

**Mathematics.**

Don't use a calculator.

1. Factorize  $25x^2 - 81$ . (Write the expression as a product.)
2. Factorize  $1 - 4x + 4x^2$ .
3. Factorize  $4 - 64x^4$ .
4. Factorize  $ka^2 + 9k + 6ak$ .
5. Factorize  $12a^3 - 108a$ .
6. Factorize  $-x^4 + 2x^3y - x^2y^2$ .
7. Factorize  $2x^2 - 2x + 0,5$ .
8. Simplify  $(\sqrt{x+h} + \sqrt{x})(\sqrt{x+h} - \sqrt{x})$
9. Simplify  $\frac{a^3 - 1}{a - 1} - a^2$
10. Simplify  $\frac{1}{a - 2} + \frac{2a}{4 - a^2}$
11. Simplify  $\left(1 + \frac{a}{b}\right)\left(\frac{a}{a+b} - 1\right)$
12. Simplify  $\frac{2}{x-1} - \frac{1}{x} - \frac{4}{x^2 - 1}$
13. Simplify  $\frac{\frac{1}{a} - \frac{1}{b}}{a - b}$
14. Simplify  $\frac{\frac{1}{y} + 1}{3 + \frac{3}{y}}$
15. Simplify  $\frac{\frac{3}{a} - 2 + \frac{a}{3}}{1 - \frac{a}{3}}$
16. Factorize  $x^3 - x^2 + x - 1$ .
17. Simplify  $\frac{x^3 - x^2 + x - 1}{x^2 + 1}$
18. Solve the equation  $x^2 + 2x - 3 = 0$ .

19. Solve the equation  $2 + x - x^2 = 0$ .
20. Solve the equation  $5 - 3x - 2x^2 = 0$ .
21. Solve the equation  $7x^2 + 13x - 2 = 0$ .
22. Solve the equation  $5x^2 - 6x - 2 = 0$ .
23. Solve the equation  $\sqrt{3x+1} = x-1$
24. Solve the equation  $45 + 8x - 4x^2 = 0$
25. Factorize the polynomial  $p(x) = 45 + 8x - 4x^2$
26. Solve the inequality  $12 - 3x \geq 6$ .
27. Solve the inequality  $45 + 8x - 4x^2 \geq 0$ .
28. Solve the inequality  $x^2 - 5x + 6 < 0$ .
29. Solve the inequality  $8 + 2x - x^2 \leq 0$ .
30. Solve the inequality  $\frac{2x(x-3)}{x+1} \leq 0$ .
31. Solve the inequality  $\frac{2x-3}{x-1} \geq 3-x$
32. Solve the inequality  $\frac{1}{2-x} \leq \frac{1}{x}$ .
33. Solve the inequality  $\frac{1}{x} \leq \frac{1}{x-2}$ .
34. Solve the inequality  $\frac{1}{2-x} \leq \frac{1}{x} \leq \frac{1}{x-2}$ . That is to find the x-values solving the two inequalities  

$$\frac{1}{2-x} \leq \frac{1}{x} \quad \text{and} \quad \frac{1}{x} \leq \frac{1}{x-2}.$$
35. Factorize  $x^3 - x^2 + x - 1$ .
36. Solve the inequality  $x^3 < x^2 - x + 1$ .
37. Simplify  $\sqrt{2} \cdot \sqrt{8}$ . (Remember; do not use a calculator.)
38. Simplify  $9^{-0,5}$ .
39. Simplify  $\sqrt{49} + \sqrt{2} \cdot \sqrt{98}$ .
40. Simplify  $0,25\sqrt{32}$ .
41. Simplify  $32^{2/5}$ .
42. Simplify  $\frac{32^{0,4}}{16^{-7/4}}$ .

43. Simplify  $(0,04)^{-0,25}$
44. Simplify  $\sqrt[3]{\sqrt{10}}$ .
45. Simplify  $\frac{(\sqrt{2})^{1,2} \cdot \sqrt[10]{8}}{(2^{0,3})^3}$ .
46. Simplify  $(3^x + 3^x + 3^x)^2$ .
47. Solve the equation  $(2^x + 2^{x+1} + 2^{x-1})^2 = 98$ .
48. Simplify  $\frac{(x^{0,4})^3 \cdot \sqrt[5]{x^2}}{(x^3)^{1/5}}$ .
49. Simplify  $\frac{(x^{3a})^3 \cdot x^{-b}}{x^{3a+2b}}$ .
50. Solve the equation  $x^h - 1 = h$ .
51. Solve the equation  $ax(ax+b)^{-2/3} + (ax+b)^{1/3} = 0$
52. Simplify  $\ln 8 + \ln 2$  (Kom ihåg att inte använda räknare.)
53. Simplify  $\ln 8 - \ln 2$
54. Simplify  $\ln 8 - \ln \sqrt{8} - \ln 2 + \ln \frac{1}{\sqrt{2}}$
55. Simplify  $\ln \frac{17}{18} + \ln \frac{18}{17}$
56. Simplify  $\ln \frac{17}{18} + \ln \frac{9}{17}$
57. Simplify  $\ln x - \ln x^2 + \ln 2x$
58. Find  $\frac{x}{y}$  when  $\ln x = 1 + \ln y$
59. Solve the equation  $\ln x = 1 - 3 \ln 2$
60. Solve the equation  $2 \ln x = \ln 2x$
61. Solve the equation  $2 \ln x = \ln 3 + \ln 12$
62. Solve the equation  $\ln x^2 = 1 + \ln x$
63. Solve the equation  $\ln(1-x) + \ln(1+x) = \ln 0,75$ .
64. Solve the equation  $\sqrt{3x+1} = x-1$
65. Find the equation for the straight line passing through the points  $(-2; 4)$  and  $(6; -2)$ .
66. Find the equation for the straight line passing through the point  $(4; 1)$  and is parallel the line to the line  $2y + 3x + 4 = 0$

67. Solve the system of equations  $\begin{cases} 7x + 2y = 13 \\ 3x - 4y = -9 \end{cases}$

68. Solve the system of equations  $\begin{cases} 230p + 450q = 1810 \\ 100p + 150q = 650 \end{cases}$

69. Solve the system of equations  $\begin{cases} 2x + 3y - z = 1 \\ -x + y + 3z = 5 \\ x - y + 2z = 2 \end{cases}$

70. Find the derivative of  $f(x) = 7x^9$

71. Find the derivative of  $f(x) = x^n$

72. Find the derivative of  $f(x) = \sqrt{x}$

73. Find the derivative of  $f(x) = \sqrt[3]{x}$

74. Find the derivative of  $f(x) = x^\alpha$

75. Find the derivative of  $f(x) = e^x$

76. Find the derivative of  $f(x) = 2e^{-3x}$

77. Find the derivative of  $f(x) = 4x^5 + 3e^{-6x}$

78. Find the derivative of  $f(x) = 7e^{-8x+3x^3}$

79. Find the derivative of  $f(x) = (x+5)^{17}$

80. Find the derivative of  $f(x) = (x+5)^{-17}$

81. Find the derivative of  $f(x) = (x^2 + 5)^{17}$

82. Find the derivative of  $f(x) = (x^2 + 5)^{-17}$

83. Find the derivative of  $f(x) = \sqrt[17]{x^2 + 5}$

84. Find the derivative of  $f(x) = \ln x$

85. Find the derivative of  $f(x) = x^7 + \ln(x+1)$

86. Find the derivative of  $f(x) = \ln(1-x^2)$

87. Find the derivative of  $f(x) = \ln(1+x^2+x^4)$

88. Find the derivative of  $f(x) = -x + x \cdot \ln x$

89. Find the derivative of  $f(x) = \sqrt[3]{1+2x^4}$

90. Find the derivative of  $f(x) = \ln \sqrt[3]{1+2x^4}$

91. Find the derivative of  $f(x) = e^{-3x^2} \cdot \ln(1+x^2)$
92. Find the derivative of  $f(x) = \frac{x}{1+x^2}$
93. Find the derivative of  $f(x) = \frac{x^2-1}{x^2+1}$
94. Find the derivative of  $f(x) = \frac{4x}{(x^2+1)^2}$
95. Consider the function  $f(x) = -x^2 + x + 6$  having the domain  $-3 \leq x \leq 2$
- Give the global maximum and minimum of the function.
  - Give the range of the function.
96. Consider the function  $f(x) = xe^{-x}$  having the domain  $-1 \leq x \leq 2$
- Give the global maximum and minimum of the function.
  - Give the range of the function.
97. Consider the function  $f(x) = -x^2 + x + 6$
- Give the largest possible domain to the function.
  - Find any zeros to the function, i.e. x-values where  $f(x) = 0$ .
  - Find the functions stationary points, i.e. x-values where where the derivative is zero.
  - Find the intervals where the function is increasing and where it is decreasing.
  - Classify the stationary points, i.e find any local minimum or maximum.
  - Find the intervals where the function is concave and convex.
  - Give any points of inflection.
  - Give the range of the function.
  - Sketch the graph.
98. Consider the function  $f(x) = x^3 + 6x^2 + 9x$  and answer questions 97. a) – i).
99. Consider the function  $f(x) = xe^{-x}$  and answer questions 97. a) – i).
100. Consider the function  $f(x) = \frac{(x-1)^2}{1+x^2}$  and answer questions 97. a) – i).
101. Consider the function  $f(x) = \frac{\ln x}{x^2}$  and answer questions 97. a) – i).

## Answers.

1.  $(5x+9)(5x-9)$
2.  $(1-2x)^2$
3.  $4(1+4x^2)(1+2x)(1-2x)$
4.  $k(a+3)^2$
5.  $12a(a+3)(a-3)$
6.  $-x^2(x-y)^2$
7.  $0,5(2x-1)^2$
8.  $h$
9.  $a+1$
10.  $-\frac{1}{a+2}$
11.  $-1$
12.  $\frac{x-1}{x(x+1)}$
13.  $-\frac{1}{ab}$
14.  $\frac{1}{3}$
15.  $\frac{3-a}{a}$
16.  $(x-1)(x^2+1)$
17.  $x-1$
18.  $x_1=1 \quad x_2=-3$
19.  $x_1=-1 \quad x_2=2$
20.  $x_1=1 \quad x_2=-2,5$
21.  $x_1=1/7 \quad x_2=-2$
22.  $x_1=\frac{3+\sqrt{19}}{5} \quad x_2=\frac{3-\sqrt{19}}{5}$
23.  $x=5$
24.  $x_1=4,5 \quad x_2=-2,5$
25.  $p(x)=-4(x-4,5)(x+2,5)$
26.  $x \leq 2$
27.  $-2,5 \leq x \leq 4,5$
28.  $2 < x < 3$
29.  $x \leq -2 \quad x \geq 4$
30.  $x < -1 \quad 0 \leq x \leq 3$
31.  $0 \leq x < 1 \quad x \geq 2$
32.  $0 < x \leq 1 \quad x > 2$
33.  $x < 0 \quad x > 2$
34.  $x > 2$
35.  $(x-1)(x^2+1)$
36.  $x < 1$
37.  $4$
38.  $1/3$
39.  $21$
40.  $\sqrt{2}$
41.  $4$
42.  $2^9$
43.  $\sqrt{5}$
44.  $10^{1/6}$
45.  $1$
46.  $3^{2x+2}$
47.  $x=1,5$
48.  $x$
49.  $x^{6a-3b}$
50.  $x=(1+h)^{1/h}$
51.  $x=-b/2a$
52.  $4 \ln 2$
53.  $2 \ln 2$
54.  $0$
55.  $0$
56.  $-\ln 2$
57.  $\ln 2$
58.  $e$
59.  $e/8$
60.  $x=2$
61.  $x=6$
62.  $x=e$
63.  $x_1=0,5 \quad x_2=-0,5$
64.  $x=5$
65.  $y=-0,75x+2,5$
66.  $y=-1,5x+7$
67.  $\begin{cases} x=1 \\ y=3 \end{cases}$
68.  $\begin{cases} p=2 \\ q=3 \end{cases}$
69.  $\begin{cases} x=0 \\ y=0,8 \\ z=1,4 \end{cases}$

70.  $f'(x) = 63x^8$
71.  $f'(x) = nx^{n-1}$
72.  $f'(x) = 0,5x^{-0,5} = \frac{1}{2\sqrt{x}}$
73.  $f'(x) = 0,2x^{-0,8}$
74.  $f'(x) = \alpha x^{\alpha-1}$
75.  $f'(x) = e^x$
76.  $f'(x) = -6e^{-3x}$
77.  $f'(x) = 20x^4 - 18e^{-6x}$
78.  $f'(x) = 7(-8 + 9x^2)e^{-8x+3x^3}$
79.  $f'(x) = 17(x+5)^{16}$
80.  $f'(x) = -17(x+5)^{-18}$
81.  $f'(x) = 17 \cdot 2x(x^2 + 5)^{16}$
82.  $f'(x) = -17 \cdot 2x(x^2 + 5)^{-18}$
83.  $f'(x) = \frac{1}{17} \cdot 2x(x^2 + 5)^{-16/17} = \frac{2x}{17(x^2 + 5)^{16/17}}$
84.  $f'(x) = \frac{1}{x}$
85.  $f'(x) = 7x^6 + \frac{1}{x+1}$
86.  $f'(x) = -\frac{2x}{1-x^2}$
87.  $f'(x) = \frac{2x+4x^3}{1+x^2+x^4}$
88.  $f'(x) = \ln x$
89.  $f'(x) = \frac{1}{3} \cdot 8x^3(1+2x^4)^{-2/3}$
90.  $f'(x) = \frac{1}{3} \cdot 8x^3(1+2x^4)^{-2/3} \cdot \frac{1}{\sqrt[3]{(1+2x^4)}} = \frac{8x^3}{3(1+2x^4)}$
91.  $f'(x) = -6xe^{-3x^2} \cdot \ln(1+x^2) + e^{-3x^2} \cdot \frac{2x}{1+x^2}$
92.  $f'(x) = \frac{1-x^2}{(1+x^2)^2}$
93.  $f'(x) = \frac{4x}{(1+x^2)^2}$
94.  $f'(x) = \frac{4-12x^2}{(1+x^2)^3}$
95. Maximum:  $f(0,5) = 6,25$     Minimum:  $f(-3) = -6$     Range:  $-6 \leq f(x) \leq 6,25$
96. Maximum:  $f(1) = \frac{1}{e}$     Minimum:  $f(-1) = -e$     Range:  $-e \leq f(x) \leq \frac{1}{e}$
97. a) Largest possible domain to the function is the real line.  
b) Zeros:  $x_1 = 3$     $x_2 = -2$   
c) Stationary point:  $x = 0,5$   
d)  $x = 0,5$  gives a maximum.  
e)  $f(x)$  is increasing in the interval  $x < 0,5$      $f(x)$  is decreasing in the interval  $x > 0,5$   
f)  $f(x)$  is concave in the domain.  
g) No inflection point.  
h) Range:  $f(x) \leq 6,25$
98. a) Largest possible domain to the function is the real line.  
b) Zeros:  $x_1 = -3$     $x_2 = 0$   
c) Stationary points:  $x_1 = -3$     $x_2 = -1$   
d)  $x = -3$  gives a local maximum.     $x = -1$  gives a local minimum.  
e)  $f(x)$  is increasing in the intervals  $x < -3$     $x > -1$   
 $f(x)$  is decreasing in the interval  $-3 < x < -1$   
f)  $f(x)$  is concave in the interval  $x < -2$      $f(x)$  convex in the interval  $x > -2$   
g) Inflection point:  $x = -2$   
h) Range: the real line.

99. a) Largest possible domain to the function is the real line.  
 b) Zero:  $x = 0$   
 c) Stationary point:  $x = 1$   
 d)  $x = 1$  gives a maximum.  
 e)  $f(x)$  is increasing in the interval  $x < 1$        $f(x)$  is decreasing in the interval  $x > 1$   
 f)  $f(x)$  is concave interval  $x < 2$        $f(x)$  is convex in the interval  $x < 2$   
 g) Inflection point:  $x = 2$   
 h) Range:  $f(x) \leq \frac{1}{e}$

100. a) Largest possible domain to the function is the real line.  
 b) Zeros:  $x = 1$   
 c) Stationary points:  $x = -1$  och  $x = 1$   
 d)  $x = -1$  gives a maximum.  $x = 1$  gives a minimum.  
 e)  $f(x)$  is increasing in the intervals  $x < -1$   $x > 1$   
      $f(x)$  is decreasing in the interval  $-1 < x < 1$   
 f)  $f(x)$  concave in the intervals  $-\sqrt{3} < x < 0$   $x > \sqrt{3}$   
      $f(x)$  convex in the intervals  $x < -\sqrt{3}$   $0 < x < \sqrt{3}$   
 g) Inflection points:  $x = -\sqrt{3}$   $x = 0$   $x = \sqrt{3}$   
 h) Range:  $0 \leq f(x) \leq 2$

101. a) Largest possible domain to the function:  $x > 0$   
 b) Zeros:  $x = 1$   
 c) Stationary point:  $x = \sqrt{e}$   
 d)  $x = \sqrt{e}$  gives a maximum.  
 e)  $f(x)$  is increasing in the intervall  $0 < x < \sqrt{e}$   
      $f(x)$  is decreasing in the intervall  $x > \sqrt{e}$   
 f)  $f(x)$  concave in the interval  $0 < x < e^{5/6}$        $f(x)$  convex in the interval  $x > e^{5/6}$   
 g) Inflection point:  $x = e^{5/6}$   
 h) Range:  $f(x) \leq \frac{1}{2e}$