this Poisson assumption, interpurchase times for a customer follow the exponential distribution and so exhibit the lack of memory property. That is, for an individual with known purchase rate α the distribution of additional time until the next purchase always follows the exponential distribution (with mean $1/\alpha$) regardless of the amount of time that has elapsed since the last purchase.

Some of the empirical support for the Poisson assumption comes from work with the NBD model, which assumes that a separate Poisson process accounts for the purchases made by each individual. It further assumes that purchase rates follow a gamma distribution across the population of individuals. With these assumptions the distribution over customers for the number of purchases made in a fixed time duration is the negative binomial—hence the name NBD for the model. Ehrenberg (1972) illustrated the effectiveness of the NBD in representing some properties of observed purchases. Additional support comes from Morrison and Schmittlein's (1981) study of conditional expectations. These are the expected number of purchases in a future time period by customers making 0, 1, 2, 3, or more purchases in an initial period. They investigated empirically the accuracy of conditional expectations estimated with the NBD model. They also analyzed the expectations generated by a model that assumed only that purchases are Poisson-process generated for each customer. That is, purchase rates can have any distribution across customers. Both models, and especially the NBD, capture the general pattern of the observed purchases.

In those instances in which the Poisson assumption is questioned, the concern usually stems from the memoryless property implied for interpurchase times. Sometimes a relatively dead period is expected immediately after a purchase, reflecting the time it takes for consumption of the brand or product. So the interpurchase times are expected to be more regular (i.e., have a smaller variance) than the exponential distribution. In this situation the Erlang distribution with shape parameter greater than 1 has been proposed as a tractable alternative for the interpurchase time distribution (Chatfield and Goodhardt 1973; Schmittlein and

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COUNTING YOUR CUSTOMERS: WHO ARE THEY AND WHAT WILL THEY DO NEXT?

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This article is concerned with counting and identifying those customers who are still active. The issue is important in at least three settings: monitoring the size and growth rate of a firm's ongoing customer base, evaluating a new product's success based on the pattern of trial and repeat purchases, and targeting a subgroup of customers for advertising and promotions. We develop a model based on the number and timing of the customers' previous transactions. This approach allows computation of the probability that any particular customer is still active. Several numerical examples are used to illustrate applications of the model.

(MARKETING; CONSUMER BEHAVIOR; POISSON PROCESS; PROBABILITY MIXTURE MODELS; NEW PRODUCT INTRODUCTIONS; MARKET SEGMENTATION; BROKERAGE FIRMS)

Modelling the Transaction Stream

A customer's relationship with a firm has two phases: they are "alive" for an unobserved period of time, then "dead."

Transaction Process:

- While alive, a customer purchases "randomly" around his mean transaction rate.
- Transaction rates vary across customers.

Latent Attrition Process:

- Each customer has an unobserved "lifetime," which is a function of their death rate.
- Death rates vary across customers.

Pareto/NBD Likelihood Function

$$L(r, \alpha, s, \beta | x, t_x, T)$$

$$= \frac{\Gamma(r+x)\alpha^r\beta^s}{\Gamma(r)} \left\{ \left(\frac{s}{r+s+x} \right) \frac{{}_2F_1(r+s+x, s+1; r+s+x+1; \frac{\alpha-\beta}{\alpha+t_x})}{(\alpha+t_x)^{r+s+x}} \right. \\ \left. + \left(\frac{r+x}{r+s+x} \right) \frac{{}_2F_1(r+s+x, s; r+s+x+1; \frac{\alpha-\beta}{\alpha+T})}{(\alpha+T)^{r+s+x}} \right\}, \text{ if } \alpha \geq \beta$$

$$L(r, \alpha, s, \beta | x, t_x, T)$$

$$= \frac{\Gamma(r+x)\alpha^r\beta^s}{\Gamma(r)} \left\{ \left(\frac{s}{r+s+x} \right) \frac{{}_2F_1(r+s+x, r+x; r+s+x+1; \frac{\beta-\alpha}{\beta+t_x})}{(\beta+t_x)^{r+s+x}} \right. \\ \left. + \left(\frac{r+x}{r+s+x} \right) \frac{{}_2F_1(r+s+x, r+x+1; r+s+x+1; \frac{\beta-\alpha}{\beta+T})}{(\beta+T)^{r+s+x}} \right\}, \text{ if } \alpha \leq \beta$$

15 years of
pretty much
nothing...

Forecasting Repeat Sales at CDNOW: A Case Study

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We conducted a modeling exercise in conjunction with the on-line music retailer CDNOW to develop a simple stochastic model of buyer behavior capable of forecasting medium-term aggregate CD purchasing by a cohort of new customers. We modeled weekly sales using a finite mixture of beta-geometric distributions with a separate time-varying component to capture nonstationarity in repeat buying. The resulting model can easily be implemented within a standard spreadsheet environment (for example, Microsoft Excel). It does a good job of describing the underlying sales patterns and produces an excellent medium-term forecast.

With the growth of e-commerce, many companies are facing challenges in figuring out how to make effective and efficient use of the detailed transaction information that they are rapidly accumulating. While some writers on database marketing and one-to-one marketing suggest using statistical models to gain managerial insights [Mulhern 1999; Forrester Report 1999], there are very few published examples to guide managers' efforts

in this direction.

We undertook an exploratory study in conjunction with CDNOW, a leading on-line music retailer, to develop an easily implementable model of buyer behavior capable of forecasting medium-term aggregate CD purchasing by a cohort of CDNOW customers. Aggregate-level forecasts are critical inputs to any attempt to value a customer base, and they serve as a diagnostic to help firms gauge the effective-

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MARKETING—BUYER BEHAVIOR
FORECASTING—APPLICATIONS

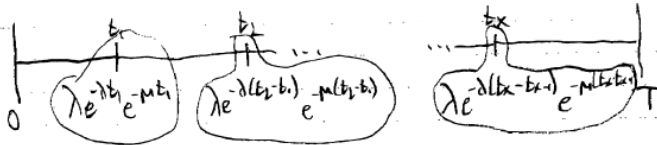
SMC For Durmies

①

Basic assumptions:

Transactions \sim exponential (λ) $\lambda \sim$ gamma ($1, \alpha$)Death \sim exponential (μ) $\mu \sim$ gamma (s, β)Observation period of length T , with X arrivals (at t_1, t_2, \dots, t_x)

Individual-level likelihood:

What is $P(\text{no purchase in } T - t_x)$?More generally, what is $P(0 | \tilde{c})$?

$$\begin{aligned}
 P(0 | \tilde{c}) &= P(\text{no purchase + survival through } \tilde{c}) \\
 &\quad + P(\text{no purchase + death at } 0 \leq u \leq \tilde{c}) \\
 &= e^{-\lambda \tilde{c}} e^{-\mu \tilde{c}} + \int_0^{\tilde{c}} e^{-\lambda u} \mu e^{-\mu u} du \\
 &= e^{-(\lambda + \mu) \tilde{c}} + \mu \int_0^{\tilde{c}} e^{-(\lambda + \mu) u} du \\
 &= e^{-(\lambda + \mu) \tilde{c}} + \frac{\mu}{\lambda + \mu} [1 - e^{-(\lambda + \mu) \tilde{c}}]
 \end{aligned}$$

$$P(0 | \tilde{c}) = \frac{\mu + \lambda e^{-(\lambda + \mu) \tilde{c}}}{\lambda + \mu}$$

This makes sense! As $\tilde{c} \rightarrow 0$, prob $\rightarrow 1$; as $\tilde{c} \rightarrow \infty$, prob $\rightarrow \frac{\mu}{\lambda + \mu}$.

SMC for Dummies

(4)

$$= \frac{\Gamma(\delta + \gamma + w + 1) \alpha^{\delta} \beta^{\delta}}{\Gamma(\gamma) \Gamma(s) (\alpha + \beta)^{(\delta + \gamma + w + 1)}} \int_0^1 p^{\delta + \gamma - 1} (1-p)^{w+1} \left[1 - \frac{\alpha - \beta}{\alpha + \beta} p \right]^{-\delta + \gamma + w + 1} dp$$

$$= \text{Remember! } {}_2F_1(a, b; c; x) = \frac{\Gamma(c)}{\Gamma(a) \Gamma(c-a)} \int_0^1 u^{a-1} (1-u)^{c-a-1} (1-xu)^{-b} du$$

Then	us
a	$\delta + \gamma + s$
b	$\delta + \gamma + w + 1$
c	$\delta + \gamma + s + w + 1$
x	$(\alpha - \beta) / (\alpha + \beta)$

$$= \frac{\Gamma(\delta + \gamma + w + 1) \alpha^{\delta} \beta^{\delta}}{\Gamma(\gamma) \Gamma(s) (\alpha + \beta)^{(\delta + \gamma + w + 1)}} \frac{\Gamma(\delta + \gamma + s) \Gamma(\gamma + w)}{\Gamma(\delta + \gamma + s + w + 1)} {}_2F_1\left[\delta + \gamma + s, \delta + \gamma + w + 1; \delta + \gamma + s + w + 1; \frac{\alpha - \beta}{\alpha + \beta}\right]$$

So for CASE I: $\alpha > \beta$

$$L = \frac{\Gamma(x + \gamma + s + 1) \alpha^{\delta} \beta^{\delta}}{\Gamma(\gamma) \Gamma(s) (\alpha + \beta)^{x + \gamma + s + 1}} \frac{\Gamma(\gamma + s + 1) \Gamma(\gamma + x)}{\Gamma(x + \gamma + s + 1)} {}_2F_1\left[\gamma + s + 1, x + \gamma + s + 1; x + \gamma + s + 1; \frac{\alpha - \beta}{\alpha + \beta}\right]$$

$$+ \frac{\Gamma(x + \gamma + s + 1) \alpha^{\delta} \beta^{\delta}}{\Gamma(\gamma) \Gamma(s) (\alpha + \beta)^{x + \gamma + s + 1}} \frac{\Gamma(\gamma + s) \Gamma(\gamma + x + 1)}{\Gamma(x + \gamma + s + 1)} {}_2F_1\left[\gamma + s, x + \gamma + s + 1; x + \gamma + s + 1; \frac{\alpha - \beta}{\alpha + \beta}\right]$$

CASE II Let $z = \delta + w$ $\alpha < \beta$

Let $q = (z - m) / z \rightarrow m = z(1 - q) \rightarrow dm = -z dq$

$$= \frac{\alpha^{\delta} \beta^{\delta}}{\Gamma(\gamma) \Gamma(s)} \int_0^1 \int_0^1 q^{w+1} (1-q)^{\delta + \gamma - 1} z^{\delta + \gamma + w + 1} e^{-[m + z(\beta - \alpha)]z} dz dq$$

Marketing Science
New Perspectives on Customer "Death" Using a Generalization of the Pareto/NBD Model
David M. McInnis
January 2007

1. Introduction
The customer death problem is a central issue in marketing. It is the problem of determining when a customer has stopped purchasing from a firm. This problem is important because it affects a firm's ability to estimate its customer base and to forecast its future sales. The Pareto/NBD model is a widely used model for customer death. However, it has several limitations. In this paper, we propose a generalization of the Pareto/NBD model that overcomes these limitations. We show that our model is more flexible and more accurate than the Pareto/NBD model. We also show that our model can be used to estimate the customer death rate and to forecast future sales.

Marketing Science
Customer-Bite Analysis in a Discrete-Time Noncontractual Setting
David M. McInnis
January 2007

1. Introduction
Customer-bite analysis is a new method for analyzing customer behavior. It is based on the idea that customers "bite" into a firm's customer base. This bite represents the customer's decision to stop purchasing from the firm. Customer-bite analysis allows us to estimate the customer death rate and to forecast future sales. In this paper, we develop a discrete-time noncontractual setting for customer-bite analysis. We show that our model is more flexible and more accurate than the Pareto/NBD model. We also show that our model can be used to estimate the customer death rate and to forecast future sales.

Marketing Science
Valuing Non-Contractual Firms Using Common Customer Metrics
David M. McInnis
January 2007

1. Introduction
Valuing non-contractual firms is a difficult task. This is because these firms do not have a contract with their customers. As a result, their customer base is not as stable as that of contractual firms. In this paper, we propose a method for valuing non-contractual firms using common customer metrics. We show that our method is more accurate than other methods. We also show that our method can be used to estimate the customer death rate and to forecast future sales.

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A Cross-Cohort Changepoint Model for Customer-Bite Analysis
David M. McInnis
January 2007

1. Introduction
A cross-cohort changepoint model for customer-bite analysis is a new method for analyzing customer behavior. It is based on the idea that customers "bite" into a firm's customer base. This bite represents the customer's decision to stop purchasing from the firm. In this paper, we develop a cross-cohort changepoint model for customer-bite analysis. We show that our model is more flexible and more accurate than the Pareto/NBD model. We also show that our model can be used to estimate the customer death rate and to forecast future sales.

Marketing Science
Customer-Based Corporate Valuation for Publicly Traded Noncontractual Firms
David M. McInnis
January 2007

1. Introduction
Customer-based corporate valuation is a new method for valuing firms. It is based on the idea that a firm's value is determined by its customer base. In this paper, we develop a customer-based corporate valuation model for publicly traded noncontractual firms. We show that our model is more accurate than other methods. We also show that our model can be used to estimate the customer death rate and to forecast future sales.

Marketing Science
The Value of a New Product: A Review of the Literature
David M. McInnis
January 2007

1. Introduction
The value of a new product is a central issue in marketing. It is the problem of determining how much a firm should invest in a new product. This problem is important because it affects a firm's ability to estimate its customer base and to forecast its future sales. In this paper, we review the literature on the value of a new product. We show that there are several different methods for estimating the value of a new product. We also show that our method is more accurate than other methods.

Marketing Science
RFM and CLV: Using Iso-Value Curves for Customer Base Analysis
David M. McInnis
January 2007

1. Introduction
RFM and CLV are two widely used methods for analyzing customer behavior. RFM is based on the idea that a customer's value is determined by their recency, frequency, and monetary value. CLV is based on the idea that a customer's value is determined by their lifetime value. In this paper, we propose a method for using iso-value curves for customer base analysis. We show that our method is more accurate than other methods. We also show that our method can be used to estimate the customer death rate and to forecast future sales.

Marketing Science
More than meets the eye
David M. McInnis
January 2007

1. Introduction
More than meets the eye is a new method for analyzing customer behavior. It is based on the idea that a customer's value is determined by their recency, frequency, and monetary value. In this paper, we propose a method for using more than meets the eye for customer base analysis. We show that our method is more accurate than other methods. We also show that our method can be used to estimate the customer death rate and to forecast future sales.

Marketing Science
ESTIMATING CLV USING AGGREGATED DATA: THE TUSCAN LIFESTYLES CASE REVISITED
David M. McInnis
January 2007

1. Introduction
Estimating CLV using aggregated data is a difficult task. This is because aggregated data does not provide as much information as individual customer data. In this paper, we propose a method for estimating CLV using aggregated data. We show that our method is more accurate than other methods. We also show that our method can be used to estimate the customer death rate and to forecast future sales.

Marketing Science
Interactive Marketing
David M. McInnis
January 2007

1. Introduction
Interactive marketing is a new method for marketing. It is based on the idea that a firm's marketing efforts should be interactive. In this paper, we propose a method for using interactive marketing for customer base analysis. We show that our method is more accurate than other methods. We also show that our method can be used to estimate the customer death rate and to forecast future sales.

Marketing Science
"Counting Your Customers" the Easy Way: An Alternative to the Pareto/NBD Model
David M. McInnis
January 2007

1. Introduction
"Counting your customers" the easy way is a new method for analyzing customer behavior. It is based on the idea that a firm's customer base is determined by the number of customers. In this paper, we propose a method for using "counting your customers" the easy way for customer base analysis. We show that our method is more accurate than other methods. We also show that our method can be used to estimate the customer death rate and to forecast future sales.

Marketing Science
Customer Lifetime Value (CLV) and Customer Lifetime Profit (CLP)
David M. McInnis
January 2007

1. Introduction
Customer lifetime value (CLV) and customer lifetime profit (CLP) are two widely used methods for analyzing customer behavior. CLV is based on the idea that a customer's value is determined by their lifetime value. CLP is based on the idea that a customer's profit is determined by their lifetime profit. In this paper, we propose a method for using CLV and CLP for customer base analysis. We show that our method is more accurate than other methods. We also show that our method can be used to estimate the customer death rate and to forecast future sales.

Marketing Science
Model Selection Using Database Characteristics: Developing a Classification Tree for Longitudinal Testhouse Data
David M. McInnis
January 2007

1. Introduction
Model selection using database characteristics is a new method for analyzing customer behavior. It is based on the idea that a firm's customer base is determined by the characteristics of its database. In this paper, we propose a method for using model selection using database characteristics for customer base analysis. We show that our method is more accurate than other methods. We also show that our method can be used to estimate the customer death rate and to forecast future sales.

Marketing Science
Chapter 11: Marketing Models for the Customer-Centric Firm
David M. McInnis
January 2007

1. Introduction
Marketing models for the customer-centric firm are a new method for analyzing customer behavior. They are based on the idea that a firm's customer base is determined by the characteristics of its database. In this paper, we propose a method for using marketing models for the customer-centric firm for customer base analysis. We show that our method is more accurate than other methods. We also show that our method can be used to estimate the customer death rate and to forecast future sales.

Marketing Science
Chapter 7: Customer Models of Buyer Behavior
David M. McInnis
January 2007

1. Introduction
Customer models of buyer behavior are a new method for analyzing customer behavior. They are based on the idea that a customer's behavior is determined by their characteristics. In this paper, we propose a method for using customer models of buyer behavior for customer base analysis. We show that our method is more accurate than other methods. We also show that our method can be used to estimate the customer death rate and to forecast future sales.

Marketing Science
Customer Lifetime Value (CLV) and Customer Lifetime Profit (CLP)
David M. McInnis
January 2007

1. Introduction
Customer lifetime value (CLV) and customer lifetime profit (CLP) are two widely used methods for analyzing customer behavior. CLV is based on the idea that a customer's value is determined by their lifetime value. CLP is based on the idea that a customer's profit is determined by their lifetime profit. In this paper, we propose a method for using CLV and CLP for customer base analysis. We show that our method is more accurate than other methods. We also show that our method can be used to estimate the customer death rate and to forecast future sales.

Marketing Science
VCLV: Examining Variance in Models of Customer Lifetime Value
David M. McInnis
January 2007

1. Introduction
VCLV is a new method for analyzing customer behavior. It is based on the idea that a customer's value is determined by their variance. In this paper, we propose a method for using VCLV for customer base analysis. We show that our method is more accurate than other methods. We also show that our method can be used to estimate the customer death rate and to forecast future sales.

Marketing Science
3. Steps: probability models for customer CLV and CE
David M. McInnis
January 2007

1. Introduction
Steps: probability models for customer CLV and CE is a new method for analyzing customer behavior. It is based on the idea that a customer's value is determined by their probability. In this paper, we propose a method for using steps: probability models for customer CLV and CE for customer base analysis. We show that our method is more accurate than other methods. We also show that our method can be used to estimate the customer death rate and to forecast future sales.

Customer-Base Analysis in a Discrete-Time Noncontractual Setting

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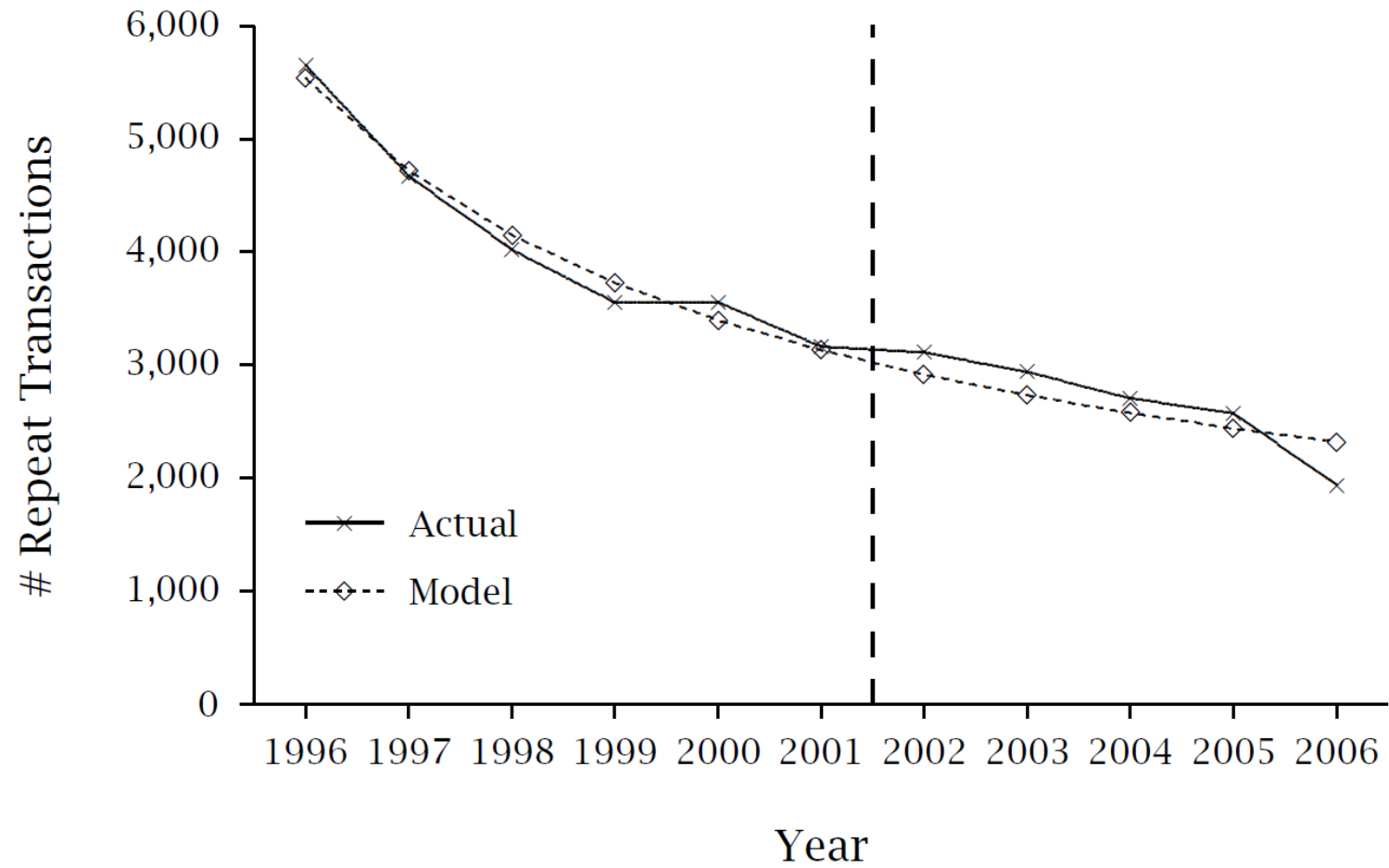
Many businesses track repeat transactions on a discrete-time basis. These include (1) companies for whom transactions can only occur at fixed regular intervals, (2) firms that frequently associate transactions with specific events (e.g., a charity that records whether supporters respond to a particular appeal), and (3) organizations that choose to utilize discrete reporting periods even though the transactions can occur at any time. Furthermore, many of these businesses operate in a noncontractual setting, so they have a difficult time differentiating between those customers who have ended their relationship with the firm versus those who are in the midst of a long hiatus between transactions. We develop a model to predict future purchasing patterns for a customer base that can be described by these structural characteristics. Our beta-geometric/beta-Bernoulli (BG/BB) model captures both of the underlying behavioral processes (i.e., customers' purchasing while "alive" and time until each customer permanently "dies"). The model is easy to implement in a standard spreadsheet environment and yields relatively simple closed-form expressions for the expected number of future transactions conditional on past observed behavior (and other quantities of managerial interest). We apply this discrete-time analog of the well-known Pareto/NBD model to a data set on donations made by the supporters of a nonprofit organization located in the midwestern United States. Our analysis demonstrates the excellent ability of the BG/BB model to describe and predict the future behavior of a customer base.

Key words: BG/BB; beta-geometric; beta-binomial; customer-base analysis; customer lifetime value; CLV; RFM; Pareto/NBD

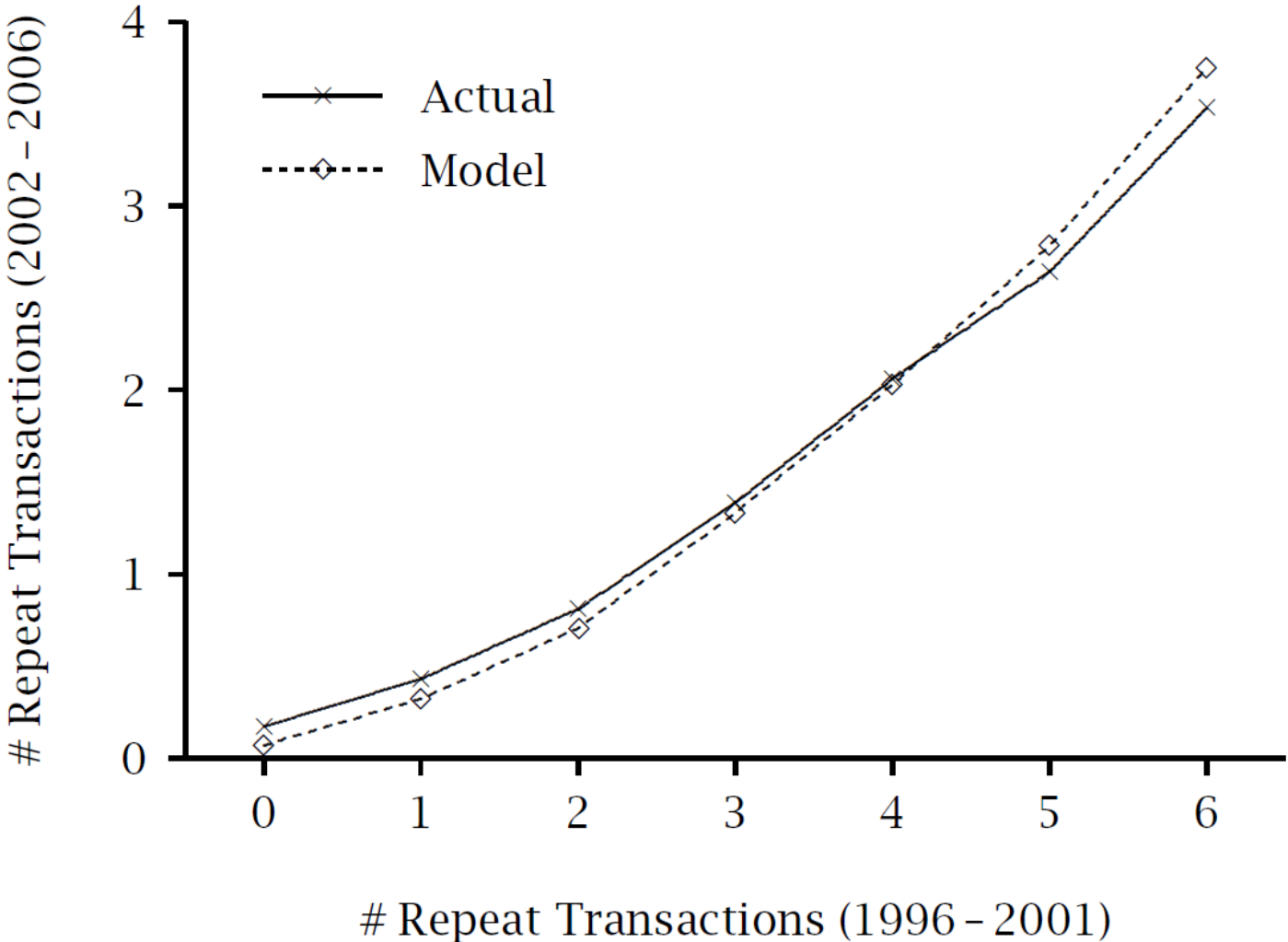
Setting

- A charity located in the Midwestern United States that is funded in large part by donations from individual supporters.
- Initial focus on 1995 cohort, ignoring donation amount:
 - 11,104 people first-time supporters.
 - This cohort makes a total of 24,615 repeat donations (transactions) over the next 6 years.
 - What level of support (# transactions) can we expect from this cohort in the future?

Tracking Annual Repeat Transactions



Conditional Expectations by Frequency



Spreading the gospel

BTYD: Implementing Buy 'Til You Die Models

This package contains functions for data preparation, parameter estimation, scoring, and plotting for the BG/BB, BG/NBD and Pareto/NBD models.

Version: 2.4
Depends: [hypergeo](#)
Imports: [Matrix](#)
Suggests: [knitr](#)
Published: 2014-11-07
Author: Lukasz Dziurzynski [aut], Edward Wadsworth [aut], Peter Fader [ctb], Elea McDonnell Feit [ctb], Daniel McCarthy [aut, cre, ctb] [ctb], Eric Schwartz [ctb], Yao Zhang [ctb]
Maintainer: Daniel McCarthy <danielmc@wharton.upenn.edu>
License: [GPL-3](#)
URL: [wcai.wharton.upenn.edu](#)
NeedsCompilation: no
Materials: [ChangeLog](#)
CRAN checks: [BTYD results](#)

Downloads:

Reference manual: [BTYD.pdf](#)
Vignettes: [Buy 'Til You Die - A Walkthrough](#)
Package source: [BTYD_2.4.tar.gz](#)
Windows binaries: r-devel: [BTYD_2.4.zip](#), r-release: [BTYD_2.4.zip](#), r-oldrel: [BTYD_2.4.zip](#)
OS X binaries: r-release: [BTYD_2.4.tgz](#), r-oldrel: [BTYD_2.4.tgz](#)
Old sources: [BTYD archive](#)

Reverse dependencies:

Reverse imports: [BTYDplus](#)

Linking:

Please use the canonical form <https://CRAN.R-project.org/package=BTYD> to link to this page.

[brucehardie.com/notes/](#)

Notes

- [Key Reference Books](#): a list of key books for those interested in applied probability modelling, with applications in marketing.
- [A Note on Implementing the Fader and Hardie \(*Interfaces*, 2001\) "CDNOW Model" Spreadsheet to accompany "A Note on an Integrated Model of Customer Buying Behavior"](#)
- [Implementing the BG/NBD Model for Customer Base Analysis in Excel](#)
- [Illustrating the Performance of the NBD as a Benchmark Model for Customer-Base Analysis](#)
- [Creating a Depth-of-Repeat Sales Summary Using Excel](#)
- [Generating a Sales Forecast With a Simple Depth-of-Repeat Model](#)
- [A Note on Implementing the Pareto/NBD Model in MATLAB](#)
- [A Note on Deriving the Pareto/NBD Model and Related Expressions](#)
- [Implementing the BG/BB Model for Customer Base Analysis in Excel](#)
- [Implementing the Pareto/NBD Model Given Interval-Censored Data](#)
- [Deriving an Expression for \$P\(X\(t\)=x\)\$ Under the Pareto/NBD Model](#)
- [Deriving an Expression for \$P\(X\(t,t+s\)=x\)\$ Under the Pareto/NBD Model](#)
- [Creating a Fit Histogram for the BG/NBD Model](#)
- [How Not to Project Customer Retention](#)
- [Fitting the sBG Model to Multi-Cohort Data](#)
- [Computing DERL for the sBG Model Using Excel](#)
- [Incorporating Time-Invariant Covariates into the Pareto/NBD and BG/NBD Models](#)
- [Technical Appendix—Customer-Base Valuation in a Contractual Setting: The Perils of Ignoring Heterogeneity](#)
- [Computing P\(alive\) Using the BG/NBD Model](#)
- [Creating an RFM Summary Using Excel](#)
- [Implementing the \$S_{\{BB\}}\$ -G/B Model in MATLAB](#)
- [Reconciling and Clarifying CLV Formulas](#)
- [The Gamma-Gamma Model of Monetary Value](#)
- [Notes on the CDNOW Master Data Set](#)
- [Overcoming the BG/NBD Model's #NUM! Error Problem](#)
- [Deriving the Conditional PMF of the Pareto/NBD Model](#)
- [Additional Results for the Pareto/NBD Model](#)
- [Computing \$P\(X\(t,t+s\)=x\)\$ Under the BG/NBD Model](#)
- [The Pareto/NBD is Not a Lost-for-Good Model](#)
- [A Spreadsheet-Literate Non-Statistician's Guide to the Beta-Geometric Model](#)
- [What's Wrong With This CLV Formula?](#)
- [A Correlated Pareto/NBD Model](#)
- [Exploring the Distribution of Customer Lifetime Value \(in Contractual Settings\)](#)
- [The Mean and Variance of Customer Lifetime Value in Contractual Settings](#)



<https://youtu.be/guj2gVEEx4s>

CLV IN E-COMMERCE

Dr. Peter Fader, a professor at Wharton, discusses new insights on CLV and their application to e-commerce.

BTYDplus: Probabilistic Models for Assessing and Predicting your Customer Base

Provides advanced statistical methods to describe and predict customers' purchase behavior in a non-contractual setting. It allows to compute quantities of managerial interest on a cohort- as well as on a customer level (Customer Lifetime Value, Customer Lifetime Value, Customer Lifetime Value) package by providing several additional buy-till-you-die models, that have been published in the marketing literature, but who NBD, MBG/NBD, BG/CNBD-k, MBG/CNBD-k, Pareto/NBD (HB), Pareto/NBD (Abe) and Pareto/GGG.

Version: 1.0.1
Depends: R (\geq 3.2.0)
Imports: [Rcpp](#), [BTYD](#) (\geq 2.3), [coda](#), [data.table](#), [mvtnorm](#), [bayesm](#), stats, graphics
LinkingTo: [Rcpp](#)
Suggests: [testthat](#), [gsl](#), [covr](#), [knitr](#), [rmarkdown](#), [lintr](#) (\geq 1.0.0)
Published: 2016-12-14
Author: Michael Platzer [aut, cre]
Maintainer: Michael Platzer <michael.platzer@gmail.com>
BugReports: <https://github.com/mplatzer/BTYDplus/issues>
License: [GPL-3](#)
URL: <https://github.com/mplatzer/BTYDplus#readme>
NeedsCompilation: yes
Materials: [README NEWS](#)
CRAN checks: [BTYDplus results](#)

Downloads:

Reference manual: [BTYDplus.pdf](#)
Vignettes: [Customer Base Analysis with BTYDplus](#)
Package source: [BTYDplus_1.0.1.tar.gz](#)
Windows binaries: r-devel: [BTYDplus_1.0.1.zip](#), r-release: [BTYDplus_1.0.1.zip](#), r-oldrel: [BTYDplus_1.0.1.zip](#)
OS X binaries: r-release: [BTYDplus_1.0.1.tgz](#), r-oldrel: [BTYDplus_1.0.1.tgz](#)

Linking:

Please use the canonical form <https://CRAN.R-project.org/package=BTYDplus> to link to this package

Navigation

- Project description
- Release history
- Download files

Project links

- Homepage

Statistics

GitHub statistics:

- ★ Stars: 762
- 🔗 Forks: 237
- 🔔 Open issues/PRs: 87

View statistics for this project via [Libraries.io](#), or by using [Google BigQuery](#)

Meta

License: MIT License (MIT)

Author: [Cam Davidson-Pilon](#)

customer lifetime value, clv, ltv, BG/NBD, pareto/NBD, frequency, recency

Maintainers



[CamDavidsonPilon](#)

Classifiers

Project description



Measuring users is hard. Lifetimes makes it easy.

pypi package 0.11.1 docs passing build passing coverage 90%

Introduction

Lifetimes can be used to analyze your users based on a few assumption:

- Users interact with you when they are "alive".
- Users under study may "die" after some period of time.

I've quoted "alive" and "die" as these are the most abstract terms: feel free to use your own definition of "alive" and "die" (they are used similarly to "birth" and "death" in survival analysis). Whenever we have individuals repeating occurrences, we can use Lifetimes to help understand user behaviour.

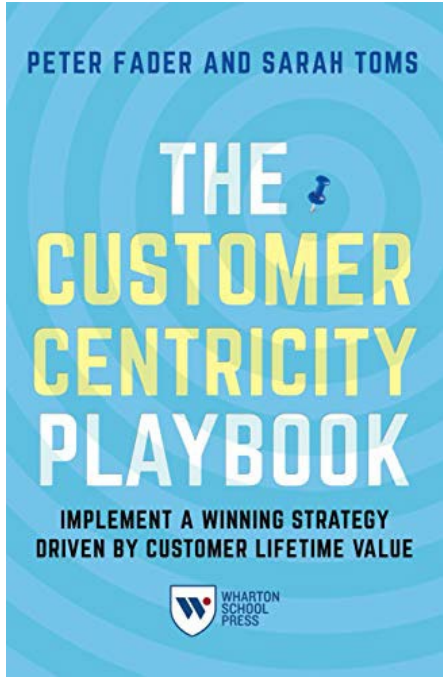
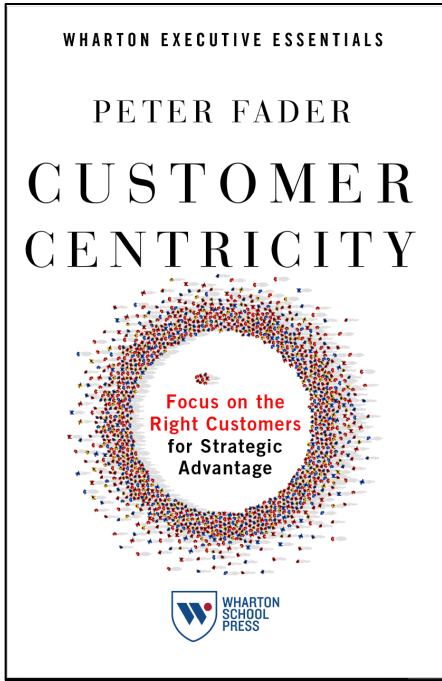
Applications

If this is too abstract, consider these applications:

- Predicting how often a visitor will return to your website. (Alive = visiting. Die = decided the website wasn't for them)
- Understanding how frequently a patient may return to a hospital. (Alive = visiting. Die = maybe the patient moved to a new city, or became deceased.)
- Predicting individuals who have churned from an app using only their usage history. (Alive = logins. Die = removed the app)
- Predicting repeat purchases from a customer. (Alive = actively purchasing. Die = became disinterested with your product)
- Predicting the lifetime value of your customers

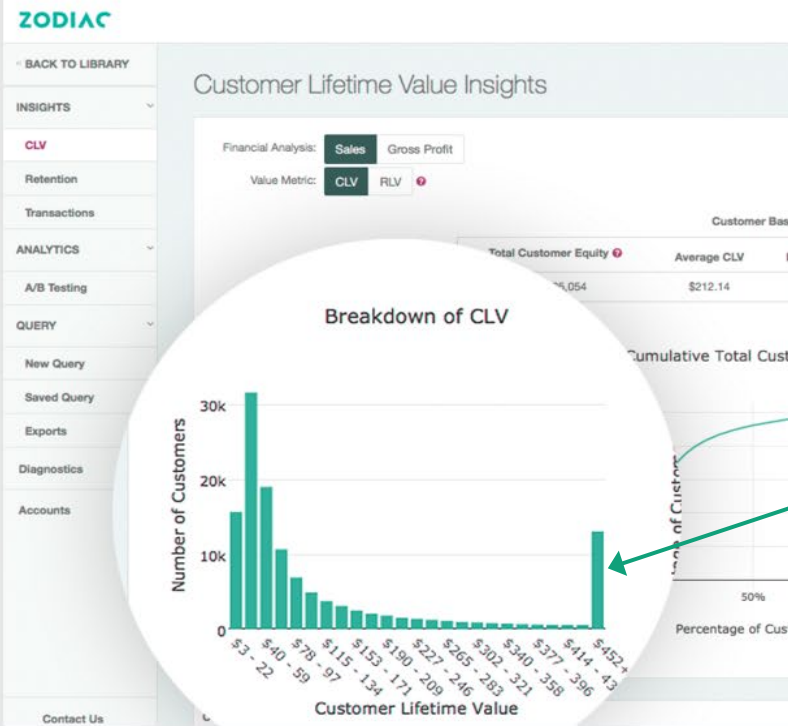
Specific Application: Customer Lifetime Value

As emphasized by P. Fader and B. Hardie, understanding and acting on customer lifetime value (CLV) is the most important part of your business's sales efforts. [And \(apparently\) everyone is doing it wrong](#). *Lifetimes* is a Python library to calculate CLV for you.

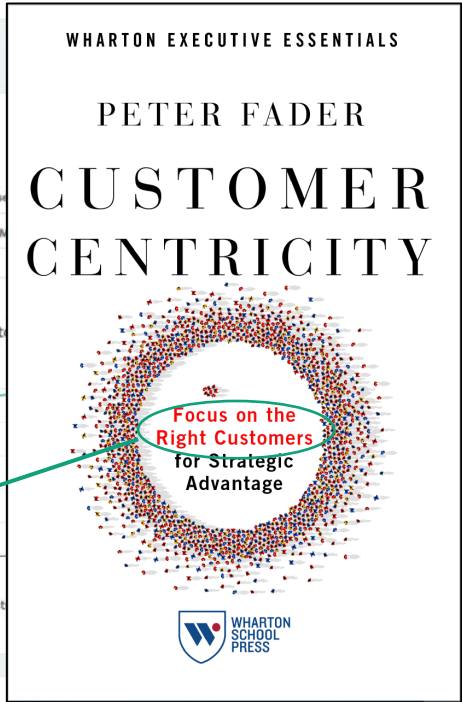


001.

LIFETIME VALUE ANALYTICS



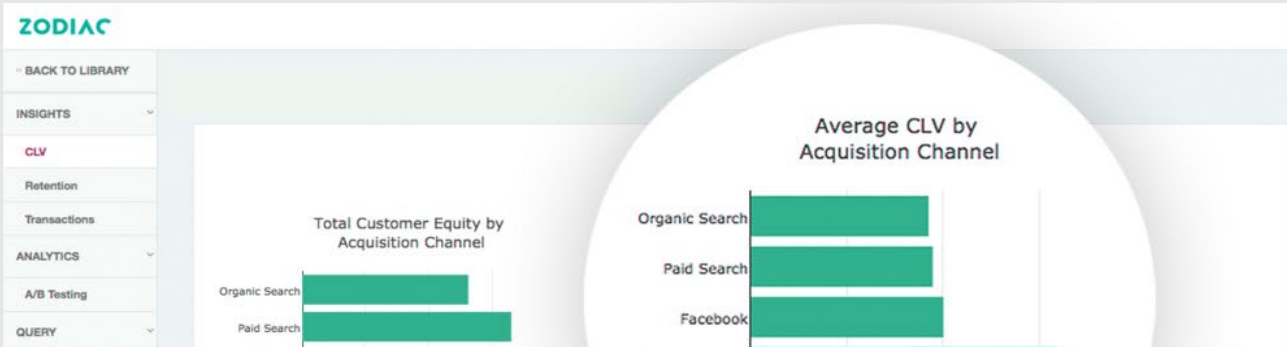
Zodiac tracks CLV insights including customer distribution by value and attribute over time and progress toward goal.



ZODIAC

002.

CUSTOMER SEGMENTATION BY VALUE.



Sample Applications

Organizations use Zodiac's platform to augment a variety of business processes including:

Customer Acquisition



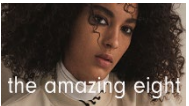
- Use CLV forecasts to more efficiently allocate multichannel marketing dollars (e.g., CLV-based ad optimization)
- Increase ROI by running lookalike campaigns based on Zodiac customer lifetime value metrics
- Lead score new prospects and recently acquired customers and alter strategies (e.g., ad spend) accordingly
- Plan and optimize marketing budgets by acquisition channels and key consumer-facing channels

Customer Retention



- Reactivate customers whose value has fallen in recent months
- Predict future churn and proactively take action to mitigate risk. Operationalize customer service
- Create winback campaigns for customers with high churn risk. Optimize clienteling strategies
- Determine customers' true brand loyalty and segment the customer base accordingly

Loyalty and Insights



- Segment customers by expected number of future transactions to more accurately assess loyalty
- Utilize different marketing levers (e.g., incentives, branding, customer service) based on loyalty tiers
- Build stronger brand loyalty through customer-centric strategies. Focus on priority KPIs
- Within low value product categories, find customers with a high "probability active" to cross-sell

Strategic Development



- Aggregate customer-level forecasts to the firm level for more accurate financial forecasts
- Leverage customer lifetime value to make business decisions that yield sustainable, longer-term profitable growth
- Calculate existing customer equity and track any changes over time
- Perform corporate valuation using a "bottoms-up," customer-based approach

Targeting Segments	Reach	Link Clicks	Total Order	Total Revenue	CVR	AOV	CPA	ROAS
	683,797	13,624	77	\$ 25,207.99	0.6%	\$ 327.38	\$116.01	\$ 2.82
	168,834	4,483	82	\$ 20,188.57	1.8%	\$ 246.20	\$ 42.24	\$ 5.83
	153,583	8,608	24	\$ 5,737.71	0.3%	\$ 239.07	\$104.86	\$ 2.28
	203,354	9,302	9	\$ 1,611.80	0.1%	\$ 179.09	\$255.53	\$ 0.70
	97,733	1,478	6	\$ 1,168.94	0.4%	\$ 194.82	\$181.41	\$ 1.07
Zodiac_Best-Cust_Segment	86,221	2,592	49	\$ 10,267.80	1.9%	\$ 209.55	\$ 19.37	\$ 10.82
	1,406	36	1	\$ 64.98	2.8%	\$ 64.98	\$ 95.98	\$ 0.68
Grand Total	1,394,928	40,123	248	\$ 64,247.79	0.6%	\$ 259.06	\$ 78.01	\$ 3.32

Nike's purchase of analytics firm Zodiac highlights focus on customer lifetime value

Nike has big digital plans as it goes direct to consumer, aims to innovate faster and build relationships. It is also beefing up its analytics team.



By Larry Dignan for [Between the Lines](#) | March 23, 2018 -- 14:31 GMT (07:31 PDT) | Topic: [Digital Transformation](#)

Nike has acquired Zodiac Inc., a consumer data analytics company, in a sign that its digital transformation plans revolve around customer lifetime value.

The athletic shoe and apparel maker, which is in a dogfight with Adidas and Under Armour, has a strategy called Consumer Direct Offense that aims to develop products faster with personalization at scale. Nike also has to focus on selling direct and owning the customer relationship since retail is a messy industry.

In 2016, Zodiac raised \$3 million in seed funding to launch predictive analytics tools based on forecasting individual customer lifetime value. The models were developed by Wharton School Professor Peter Fader and a team of data scientists at the University of Pennsylvania.

Zodiac's mission is to understand the value of an individual customer to boost revenue and retention with the right marketing, recommendations and offers.

In November, Nike outlined plans to juice its growth in the years ahead by scaling new product platforms quickly and then going direct to consumer via its retail outlets, mobile apps and e-commerce partners.

Mark Parker, speaking on Nike's third quarter earnings conference call, outlined the company's progress across key areas:

- 2X Innovation, which revolves around developing new platforms (types of shoes and technologies).
- 2X Speed, which revolves around investing in digital to serve consumer demand faster. There's also a heavy dose of investment in robotics and automation.
- 2X Direct, which leads with digital channels as well as Nike's own retail outlets.

Analytics will be critical to multiple efforts. Parker added that Zodiac and its "proprietary tools will help us deepen relationships with consumers all over the world with a primary focus on our NikePlus members."

Sample Applications

Organizations use Zodiac's platform to augment a variety of business processes including:

Customer Acquisition



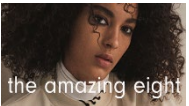
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Predictive Customer And Corporate Valuation

See what customers will do tomorrow, so
you can make better decisions today

GET THE DEMO



 **FOR INVESTORS**



 **FOR BUSINESSES**

Lyft IPO: Despite Positive CLV, Hard to Justify Its \$20-25B Target Valuation



In [our previous post on Lyft](#), we assessed the firm's unit economics by analyzing its pre-IPO filing. We concluded that while losses are currently very large, it is possible for Lyft to grow itself into profitability because it acquires customers profitably, with strong long-term repeat buying from a relatively small segment of highly loyal customers.

In this post, we move from unit economics to overall financial valuation so that we can give our customer-based corporate valuation (CBCV) view on whether their target IPO valuation of **\$20-25B** is justified by the stream of future cash flows we can expect from the firm. In doing so, we take the model that we fit earlier and use it to project what future customer acquisitions will be, the stream of revenue we can expect from those customers after they have been acquired, how that revenue will flow through into variable profits, and how much of that variable profitability the firm will be able to keep after deducting fixed costs.

Cutting to the chase, our results suggest that **Lyft's target valuation is not justified by their fundamentals under any practically realistic future state of the world**. Our more optimistic scenario implies a fair valuation of about **\$7B**, while our base case scenario implies a fair valuation of **\$4.5B**.

While we will discuss every one of the contributing factors that lead us to these conclusions, some of the main reasons for this cautionary view of their valuation are the following:

1. **Lyft's fixed costs are too high.** CLV is a variable profitability measure, so the fact that Lyft has a positive average CLV implies nothing about Lyft's fixed cost structure (if anything, average CLV at a company may be positively correlated with the amount of fixed costs a business has, because companies that invest heavily in automation improve the

FINANCIAL TIMES

IPOs US & Canadian companies Technology sector

Lyft's insurance problem



22

MARCH 28, 2019 By: **Jamie Powell**

Lyft, the "nice" ride-sharing company, is IPO'ing on Nasdaq Friday. It's expecting to be valued above \$23bn.

As with all US IPOs, the company submitted an [S-1 filing](#) with the Securities and Exchange Commission over a month ago.

Alphaville thought about doing a take. Then we didn't. And then everyone else did. So, in our unbearably contrarian ways, we held off.

(The best analysis, in our humble opinion, is a two-part done by Theta Equity Partners. It can be read [here](#) and [here](#).)

But to our surprise, one part of Lyft's filing has gone largely unnoticed by the finance media: how fundamental insurance is to its current operating model and its future

Slack's Fair Equity Valuation: \$22-27B With Upside Potential



Lyft and Uber's stocks have performed very poorly post-IPO (Lyft down 30% from its IPO 14, 2019). They have performed so poorly, in fact, that many popular media outlets are [have been "poisoned"](#) by these firms. Investors are wondering if this means companies IPOs will all face similar struggles now -- if so, Slack would have trouble getting a high v losing money. And [unlike Uber](#), Slack won't have underwriters supporting its stock thro because it is going public through a direct public offering (DPO) instead of an IPO.

In [our previous article about Slack](#), we assessed the firm's unit economics by analyzing customer data from its pre-DPO filing. We concluded that it has very strong customer profitability (more than 1,000% marketing ROI), even though it takes about 3 years on average to pay back the initial investment in customer acquisition, and much of the value will be coming from just 1% of the customers they acquire. In contrast, we estimated [Lyft's return on CAC](#) at a much lower ~60% (we couldn't perform a similar analysis on Uber because they chose not to disclose critical customer metrics such as cohort-level repeat usage or customer acquisition data).

It makes intuitive sense that Slack's strong CLV numbers should be directionally positive for their valuation, and in this post we show exactly how into an estimate of th take the model that v revenue we can expect variable profits, and h

Cutting to the chase, more recent private m 60%+ upside opportu acquired customers, s

Slack Technologies: It's Not Too Late to Buy the Stock at a Discount

Alex Durhortier, CFA, The Motley Fool
Motley Fool June 26, 2019

Following [Lyft](#), [Pinterest](#), and [Uber](#), another "unicorn" (a start-up with a private valuation above \$1 billion) has been set free into the public markets. **Slack Technologies** (NYSE: WORK) completed its direct public offering last Thursday, and *unlike* Lyft and Uber -- which are now "busted" IPOs -- shares of the seemingly ubiquitous workplace collaboration software provider have performed well. Monday's \$35.76 closing price is at a 38% premium to the stock's \$26 per share reference price. Many may be thinking that this pop has eliminated any opportunity for fundamental investors to pick up Slack at a reasonable price. Well, think again, for as one value investor quipped recently, "The three most dangerous words in investing are 'I missed it.'"

Even at their current price, there is evidence to suggest that Slack Technologies shares remain undervalued. I have highlighted the work of Theta Equity Partners before, and the research firm has already distinguished itself in regard to [unicorn share offerings](#). Theta has pioneered a powerful approach to [customer lifetime value analysis](#) that takes a deep, data-driven look at customer behavior, and it ran Slack's numbers through its model.

Slack's numbers are compelling

I won't keep you in suspense -- here's what Theta concluded in regard to Slack's valuation prior to its share offering (emphasis in the original):



Salesforce buys Slack in a \$27.7B megadeal

Ron Miller, Alex Wilhelm / 4:07 PM EST • December 1, 2020

Comment

weighted cohort, and of Slack's n to 11%, ... average of \$30B. r Bloomberg, and a 40% (ched).

Warby Parker: While the Stock Pendulum Swings, the Unit Economics Hold Steady

🕒 July 26, 2022 📁 CBCV, Warby Parker

Under the assumptions outlined above, our **base case valuation of Warby Parker's equity is at \$2.4 billion, or \$21 per share**. The valuation is slightly lower than our IPO analysis, where we valued Warby Parker's equity at \$2.5 billion, or \$22 per share. That said, it is still well above the aforementioned \$11 at which the company is currently trading. In other words, **if our analysis is correct, Warby Parker is trading at an attractive valuation with meaningful upside potential right now.**

<https://thetaclv.com/resource/warby-parker-while-the-stock-pendulum-swings-the-unit-economics-hold-steady/>

Hypothetical trading strategy

(McCarthy 2020)

- Revisit seven “live” CBCV analyses

McCarthy and Fader (JMR 2018)



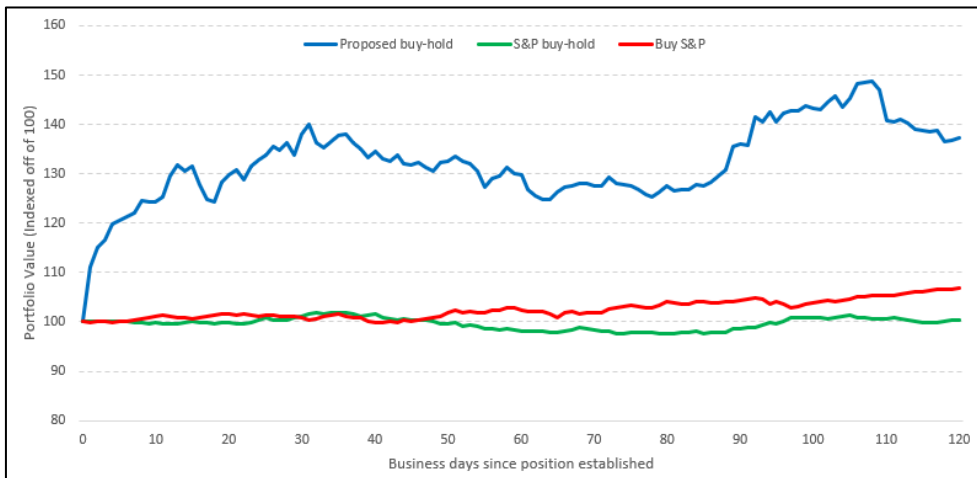
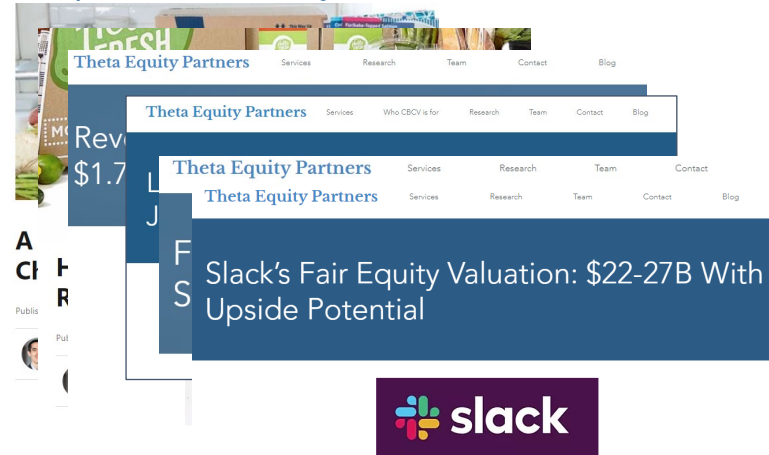
REVOLVE



FARFETCH



Every previously posted (but unpublished) CBCV analyses





When Data Creates Competitive Advantage

...and when it doesn't
94



CURRENT ISSUE

January–February 2020

When data creates competitive advantage...and when it doesn't

Spotlight

Are You Undervaluing Your Customers?

CUSTOMERS MAGAZINE ARTICLE by Rob Markey

It's time to start measuring and managing their worth.

Save Share

How to Value a Company by Analyzing Its Customers

CUSTOMERS MAGAZINE ARTICLE by Daniel McCarthy and Peter Fader

A step-by-step guide to customer-based corporate valuation

Save Share

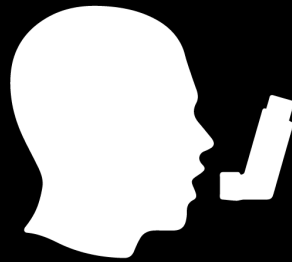
Read it here:

https://bit.ly/HBR_FASB

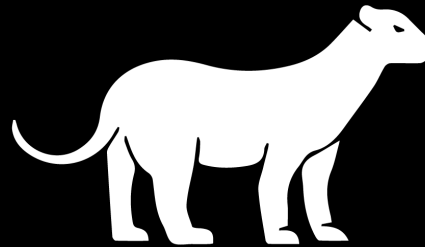
Other applications of BTYD models



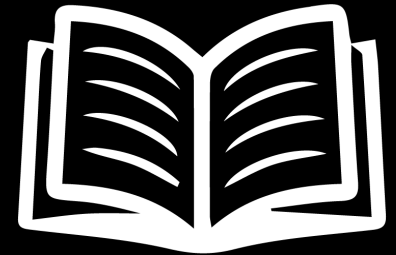
**Doctors Without
Borders**



Pediatric Asthma



Animal Tracking



Library Books

(See more in TEDxPenn talk: <https://youtu.be/dHRoO4kbtug>)

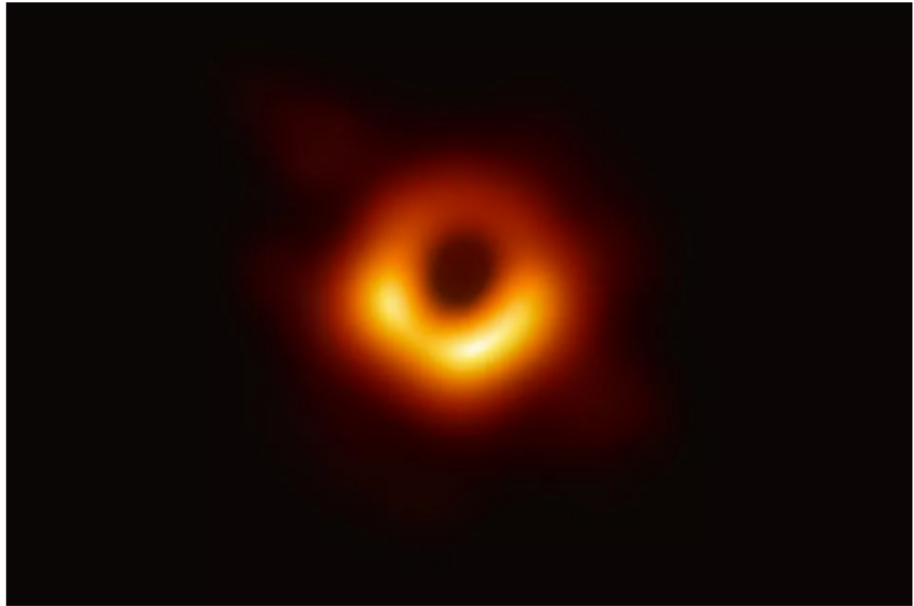


Historic 1st Photo of a Black Hole Named Science Breakthrough of 2019

By Charles Q. Choi December 19, 2019

It was once thought impossible. Then it happened.

 Comments (0)



The Event Horizon Telescope, a planet-scale array of eight ground-based radio telescopes forged through international collaboration, captured this image of the supermassive black hole in the center of the galaxy M87 and its shadow. (Image: © EHT Collaboration)

The [first image of a black hole](#), previously thought nigh impossible to capture, was named the top scientific breakthrough of 2019 by the journal Science.

Press Release: The Nobel Prize in Physics 2017

3 October 2017

The [Royal Swedish Academy of Sciences](#) has decided to award the Nobel Prize in Physics 2017 with one half to

Rainer Weiss
LIGO/VIRGO Collaboration

and the other half jointly to

Barry C. Barish
LIGO/VIRGO Collaboration

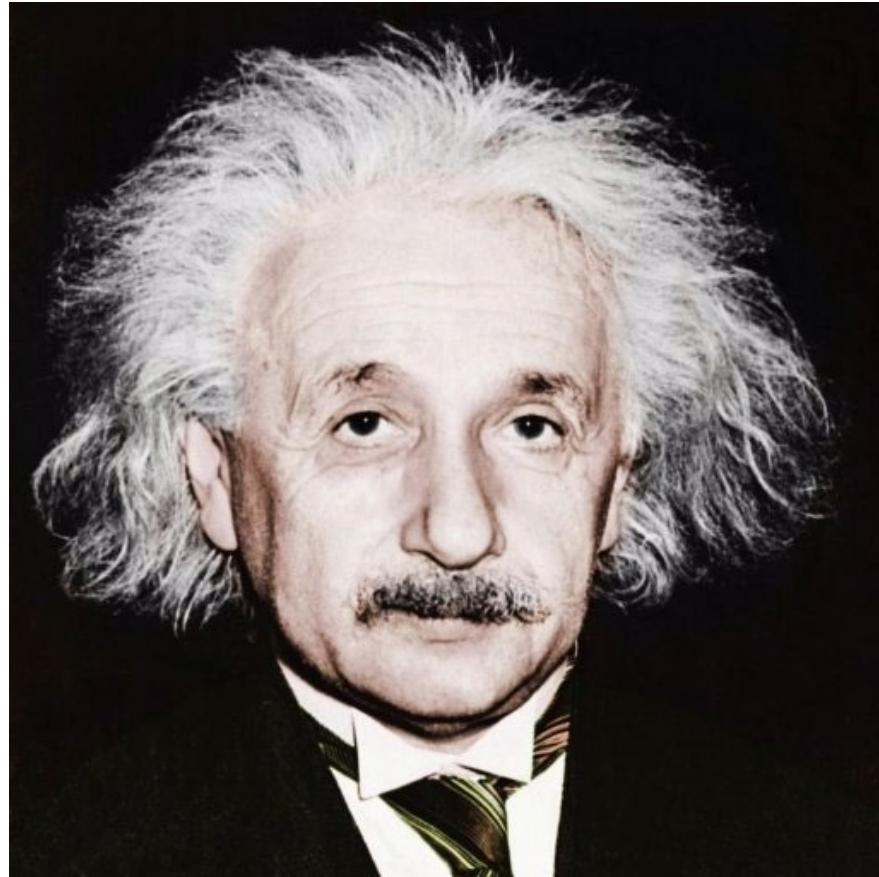
and

Kip S. Thorne
LIGO/VIRGO Collaboration

"for decisive contributions to the LIGO detector and the observation of gravitational waves"

Gravitational waves finally captured

On 14 September 2015, the universe's gravitational waves were observed for the very first time. The waves, which were predicted by Albert Einstein a hundred years ago, came from a collision between two black holes. It took 1.3 billion years for the waves to arrive at the LIGO detector in the USA.



“Make everything as simple as possible, but not simpler”

Further resources

- Follow/connect with Dan McCarthy: daniel.mccarthy@emory.edu, @d_mccar
- (And me: faderp@wharton.upenn.edu, @faderp)
- The two main CBCV papers: <http://whr.tn/CorpValPaper1> & <http://whr.tn/CorpValPaper2>
- Theta: <https://www.thetaCLV.com/>

