The Length of Contracts and Collusion

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Many commodities (including energy, agricultural products and metals) are sold both on spot markets and through long-term contracts which commit the parties to exchange the commodity in each of a number of spot market periods. This paper shows how the length of contracts affects the possibility of collusion in a repeated price-setting game. Contracts can both help and hinder collusion, because reducing the size of the spot market cuts both the immediate gain from defection and the punishment for deviation. Firms can always sustain some collusive price above marginal cost if they sell the right number of contracts, of any duration, whatever their discount factor. As the duration of contracts increases, however, collusion becomes harder to sustain.

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1 Introduction

Many commodities are sold both through long-term contracts and on spot markets. Empirical evidence suggests that most forward contracts last for many spot market trading periods. For example, two-thirds (by volume) of the gas trades reported in Britain on December 3, 2004 were for deliveries spread over a month or more, as were more than 90% of the electricity trades (Heren, 2004a,b). Fifty percent of California winegrape producers have contracts of more than one year with an average length of 3.5 years, the most frequent contract lengths being 3, 5 and 10 years (Goodhue *et al.*, 1999). In the U.S. agricultural industry, many contracts are signed for 3 months (20% for cattle and 35% for poultry) or for 3-12 months (80% and 49% respectively) (USDA, 1993). For non-ferrous metals traded on the London Metal Exchange similarly, contracts range from one day to many years (Slade and Tille, 2004 or www.lme.com).

The seminal result on the interaction between spot and forward markets (Allaz and Vila, 1993) is that the presence of a forward market will make the spot market more competitive – firms are competing only over the unsold portion of the overall demand, and face a more elastic residual demand curve. More recently, however, some authors have suggested ways in which forward markets could be used to make the spot market less competitive. One possibility is that firms could buy in the forward market to increase their exposure to the spot market, and make their residual demand less elastic (Mahenc and Salanié, 2004). Another is that contracts could increase the likelihood and severity of collusion (Ferreira, 2003, Le Coq, 2004, Liski and Montero, 2006). The key intuition of these papers is that a firm which defects from a collusive agreement will not be able to capture the demand already covered by contract sales. Compared to the case with no contracts, this reduces the gains from defection without changing the punishment path, and therefore makes collusion easier to sustain. An important feature of these papers is that the contracts studied only last for a single spot period.

The aim of this paper is to investigate how the length of forward contracts (i.e. the number of spot periods with delivery commitment) affects the firms' ability to collude on the spot market. We show that the longer the contracts last, the more difficult it is to sustain collusion. At the same time, however, every firm can sustain some collusive price above marginal cost for any length of contracts. This includes firms that would not be able to collude in the absence of contracts – we show that they will need to cover a high proportion of their output with contracts to make collusion possible.

The mechanism works as follows. We adapt the setting used in Liski and Montero (2006) where firms repeatedly alternate between selling contracts and competing in price on the spot market, but with several spot periods between each round of forward trading.[1] A contract is thus an agreement where a firm (buyer) commits to sell (to buy) a fixed quantity at a fixed price in each of a number of spot market periods. The length of the contract is defined as the number of spot periods for which the agreement stands, and is set exogenously. Assume that firms collude on a price above marginal cost with a trigger strategy. Any contract reduces the demand to be met in the spot market, reducing the short-term gain from defection, and therefore makes collusion more attractive. The contracts affect the payoff during the punishment phase, however. As soon as deviation occurs, the punishment is to offer the competitive price in the spot market in every period afterwards, and sell contracts at this price from the next contract round onwards. This punishment immediately affects profits in the spot market, but the deviating firm still receives profits from its contracts, until the next contract round. While the defecting firm cannot take away the sales its rival has covered by contracts in the period in which it defects, its rival cannot take away the sales the firm has covered by contracts made before the defection. This reduces the severity of the punishment that can be inflicted for defection. The longer the contracts last for, the greater the reduction of the punishment. The contracted quantities act as a "protection" during the length of the contract. The paper discusses how these two effects – the lower gain from deviation and the reduced punishment afterwards – interact as the length of the contracts varies. We call the first effect the "gain-cutting effect" and the second the "protection effect".

While we are not aware of other papers that have paid attention to the length of the contract in an oligopolistic game, Liski and Montero (2006) briefly consider what would happen if firms traded one-period contracts many periods in advance, which is in some ways equivalent to trading a multi-period contract. They point out that in these later periods, the contracts

[1] The terminology used for forward market and spot market can differ between industries. In some industries there is no obvious spot or forward market, but short-term and long-term contracts coexist. Our model is applicable to the interaction between sales of short-term and long-term contracts in these industries, as well as to those in which there is a (more or less) organised spot market. However, we will always use the term "spot" market to refer to the short-term market and forward contract to refer to the longer-term commitments.

actually make collusion harder to sustain, and so the firms would not want to sell contracts so far in advance that they were unable to collude. If each period has its own contract, this is easy to achieve. Their paper therefore naturally concentrates on the key result that short-term contracting makes collusion easier, and demonstrates this result with collusive paths supported by contracts sold a single period in advance.

In practice, firms may not be able to choose exactly how far in advance (or for how long) they sell their output. Market conventions generally dictate that forward contracts will last for a week, a month, a quarter or a year. Moreover, liquidity is concentrated in the contracts which are closest to delivery. We therefore assume that a firm cannot manufacture, say, an eleven-month contract by selling a year-long contract and simultaneously buying an identical contract for the final month of that year. The firm might be able to unwind part of its commitment later in the year, once there is more liquidity in the market for the final month. This would only affect the firm's payoffs if it happened before a deviation has occurred, however, and we will show that any deviation will take place at the start of the contract's life. The firms in our model have to take the length of contracts as given, but still wish to maintain a collusive equilibrium.

We are considering forward contracts for physical delivery, which remove part of the market demand from a defecting firm. If the firms had sold contracts that are financially settled, such as most futures contracts, the defector would still be able to cover the entire physical market demand at a defection price below the collusive price. Furthermore, the defector would still receive the higher price on that part of their output covered by contracts, raising their deviation profits, relative to the situation with no contracts.[2] Financially settled contracts therefore make collusion harder than the forward contracts we study.

With different contract lengths and an infinitely repeated game we offer a general setup described in the next section. In section 3, we give the general conditions for collusion to be sustainable. Section 4 shows how the discount factor needed to sustain the collusive price varies with the proportion of the collusive output covered by contracts, and the duration of those contracts. This allows us to demonstrate the two effects at work. Section 5 discusses how the collusive price varies with the discount factor, the proportion of output sold through contracts, and their duration, and shows that firms can maintain some collusive price above marginal cost for *any* positive discount factor, given appropriate contracts. Section 6 illustrates our results by using a linear demand example. Section 7 concludes.

2 The model

Consider two firms, *1* and *2*, who produce a homogeneous good with a constant marginal cost c and no capacity constraints. They can sell their good on either the contract market or the spot market. The firms have a common discount factor, δ.

Timing. Firms compete in price repeatedly on the spot market (taking place in all periods t =1,2,3,...). There is a contract round, which takes no time, before the first spot period. The contracts sold in this round will last for $\lambda > 0$ spot periods. As soon as they expire, there is another contract round, followed by λ spot periods, and so on.[3]

Demand. In each period, the demand is given by *D(p)*, which is a decreasing and continuous function of the spot price p. This demand can be met either by sales in the spot market or by commitments made under forward contracts. The price paid for forward contracts does not affect this demand – if the prices differ, buyers effectively receive a lump sum gain or loss from their contract holdings, but there is no income effect.

Contract market. In each contract round, the two firms simultaneously choose the amount of forward contracts they want to sell in the forward market that call for delivery of an equal volume in each of the next λ spot periods. Payment is made at the time(s) of delivery, avoiding the need to use different amounts of discounting for spot and contract sales. It is convenient to work in terms of the proportion of output sold in the contract market, rather than its volume. We define $x \in [0,1]$ as the proportion of contract sales, relative to the total output the firm would sell in the

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[2] This distinction is also noted by Liski and Montero (2006), footnote 5.

[3] Our main qualitative results would also hold if contracts were traded before every spot period, with $1/\lambda$ of the stock being renewed each time. The gain-cutting effect would be unchanged, while the protection effect would be weakened, as the number of contracts held during the punishment phase would fall continuously. In practice, however, many industries have contracts which start on standard dates, such as the first day of the month, which fits the simpler formulation we use. The qualitative results would also hold if some contracts were traded further in advance (e.g. March contracts were traded in January), but the protection effect would be greater, lasting until all the contracts already signed at the time of a deviation had expired.

collusive equilibrium (spot and forward sales).[4] We treat λ and x as fixed exogenously, although they could be chosen as part of a collusive agreement. The firms' contract positions become common knowledge at the end of each contract round (i.e., before the first spot period for which those contracts apply).

No arbitrage. We assume that the contracts will only be accepted by purchasers if the contract price is equal to the expected price in the spot market. In other words, the firms will be able to sell contracts at a particular collusive price, if and only if they will be able to sustain this price in the spot market, given their discount factors and contract sales.

Spot market. In each spot period, the firms simultaneously bid prices, and the firm with the lower price serves the entire spot demand – the total demand at that price, less the forward commitments by each firm. If this would be negative (demand at the spot price is less than the forward sales), there are no spot market transactions.[5] We assume an efficient-rationing rule; if the firms submit the same price bid, they share the spot demand equally.

General profit functions. Aggregate profits are given by $\pi(p) = (p - c)D(p)$ and are assumed to be single peaked with a unique maximum at $\pi(p^m)$. Let $\pi^m (= \pi(p^m))$ denote the per-period profit earned by a monopoly firm. Let $\pi^c (= \pi(p^c))$ denote the per-period profit earned by each firm from collusion at the price p^c. And finally π^d denotes the firm's profit during the deviation period, taking its spot and contracted output together.

3. Sustaining Collusion

We restrict our attention to stationary collusive agreements supported by trigger strategies. This greatly simplifies the analysis and its exposition and does not restrict the scope of the results.[6]

[4] We follow Liski and Montero's notation, but we do not allow for over-contracting or forward purchases, requiring that $x \in [0,1]$.

[5] This will not be the case in equilibrium, or along the deviation paths we consider, but we state it for completeness.

[6] Players revert to the static Nash equilibrium and remain there forever after any deviation. Exactly as in a repeated Bertrand competition, unrelenting trigger strategies are "optimal punishments" in our setting, since the players are at their security levels. Expressed differently, no complex punishment mechanism can enlarge the set of sustainable equilibria (Abreu, 1986).

In the first contract round, each firm offers long-term contracts to sell $xD(p^c)/2$ at the collusive price p^c, where $D(p^c)/2$ is the output it would produce if it shared the (overall) market equally at this price. If both firms have followed the collusive equilibrium so far, then in each spot period, the firm will bid a spot price of p^c, sharing the spot market demand equally with the other firm. If at any point in time anyone is detected cheating in any previous contract round or spot period, then both firms will bid marginal cost, c, in each subsequent spot period. In every subsequent contract round after a deviation has occurred, the firms will sell an arbitrary volume of contracts, at the same price, c, as long as their combined sales do not exceed the market demand at marginal cost, $D(c)$.

We now investigate whether these strategies can sustain collusion against an optimal deviation. In principle, a firm can deviate by either increasing its forward sales on the contract round or undercutting its spot price. Deviating during the contract round is never optimal, as shown by Liski and Montero (2004). Contracts are fully observable, and buyers know the firms' strategies. If one firm deviates in a contract round, the spot price in every succeeding spot period will be equal to c. Knowing this, no buyer would pay more than c in the forward market to a seller that is attempting to deviate from its collusive strategy. The profit from deviation will thus be zero. Any collusive strategy that gives a profit of more than zero is thus proof against collusion occurring in a contract round. We can then state:

Lemma 1. *It is never optimal to deviate during a contract round.*

Thus, we need only to concentrate on deviations in the spot market. If there were no contracts, by slightly undercutting $p^c \in (c, p^m]$, a firm can earn approximately the aggregate collusive profit. Given that at the opening of the spot market in period t there is an already secured supply of $xD(p^c)$ units coming from firms' forward obligations signed in the most recent contract round, this may not be the optimal deviation in our model. If the residual demand in the spot market is too low, p^c may be above the monopoly price associated with that demand, and hence too high to be an optimal deviation.

Define p^{Rm} as the monopoly price associated with the residual demand[7], and given by

$$p^{Rm} = p^{Rm}(x, p^c) \equiv \arg\max(p-c)(D(p) - xD(p^c)) \tag{1}$$

The optimal deviation price is then defined as

$$p^d = p^d(x, p^c) \equiv \min\left[\sup(p \mid p < p^c),\ p^{Rm}\right] \tag{2}$$

The firm's spot market profits from charging the optimal deviation price are given by π^{sd}, where

$$\pi^{sd} = \pi^{sd}(x, p^c, p^d) = (p^d - c)\left(D(p^d) - xD(p^c)\right) \tag{3}$$

The firm's total profits in the spot period during which it deviates are thus equal to

$$\pi^d = \pi^d(x, p^c, p^d) = \pi^{sd} + x\pi^c \tag{4}$$

In the punishment phase, starting just after the deviation period, the firms offer c on the spot market forever and offer forward contracts at that price in all following contract rounds. As a result, every subsequent spot profit is equal to zero. However, until the current contracts expire, firms still sell their contracted quantities at the collusive price, p^c. The punishment does not affect those quantities until the next contract round starts.

This affects the optimal timing of a deviation. The present value of the profits from collusion continued forever equals $\pi^c/(1-\delta)$ in any spot market period. This includes profits of $x\pi^c$ from forward sales and $(1-x)\ \pi^c$ from sales in the spot market. The present value of the profits from a deviation depends on the number of spot market periods remaining before the next contract round. Consider a deviation with $\tau \in \{1, 2, \ldots \lambda\}$ periods remaining until the next contract round. We can write the profits during the punishment phase starting after the deviation period, discounted to the deviation period, as $x\pi^c \sum_{t=1}^{\tau-1} \delta^t$. The present value of the overall profits from deviation is thus equal to

$$PV^{deviation} = \pi^d + x\pi^c \frac{\delta - \delta^\tau}{1-\delta} \tag{5}$$

This allows us to state:

Lemma 2. *A deviation is the most profitable when it occurs in the spot period immediately after a contract round.*

[7] We assume that $(p-c)(D(p) - xD(p^c))$ is a single peaked function with a unique maximum p^{Rm}.

Proof: Immediate, from inspection of equation (5), which is maximised when τ is at the highest possible value, implying $\tau = \lambda$. Since the profits from continued collusion do not depend on time, choosing the highest level of deviation profits is sufficient to choose the best time to deviate. ■

Using the one-stage deviation principle for infinite-horizon games, the collusive agreement described above is sustainable as a subgame-perfect equilibrium as long as neither firm has an incentive at any period to defect unilaterally from the collusive agreement. The punishment phase (i.e., reversion to marginal cost forever) is subgame perfect, so we need to find the condition under which deviation from the collusive path is not profitable for either firm. Setting $\tau = \lambda$ in equation (5), this condition is equivalent to

$$\frac{\pi^c}{1-\delta} \geq \pi^d + \frac{\delta - \delta^\lambda}{1-\delta} x\pi^c \tag{6}$$

The first part of the RHS of the inequality (6) gives the first-period deviation profit, which includes both profit from the residual spot demand and the forward sales in the deviation period. The second part of the RHS gives the profits that the firm will obtain from its forward sales after the deviation occurs until the period immediately before the next contract round.

We can use the fact that $\pi^d = \pi^{sd} + x\pi^c$ to rewrite equation (6) as:

$$\left[1 - \left(1 - \delta^\lambda\right)x\right]\pi^c \geq (1-\delta)\pi^{sd} \tag{7}$$

Let us define

$$h(x, \lambda, \delta, p^c) = \left[1 - \left(1 - \delta^\lambda\right)x\right]\pi^c - (1-\delta)\pi^{sd} \tag{8}$$

We define the minimum discount factor as $\underline{\delta} = \underline{\delta}(x, \lambda, p^c)$ such that $h(x, \lambda, \underline{\delta}, p^c) = 0$. Hence we can state

Proposition 1. *The grim trigger strategies described above constitute a subgame perfect equilibrium if and only if* $\delta \geq \underline{\delta}(x, \lambda, p^c)$, *where* $h(x, \lambda, \underline{\delta}, p^c) = 0$.

Proof. The collusion is sustainable for any discount factor satisfying inequality (7). From the definition of *h* (equation (8)), we know that this is equivalent to $h(x, \lambda, \delta, p^c) \geq 0$. We also have:

$$\frac{\partial h(x, \lambda, \delta, p^c)}{\partial \delta} = \lambda \delta^{\lambda - 1} x \pi^c + \pi^{sd} > 0 \quad \forall p^c > c \tag{9}$$

This implies that any discount factor greater than $\underline{\delta}$ will give $h > 0$. ■

Proposition 1 sets out the sufficient condition for a collusive equilibrium. In the following sections, we discuss its implications, concentrating first on the interaction between the minimum discount factor and the length of contracts, and second on the price that can be sustained through collusion.

4. The critical discount factor

Why does the level of forward contracting affect the critical discount factor at which collusion can just be sustained? There are two effects playing in opposite directions. First there is a pro-collusive effect (captured by the variable π^d in the inequality (6)) which we call the *gain-cutting effect*. Second there is a pro-competitive effect (captured by the term $(\delta - \delta^\lambda) / (1 - \delta) x \pi^c$ in the inequality (6)) which we call the *protection effect*.

Assume that firms have signed contracts ($x > 0$). A firm which defects from a collusive agreement will not be able to capture the demand already covered by contract sales. This reduces the gains from defection, for a given collusive price and discount factor, allowing the firms to sustain a higher collusive price, or making collusion possible with a lower discount factor. We say that the forward sales have a *gain-cutting effect* that increases with the amount of forward sales (because we have $\partial \pi^d / \partial x < 0$). The more forward sales there are, the less the deviation profit is, which tends to make it easier to collude.

Forward sales can have a second impact on collusion, however, if they last for more than one period. While the defecting firm cannot take away the sales its rival has covered by contracts in the period in which it defects, its rival cannot take away the sales the firm has covered by contracts made before the defection. Hence if $\lambda > 1$, a firm that defects in the first spot period after a contract round will continue to receive the fixed forward price for part of its output, until the next contract round occurs. This reduces the severity of the punishment that can be inflicted for defection. The longer the length of the contracts, the greater this reduction, and the more protected are the firms. This *protection effect* increases with the length of the contract (since the coefficient $(\delta - \delta^{\lambda})/(1-\delta)$ increases in λ) as well as with the contracted quantity.

It is important to notice that the two effects do not play at the same time. The gain-cutting effect appears (during the deviation period) before the protection effect. Moreover the gain-cutting effect always exists as soon as $x > 0$, while the protection effect only appears when $\lambda > 1$. The overall impact on the sustainability of collusion depends upon the interaction of the two effects.

We now explore how the minimum discount factor $\underline{\delta}$ varies with the length of the contract (λ) and with the contracted quantity (x). Before exploring the general results of Proposition 1, we discuss two simple examples that allow us to relate to previous literature.

No contracting case (Repeated Bertrand game). Assume that firms hold no contracts $(x = 0)$ and agree to collude on the monopoly price; the game is identical to the traditional repeated price game. By sticking to the agreement, a firm receives $\pi^m / 2$ which corresponds to half the aggregate monopoly profit. When $x = 0$, the optimal deviation is to undercut the collusive price by a fractional amount, capturing the entire market demand at this price, and obtaining twice the collusive profit in that period. In all future periods, the unilateral deviation triggers retaliation from the other firm. As a result, in all subsequent periods firm *i* earns the static Nash equilibrium profits, that is $\pi(c) = 0$. We can rewrite the inequality (7) as $\pi^m/(1-\delta) \geq 2\pi^m$. From Proposition 1, if there are no contracts, firms can sustain the monopoly price if their discount factor is ½ or more, and cannot sustain any price above marginal cost for any lower discount factor (Tirole, 1988). With no contracting, neither the gain-cutting effect nor the protection effect exists.

One period contracting case. Assume firms offer one-period contracts ($x > 0$ and $\lambda = 1$) and agree to collude on the monopoly price p^m, so that $\pi^c = \pi^m/2$. It is straightforward to show that whenever x>0 and $p^c = p^m$ the optimal deviation price is the residual monopoly price p^{Rm} defined by equation (1). Since contracts only last for one period, the punishment phase starts immediately after the deviation period. As a result, in all subsequent periods firm *i* earns the static Nash equilibrium profits, that is $\pi(c) = 0$, and there is no protection effect. Collusion is sustainable whenever $\pi^m/(1-\delta) \geq \pi^{sd} + x\pi^m$. Recall that from equation (3) we have $\pi^{sd} = 2(1-x)\pi^m$, and so the condition can be rewritten as $1/(1-\delta) \geq 2 - x$. Since the gain-cutting effect becomes stronger as the level of contracting rises, the critical discount factor is strictly decreasing in the level of one-period contracting. This explains the main result with one-period contracting (studied first by Liski and Montero, 2006): forward trading allows firms to sustain collusive profits that otherwise would not be possible. When we allow for longer contract lengths this result does not always hold.

General case. For any length of the contract (λ), we have:

Proposition 2. *The critical discount factor is strictly increasing in the length of the contract.*

Proof. We can write the partial derivative of *h* with respect to δ as $h_\delta = \lambda\delta^{\lambda-1}x\pi^c + \pi^{sd}$, which is clearly positive for prices above marginal cost. The partial derivative of *h* with respect to λ is $h_\lambda = x\pi^c\delta^\lambda \ln(\delta) < 0$, since $\delta < 1$. By the implicit function theorem, we have $d\underline{\delta}/d\lambda = -h_\lambda/h_\delta$. Using the above inequalities we can conclude that the critical discount factor $\underline{\delta}$ is strictly increasing in the length of the contract ($d\underline{\delta}/d\lambda > 0$).■

With one period contracting, the firm knows that in every period after its defection its profits will be reduced to zero. Proposition 2 arises because if the contract lasts for more than one period ($\lambda > 1$), however, a firm that defects in the first spot period after a contract round will continue to receive the fixed forward price for part of its output, until the next contract round occurs. This reduces the severity of the punishment that the firm will receive – the longer the contracts last, the less the firm's profits will be reduced. Hence the greater the length of the contract, the longer the firm is protected against harsh punishment. This increases the temptation to deviate.

Proposition 3. *The critical discount factor may rise or fall as the proportion of contracted output increases.*

Proof. We have $h_x = -\left(1-\delta^{\lambda}\right)\pi^c - (1-\delta)\partial\pi^{sd} / \partial x$, and note that $\partial\pi^{sd} / \partial x < 0$, since increases in contract cover reduce the residual demand available in the spot market to a deviating firm. It is easy to show that depending on the values of x, p^c, λ and δ, the function h_x can be positive or negative. In Section 6 we give examples of these cases when demand is linear. By the implicit function theorem, we have $d\underline{\delta}\big/dx = -h_x\big/h_\delta$. This implies Proposition 3. ■

The reason for this result is that the level of contracted output influences both of our effects. When the contracted quantity increases, the residual demand decreases and therefore the gains from deviation are lower (the gain-cutting effect becomes stronger). At the same time, when the contracted quantity increases, the forward profit increases, implying that the protection effect becomes stronger. This gives more incentive to deviate from the collusive agreement. Since the two effects are working in opposite directions, we get an ambiguous overall impact on the sustainability of collusion.

5. The level of the collusive price

To explore the impact of the level of the collusive price upon the sustainability of collusion, we start by asking whether firms would deviate with a small or a large reduction in the spot price. We already defined $p^d(x, p^c)$ as the optimal deviation price given p^c and x where $p^d(x, p^c) = \min\left\{p^c - \varepsilon, p^{Rm}(x, p^c)\right\}$.

Note that the residual monopoly price strictly falls with x and strictly rises with p^c. Moreover $p^{Rm}(0, p^c) = p^m$, $p^{Rm}(1, p^c) < p^c$, and $p^{Rm}(1, c) = c$. By continuity, it follows that there exists a unique $x^*(p^c) \in [0,1]$ such that $p^{Rm}\left(x^*(p^c), p^c\right) = p^c$.

Assume that $p^{Rm}(x, p^c)$ is differentiable with

$$\frac{\partial p^{Rm}(x, p^c)}{\partial x} < 0 \quad \forall x \in [0,1) \tag{10}$$

and

$$\frac{\partial p^{Rm}(x, p^c)}{\partial p^c} > 0 \quad \forall p^c \in [c, p^m) \tag{11}$$

Furthermore:

$$\begin{cases} p^c \leq p^{Rm}(x, p^c) & \forall x \in [0, x^*] \\ p^c > p^{Rm}(x, p^c) & \forall x \in (x^*, 1] \end{cases}. \tag{12}$$

If the collusive price is high and the level of contracting is low, the residual monopoly price will be greater than the collusive price. In that case, the optimal deviation will be to undercut the collusive price by a small amount, as it is when firms are trying to collude on the monopoly price with no contracts.

We can rewrite the definition of the optimal deviation price as follows:

$$p^d = p^d(x, p^c) = \begin{cases} p^c - \varepsilon & \text{if} \quad x \leq x^*(p^c) \\ p^{Rm}(x, p^c) & \text{if} \quad x > x^*(p^c) \end{cases} \tag{13}$$

where $x^*(p^m) = 0, x^*(c) = 1$ and $\dfrac{\partial x^*(p^c)}{\partial p^c} < 0$.

Figure 1 shows the two regions, in which deviating with a price slightly under p^c, and by setting p^{Rm}, is optimal. We call these region 1 and region 2, respectively. We plot the frontier between these two kinds of deviation, by equating p^c and p^{Rm}. It leaves the vertical axis at p^m, and reaches a value of 1 at a price of c. We had already seen that slightly undercutting $p^c \in (c, p^m]$ is optimal when there were no contracts. The right-hand end of the line implies that this is also the case when the collusive price is close to marginal cost, unless there is a very high degree of contracting. Covering more output with forward sales makes the residual demand in the spot market more elastic, favouring a lower deviation price. Setting a lower collusive price makes the residual demand at that price less elastic, favouring a high deviation price.

Figure 1 about here.

The distinction between these two regions is important, because the firm's spot market profits after deviation are different. In both regions, the firm's forward profits are equal to $\pi^f = x\pi^c$, and its spot market profits from collusion are equal to $\pi^s = (1-x)\pi^c$. In region 1, the optimal deviation of slightly undercutting p^c gives the firm double the spot market profits that it would gain from collusion. We use this straightforward relationship in the proof of:

Proposition 4: *For any positive discount factor and any length of contracts, given an appropriate level of forward contracting a firm can sustain collusion at some price above marginal cost.*

Proof. To prove Proposition 4, it is enough to show that there exist a pair (p^c, x) in Region 1 and a discount factor $\delta \in [0,1]$ such that collusion at $p^c > c$ is sustainable. In region 1, by slightly undercutting p^c the deviating firm receives $\pi^{sd} = 2(1-x)\pi^c$ as its spot market profits. From equation (7), collusion is sustainable if and only if $\frac{\pi^c}{1-\delta} \geq (2-x)\pi^c + \frac{\delta - \delta^\lambda}{1-\delta} x\pi^c$. We can rewrite this inequality as $1-(1-\delta)(2-x)-\left(\delta-\delta^\lambda\right)x > 0$, which is equivalent to $(2\delta-1)(1-x)+x\delta^\lambda > 0$. The last inequality is always true for $\delta > 1/2$ or if

$$x\delta^\lambda > (1-2\delta)(1-x) \tag{14}$$

Note that as x goes to 1, the above inequality is true for any $\delta > 0$, whatever the value of λ. ■

This proposition shows that forward contracts make it possible for firms to maintain some level of collusion at a price above marginal cost, however long those contracts last for, and however low their discount factor. Without contracts, collusion would only be possible for a discount factor of ½ or more, and so this result implies that contracts can have a pro-collusive effect, just as for Liski and Montero (2004). It is worth pointing out, however, that if the discount factor is low, or λ is large, a high level of contract cover will be required. The proof, which is based on a sufficient but not a necessary condition, requires that the firm is in region 1, implying that p^c must not be too high for the level of x, and that the highest p^c falls as x rises.

We define the maximum sustainable collusive price as $\overline{p^c} = \overline{p^c}(x, \lambda, \delta)$ such that $h\left(x, \lambda, \delta, \overline{p^c}\right) = 0$. Within region 1, the level of the price does not affect the sustainability of collusion, implying that the highest sustainable price must be (weakly) within region 2. This in turn implies that the deviation price is set by maximising profits, given the residual demand in the spot market, rather than by slightly undercutting $\overline{p^c}$. We will use this in the proof of:

Proposition 5: *Assume that* $\partial^2 D(p)/\partial p^2 < 0$. *The maximum sustainable collusive price is increasing in the discount factor.*

Proof. Consider the maximum collusive price $\overline{p^c}$. By the implicit function theorem, we have $\partial\overline{p^c}/\partial\delta = -h_\delta/h_{p^c}$. We have $h_{p^c} = \left(1-\left(1-\delta^\lambda\right)x\right)\dfrac{\partial\pi^c}{\partial p^c} - (1-\delta)\dfrac{\partial\pi^{sd}}{\partial p^c}$. Since $h\left(x,\lambda,\delta,\overline{p^c}\right)=0$, we have $1-\left(1-\delta^\lambda\right)x = (1-\delta)\dfrac{\pi^{sd}}{\pi^c}$, and so $h_{p^c}\big|_{\overline{p^c}} = (1-\delta)\left(\dfrac{\pi^{sd}}{\pi^c}\dfrac{\partial\pi^c}{\partial p^c} - \dfrac{\partial\pi^{sd}}{\partial p^c}\right)$.

With $\dfrac{\partial\pi^c}{\partial p^c} = \dfrac{(p^c-c)}{2}\dfrac{\partial D(p^c)}{\partial p^c} + \dfrac{D(p^c)}{2}$ and $\dfrac{\partial\pi^{sd}}{\partial p^c} = -\left(p^d-c\right)x\dfrac{\partial D(p^c)}{\partial p^c}$, we then have

$$h_{p^c}\big|_{\overline{p^c}} = (1-\delta)\left(p^d-c\right)\left(\frac{D\left(p^d\right)}{D(p^c)}\frac{\partial D(p^c)}{\partial p^c} + \frac{D\left(p^d\right) - xD(p^c)}{p^c-c}\right).$$

Since we are in region 2, and π^{sd} (given in equation (3)) has been maximised with respect to p^d, we know that

$$D\left(p^d\right) - xD(p^c) + \left(p^d-c\right)\frac{\partial D\left(p^d\right)}{\partial p^d} = 0$$

This gives us:

$$h_{p^c}\big|_{\overline{p^c}} = (1-\delta)\left(p^d-c\right)\frac{\partial D(p^c)}{\partial p^c}\left[\frac{D\left(p^d\right)}{D(p^c)} - \frac{p^d-c}{p^c-c}\frac{\partial D\left(p^d\right)}{\partial p^d}\Big/\frac{\partial D(p^c)}{\partial p^c}\right]$$

Since $p^d \le p^c$, the first fraction in the square brackets is greater than one, and the second fraction is less than one. Since $\partial D(p)/\partial p < 0$ and $\partial^2 D(p)/\partial p^2 < 0$, the ratio of the two derivatives is also less than one. The term in square brackets is therefore positive, and $h_{p^c}\big|_{\overline{p^c}} < 0$. Recalling that $h_\delta > 0$, we can conclude that the maximum sustainable price is strictly increasing in the critical discount factor δ. ■

We can also prove:

Proposition 6: *Assume that* $\partial^2 D(p)/\partial p^2 \geq 0$. *The maximum sustainable collusive price is decreasing in the length of contracts.*

Proof. Consider the maximum collusive price $\overline{p^c}$. By the implicit function theorem, we have $\partial\overline{p^c}/\partial\lambda = -h_\lambda/h_{p^c}$. Recalling that $h_\lambda < 0$ and $h_{p^c}|_{\overline{p^c}} < 0$, we can conclude that the maximum sustainable price is strictly decreasing in the length of contracts λ.■

Proposition 7: *The maximum sustainable collusive price may fall or rise as the proportion of output covered by contracts rises.*

Proof. By the implicit function theorem, we have $\partial\overline{p^c}/\partial x = -h_x/h_{p^c}$. It is easy to show that depending on the values of x, $\overline{p^c}$, λ and δ, the function h_x can be positive or negative. In Section 6 we give examples of these cases when demand is linear. This implies Proposition 7. ■

In the next section, we use a linear example to illustrate the relationships between the length of the contract, the maximum sustainable price, the discount factor and the amount of contracted quantity.

6. A linear demand example

We give an example of the interaction of the level of contract cover, the contract length, the discount factor, and the collusive price, by considering the same linear demand as Liski and Montero (2004). The total demand for the good is given by $a - p$, where p is the price in the spot market. In addition, we denote the price, quantity and profit associated with the one-period monopoly solution by $p^m = (a+c)/2$, $q^m = (a-c)/2$ and $\pi^m = (p^m - c)\, q^m = (a-c)^2/4$, respectively.
Assume first that the firms collude on the monopoly price, so that $p^c = p^m$. It is easy to show that the monopoly price associated with the residual demand is given by $p^{Rm} = \dfrac{a + c - xq^m}{2}$.
We can insert these, and the other relevant formulae, into equation (7) and obtain:

Proposition 8: *Collusion at the monopoly price can be sustained through the use of grim trigger strategies, if and only if* $2x\delta^{\lambda} + \delta(2-x)^2 \geq 1 + (1-x)^2$.

It is interesting to note that with full contracting (x = 1), the condition becomes $2\delta^{\lambda} + \delta \geq 1$. For $\lambda = 1$, this gives us $\delta \geq 1/3$, as in Liski and Montero (2004). With $\lambda = 2$ and $\mathrm{x} = 1$, the critical discount factor is again ½, just as if no contracts had been sold. The gain-cutting effect is exactly compensated by the protection effect. Figure 2 shows how the critical discount rate varies with x and the duration of the forward contracts. The two lowest lines, for contracts lasting one or two periods, are everywhere weakly below ½, confirming that these short-lived contracts make collusion easier to sustain. The lines for contracts lasting for three or more periods, however, rise above ½ at their right-hand ends. The top line shown, representing contracts lasting for 30 spot periods (which might imply a month-long contract overlying a spot market repeated each day), shows that the critical discount rate rises to more than 0.9 if the firms are fully contracted.

Assume now that the firms collude on $p^c \in (c, p^m]$. It is also possible, with some algebra, to calculate the maximum sustainable collusive price for values of x, δ and λ. We insert expressions for the collusive profits and the deviation profits in the spot market into equation (8):

$$h\left(x,\lambda,\delta,\overline{p^c}\right) = \frac{a-\overline{p^c}}{2}\left(\overline{p^c}-c\right)\left(1-x+x\delta^{\lambda}\right) - \left(\frac{a-c-x(a-\overline{p^c})}{2}\right)^2\left(1-\delta\right) = 0 \qquad (15)$$

We can solve this quadratic equation for $\overline{p^c}$:

$$\begin{aligned}\overline{p^c} &= \overline{p^c}(x,\lambda,\delta) \\ &= c + (a-c)\left(\frac{1}{2} - \frac{x(2-x)(1-\delta) - 2\sqrt{\left(1-x+x\delta^{\lambda}\right)\left(x\delta^{\lambda}-(1-x)(1-2\delta)\right)}}{4\left(1-x+x\delta^{\lambda}\right)+2(1-\delta)x^2}\right)\end{aligned} \qquad (16)$$

We plot the results in Figures 3 to 5. These show how the maximum sustainable prices vary with the absolute volume of contracted sales, for different discount factors and three contract lengths – one period, two periods, and four periods. The figures are calibrated for $a = 8$ and $c = 0$, so that the monopoly price is equal to 4. The horizontal dashed line shows this price. Because we are plotting the absolute volume of sales, rather than x, on the horizontal axis, we can also show the firm's total output at a given collusive price, assuming that it is sharing the market equally. This is given by the dotted line, which can also be described as a half-demand curve (it shows half of the market demand). The line with dots and dashes is the border between region 1 (below the line - undercut the collusive price by ε) and region 2 (above the line – undercut by charging the residual monopoly price). With linear demand and constant costs, this border is also linear.

The solid lines give the frontier linking the highest sustainable collusive price and the level of contracted output, for a range of discount factors. In each case, prices below and to the right of the frontier (and to the left of the half-demand curve) are also sustainable.

Figure 3 shows that when contracts last for a single spot period, firms with a discount factor of ½ or more can sustain collusion at the monopoly price, whether or not they sell any contracts. Firms with a discount factor of between ½ and ⅓ can sustain the monopoly price if they have covered enough of their output with forward contracts. For example, a firm with a discount factor of 0.4 will be able to sustain the monopoly price if it sells one unit in the forward market, at point A, but it would not be able to sustain a price of 1, at point B. At this lower price, the single unit sold in advance would be too small a proportion of the firm's overall sales. From equation (14), with $\delta = 0.4$ and $\lambda = 1$, collusion is sustainable if $x \geq 1/3$. Contract sales of 1 unit would give $x = 1/2$ at a price of 4 (for the firm would have total sales of 2), but $x = 0.29$ at a price of 1, when the firm would be selling 3.5 units. Finally note that firms with a discount factor of less than ⅓ cannot sustain the monopoly price, but will be able to sustain prices above marginal cost if they sell in the forward market.[8]

Figure 3 about here.

Figure 4 shows that firms with a discount factor of more than ½ can sustain collusion at the monopoly price for any level of contract cover, when the contracts last for two spot market periods. With a discount factor of ½ or less the frontiers are shifting downwards and to the right,

[8] Note that in region 1 (the area below the line of dots and dashes) the frontiers are bounded by straight segments – if extended, these would all converge on the vertical axis at a price of 8, the value taken by a in our example. This is also the case in figures 4 and 5, and would be for any other value of λ.

showing that more cover is needed to make collusion sustainable, and that the maximum sustainable prices are falling. Firms with discount factors of close to ½ can still sustain the monopoly price for some levels of contract cover, however, and a frontier for a firm with $\delta = 0.47$ has been added to illustrate this.

Figure 4 about here.

Figure 5 illustrates the impact of contracts that last for four spot periods. With these contracts, the maximum sustainable prices for firms with discount factors of less than ½ are much lower, and only one case is shown on the figure. Without contracts, however, collusion would not be possible at all. A firm with a discount factor of ½ could sustain the monopoly price if it had no contracts, and could sustain a slightly higher price with a low level of contract cover – although it should not want to, as the higher price would reduce its profits! As its contract cover increases, however, the maximum sustainable price for this firm falls – if it covers all of its output in the forward market, the maximum price is less than half of the monopoly price. A firm with a discount factor of just over ½ will also have a non-monotonic frontier – it might maximise its sustainable price with a small amount of contract sales. Firms with higher discount factors, however, maximise their sustainable price if they do not sell any contracts, and the price that they can sustain falls as their contract sales increase. A firm with a discount factor of 0.645 is (just) able to sustain the monopoly price with full contracting.

Figure 5 about here.

Figures 3-5 clearly show propositions 2 – 7 in action. The frontiers rise with higher discount factors, and fall as the length of the contracts increases. As contracts become longer, the protection effect becomes more important, relative to the gain-cutting effect, and collusion becomes harder to sustain for firms which would be able to sustain the monopoly price in the absence of contracts. In the bottom right-hand corner, however, contracts allow firms with low discount factors to sustain a collusive price, albeit a low one.

Firms with higher discount factors can sustain higher collusive prices. As the length of contracts increases, the highest sustainable price falls, or a higher discount factor is required to maintain a given collusive price. As the proportion of output covered by contracts increases, however, the highest sustainable price will sometimes rise, although it falls for high levels of contract cover when $\lambda > 1$.

7. Conclusion

We have shown that long-term contracts have an ambiguous impact on collusion. In some cases, they make collusion on a price above marginal cost possible when it would not be possible without them. In other cases, collusion would be possible without contracts, but becomes impossible (at a given price) as the level of contracting, or their length, becomes too great. We have shown that this ambiguity is due to the interaction of two effects, the gain-cutting effect, which reduces the immediate gain from defection, and the protection effect, which reduces the amount of punishment that deviators can receive. These effects are implicit in the work of Liski and Montero (2006), and we characterise them more fully in this paper.

We have taken the length of contracts, and the level of contracting, as exogenous. This was to allow us to explore the impact of changing these variables, but does open the question of how they would be chosen. Contracts will often last for a "natural" period of calendar time, such as a week, a month or a year. In a competitive market, a particular contract design will only last if it meets a need, and for the length of a contract, this implies a trade-off between the convenience of not having to trade too frequently, and the ability to match a contractual position with a physical one. For example, month-long contracts will be well-placed in a market where demand does not change significantly from week to week, but does vary predictably over the course of a year. In some electricity markets, regulators have mandated "vesting contracts" in the hope of reducing incumbents' market power at the start of competition. From the point of view of the firms, their length is exogenous, but such contracts are not a permanent market feature.

The more interesting question is why firms would offer contracts if they make it harder for the firms to collude. For some firms, the premise of the question is incorrect, for collusion is only possible if they have sold contracts, and in these circumstances, they would wish to sell the amount of contracts that maximised the collusive price that they could achieve. Other firms, however, will potentially lose out by selling contracts, assuming that the possibility of collusion is in fact an attractive one for them. In a different model, with uncertain demand, covering some output with forward contracts would reduce the variability of the firms' profits. Hedging their profits in this way might be sufficient motivation for them to sell contracts. That remains an area for further research.

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Figure(s)

Figure 1: Deviation types

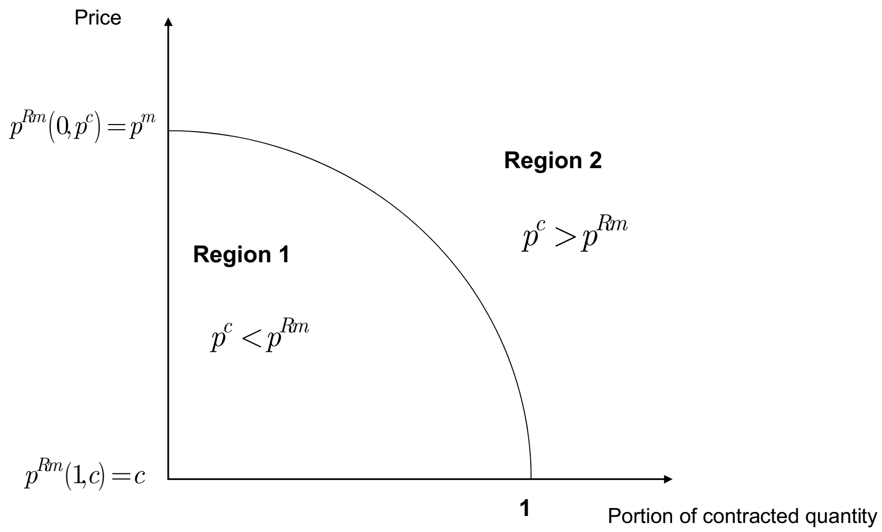

Figure 2: Critical discount factors to sustain collusion at the monopoly price, linear example

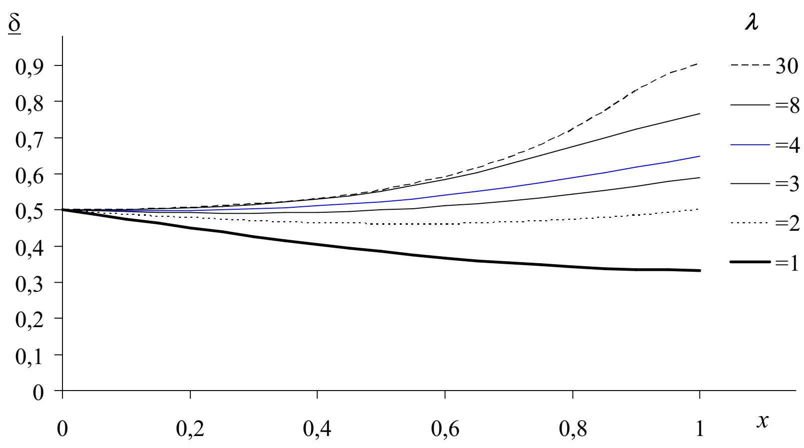

Figure 3: Sustainable collusive prices with $\lambda = 1$, linear example

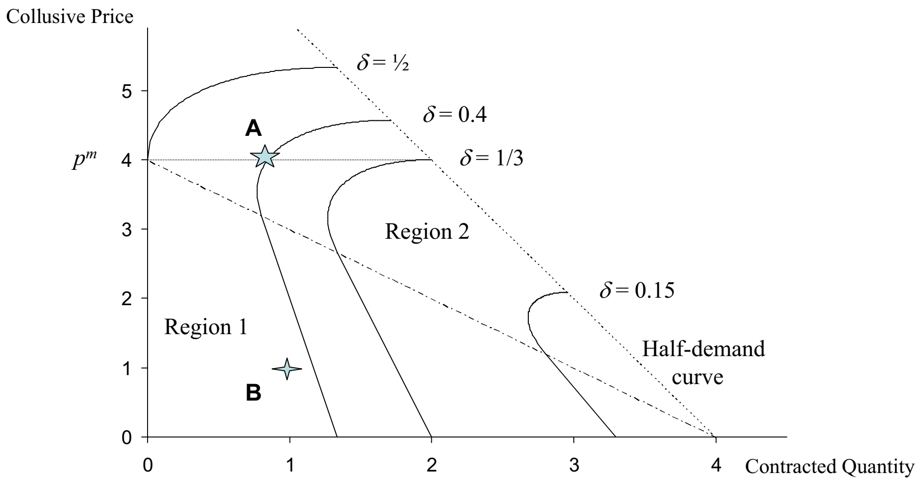

Figure 4: Sustainable collusive prices with $\lambda = 2$, linear example

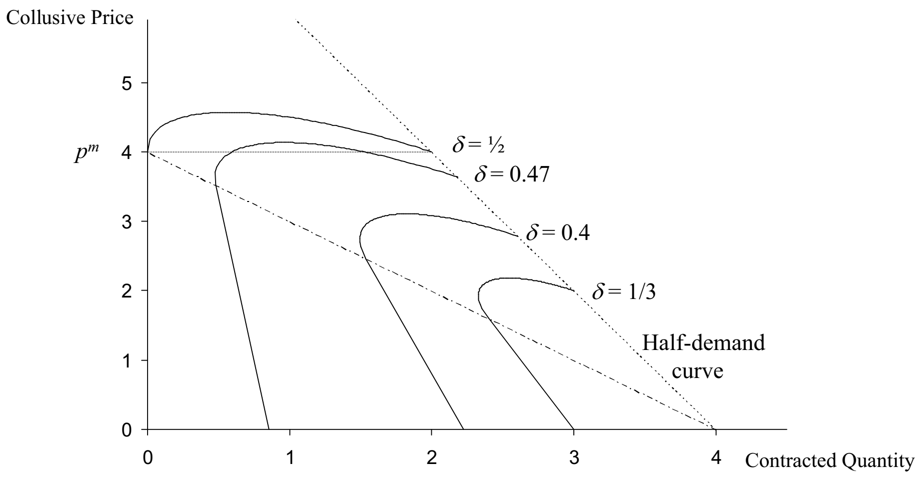

Figure 5: Sustainable collusive prices with $\lambda = 4$, linear example