Asymmetric information and the pecking (dis)order

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Abstract

In this paper we revisit the pecking-order theory of Myers and Majluf (1984) in a real options framework, where asymmetric information is the only contracting friction. We show that when insiders are relatively better informed on the assets in place of their firm, rather than on the new growth opportunities, equity financing can dominate (i.e., be less dilutive than) debt financing, creating a pecking (dis)order. We find that equity is more likely to dominate debt for younger firms that have larger investment needs, and with riskier, more valuable growth opportunities. Thus, our model can explain why high-growth firms may prefer equity over debt, and then switch to debt financing as they mature. Finally, we study the optimal security design problem, and we provide conditions for which convertible debt and warrants emerge as optimal securities.

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1 Introduction

Raising capital under asymmetric information exposes firms to potential value dilution. When insiders have better information than investors on firm value, firms of better-than-average quality will find that the market prices their securities below their fundamental value. Under these circumstances, Myers and Majluf (1984) suggest that firms can reduce dilution (i.e., mispricing) by issuing debt rather than equity, an intuition known as the pecking order theory. The reason for the pecking order preference, as Myers (1984) argues, is that the value of debt, by virtue of being a more senior security than equity, is less sensitive to private information.\(^1\)

Important deviations from the pecking-order theory emerged in several recent empirical studies. For example, Frank and Goyal (2003) and Fama and French (2005) document that small, high-growth firms, a class of firms which is presumably more exposed to the effects of asymmetric information, typically rely heavily on financing through outside equity, rather than debt. Leary and Roberts (2010) conclude that “the pecking order is never able to accurately classify more than half of the observed financing decisions.”\(^2\) This evidence has led researchers to conclude that asymmetric information is not a first-order determinant of corporate capital structures (Fama and French, 2002).

Failure of the pecking order theory in empirical tests may be due to the fact that asymmetric information is not a first-order driver of capital structure choices, but it may also be a sign that the conditions under which that theory holds are not met. The precise conditions under which the pecking theory holds have been object of considerable research. In a seminal paper, Nachman and Noe (1994) show that the original Myers and Majluf conjecture holds only under very special conditions.\(^3\) While the circumstances under which the pecking order

\(^1\)Myers and Majluf (1984) also suggest that internal financing dominates external financing, where external financing can either be in the form of debt or equity. In this paper, we do not consider internal financing because it would dominate external financing, just as in Myers and Majluf (1984). In addition, by design, we do not consider the possibility of (partially) separating equilibria, and thus, the possibility of the “announcement effects” also discussed in Myers and Majluf (1984).

\(^2\)Leary and Roberts (2010) also note that most of the empirical evidence is inconclusive, and write: “Shyam-Sunder and Myers (1999) conclude that the pecking order is a good descriptor of broad financing patterns; Frank and Goyal (2003) conclude the opposite. Lemmon and Zender (2010) conclude that a ‘modified’ pecking order—which takes into account financial distress costs—is a good descriptor of financing behavior; Fama and French (2005) conclude the opposite. Frank and Goyal (2003) conclude that the pecking order better describes the behavior of large firms, as opposed to small firms; Fama and French (2005) conclude the opposite. Finally, Bharath, Pasquariello, and Wu (2010) argue that firms facing low information asymmetry account for the bulk of the pecking order’s failings; Jung, Kim, and Stulz (1996) conclude the opposite.”

\(^3\)In particular, they show that debt emerges as the solution of an optimal security design problem if and only if the private information held by firm insiders orders the distribution of firm value by Conditional Stochastic Dominance (CSD). The Statistics and Economics literature also often uses the term Hazard Rate Ordering to refer to CSD, and we shall use both terms interchangeably. An important class of distributions that satisfies these conditions is the lognormal distribution (or, equivalently, a Geometric Brownian Motion, GBM), where the mean of the distribution (or the drift) is private information.
theory holds are now well understood, considerably less is known with respect to those cases where such conditions are not met.

In this paper, we show that Myers and Majluf conjecture can be violated in many natural situations, and we derive comparative statics results that characterize such cases. In our model, equity financing can dominate (i.e., be less dilutive than) debt financing when asymmetric information is the only friction between management and outsider investors. This happens when the insiders are relatively better informed than investors on the assets in place of their firm, rather than on the growth opportunities — in other words, when a firm’s assets in place are relatively more exposed to asymmetric information. In particular, we find that equity is more likely to dominate debt for young firms that have greater investment needs, and that have access to riskier and more valuable growth opportunities. Thus, our model can explain why high-growth firms may initially prefer equity over debt, and then switch to debt financing as they mature and exploit their growth opportunities.

At the beginning of the period, the firm must raise funds to finance an initial investment in capital markets characterized by asymmetric information.\textsuperscript{4} We model firm value as an exchange (or “rainbow”) option: by making the early investment, the firm acquires the option to “exchange,” at a later date, its existing assets in place with new assets that embed the growth opportunity.\textsuperscript{5} The value of assets in place and the assets embedding the growth opportunity are both characterized by lognormal distributions, where the growth opportunity is riskier than the assets in place (i.e., it has greater variance). We model asymmetric information by assuming that the firm insiders have private information on the means (or the drifts) of the distributions, while their second moments are common knowledge. We show that equity can be less dilutive than debt when the asset that has greater risk (the growth opportunity) is less exposed to asymmetric information relative to the asset with lower volatility (the assets in place). We also show that equity can be less dilutive than debt when the firm must raise more capital, when the growth opportunity has longer time to expiration, and when it represents a greater proportion of overall firm value.

Intuitively, our results depend on the fact that the right-tail properties of the distribution of overall firm value are determined by the asset with higher volatility and/or greater mean.\textsuperscript{6}

\textsuperscript{4}Note that, in the spirit of Myers and Majluf (1984), we rule out of the possibility that firms finances their growth opportunities separately from the assets in place, i.e., by “project financing.”

\textsuperscript{5}We adopt the exchange option framework because it is a well understood paradigm in option pricing theory for which a closed form solution is available (see Rubinstein, 1991), and it is a natural description of an investment decision in a firm (Stulz, 1982). In section 5 we will also examine the case in which the growth opportunity requires an additional investment at the time the option is exercised. This case will be discussed with simple numerical simulations, since this class of (“double strike”) options does not have a closed form solution.

\textsuperscript{6}Technically, our results reflect the rather limited closure properties of the conditional stochastic dominance
Thus, when the asset that is relatively less mispriced (that is, less affected by asymmetric information) also has greater volatility, issuing a security with exposure to payoffs in the right tail of the firm-value distribution, such as equity, can be less dilutive than a security which lacks such exposure.

We also show that the Myers and Majluf conjecture can be further violated in the case of a multidivisional firm composed by two segments. The distribution of the value of individual divisions is described again by lognormal distributions. Since the (weighted) average of two lognormal distributions is not itself a lognormal distribution, the pecking order may fail for the overall firm, even if it would hold for the two divisions individually. We show that equity can dominate debt when the division that has lower exposure to asymmetric information has also a higher variance. Thus, our model generates new predictions on the cross-sectional variation of firm capital structures of multidivisional firms.

An example with a potential for the pecking order reversal is offered by a firm whose assets in place have been obtained by the exploitation of past growth opportunities (such as the outcome of R&D activities) and that is also endowed with new growth opportunities (such as new R&D activities). In these situations it is plausible that the firm has relatively more accurate information on its assets in place, on which more information has become available over time to its insiders, rather than on the new growth opportunities, where critical information on their true value still has yet to be revealed. If the new growth opportunities have greater volatility, our model shows that the original Myers and Majluf conjecture may not hold.

A second example is a firm with assets in place that has the option to acquire another firm. The firm wishes to raise capital to pay for the acquisition. Firm insiders have private information on both assets in place and the value of the target firm’s assets. In this setting, it is again plausible to expect that the firm has relatively better information on the assets in place, that are already under the firm’s control, than on the new assets that still have to be acquired. The volatility of the assets in place may be lower than the volatility of the target firm’s assets, a situation that we show can generate a reversal of the pecking order.

An additional contribution of our paper is to study the case when the firm has pre-existing debt in its capital structure. We show that, interestingly, firms that already have debt outstanding are relatively more likely to prefer equity over debt, solely driven by informational considerations. This happens because low realizations of future firm value have already been pledged to the existing bondholders, making new debt financing less attractive than equity.

order. See section 1.B.3 of Shaked and Shanthikumar (2007). Note that we shall use both the terms hazard rate ordering and conditional stochastic dominances in our discussion to refer to the same statistical order. Both terms are standard in the literature.
This property of our model implies that equity financing can alleviate the debt overhang problem of Myers (1977). In addition, it suggests that (pre-existing) high leverage may lead to more equity financing. Thus, asymmetric information may in fact lead to “mean reversion” in leverage levels, as it is often documented in the empirical literature on capital structure (see Frank and Goyal, 2003; Fama and French, 2005; Leary and Roberts, 2005). These predictions are novel within models based on informational frictions, and invite for further research.\footnote{Note that our model is essentially a “convex combination” of Black and Scholes (1973)/Merton (1974) and Myers and Majluf (1984). As such, it has a static capital structure choice, even if the model lends itself to a dynamic specification (note that, in a similar framework, also Leland, 1994, allows for a static financing decision). Further research focusing on dynamic capital structure choices is suggested by the fact that the existing set of securities in a firm’s balance sheet affects the optimal financing choice (see Section 6).}

We conclude the paper with considering an explicit optimal security design problem, where the firm can issue other securities than equity and debt. Feasible securities include convertible bonds, warrants, as well as equity and debt, among others. Our main conclusions extend to the more general security design problem: we show that when a certain “low-information-cost-in-the-right-tail” condition holds, straight (but risky) debt is optimal when the firm needs to raise low levels of capital, but “equity-like securities” — such as convertible debt — emerge as the optimal securities when the firm must raise larger amounts of capital. Furthermore, we find that warrants can be optimal securities in the presence of pre-existing debt.

Our paper is also linked to several other papers belonging to the ongoing research on the pecking order and, more generally, the security design literature. In addition to Nachman and Noe (1994), subsequent research has focused on different aspects of the security design problem. DeMarzo and Duffie (1999) consider the \textit{ex-ante} security design problem faced by a firm before learning its private information, rather then the \textit{interim} security design problem (that is, after becoming informed) studied by Nachman and Noe (1994). DeMarzo (2005) considers both the \textit{ex-ante} and the \textit{interim} security design problems, and examines the question of whether (or not) to keep multiple assets in a single firm (pooling), and the priority structure of the securities issued by the firm (tranching). DeMarzo, Kremer, and Skrzypacz (2005) examine the security design problem in the context of auctions. Chakraborty and Yılmaz (2009) show that when investors have access to noisy public information on the firm’s private information, the dilution problem can be costlessly avoided by issuing securities having the structure of callable, convertible bonds. Chemmanur and Fulghieri (1997) and Chakraborty, Gervais, and Yılmaz (2011) argue that warrants may be part of the optimal security structure. Finally, a growing literature considers dynamic capital structure choice (Fischer, Heinkel, and Zeckner, 1989; Hennessy and Whited, 2005; Strebulaev, 2007; Hennessy, Livdan, and Miranda, 2010; Morellec and Schürhoff, 2011). We conjecture that the economic forces of our
static framework will play a first-order role in a dynamic version of the model.

There are other papers that challenge Myers and Majluf (1984) and Myers (1984) by extending their framework. A series of papers shows that a wider range of financing choices, which allow for signaling with costless separation, can invalidate the pecking order (see, e.g., Brennan and Kraus, 1987; Noe, 1988; Constantinides and Grundy, 1989). However, Admati and Pfleiderer (1994) point out that the conditions for a fully revealing signaling equilibrium identified in these papers are rather restrictive. Cooney and Kalay (1993) relax the assumption that projects have a positive net present value (NPV). Fulghieri and Lukin (2001) relax the assumption that the informational asymmetry between a firm’s insiders and outside investors is exogenous, and allow for endogenous information production. Finally, Dybvig and Zender (1991) study the effect of optimally designed managerial compensation schemes, and Edmans and Mann (2012) look at the possibility of asset sales for financing purposes. Unlike these papers, our framework is closest to the original one in Myers and Majluf (1984), in that the only friction is the asymmetric information between insiders and financiers.

The remainder of the paper is organized as follows. We start by providing in Section 2 a simple example that illustrates the basic results of our paper and its intuition. Section 3 presents the basic model. Section 4 studies the debt-equity choice. Section 5 considers the real options model that illustrates the main results. Section 6 extends the analysis to pre-existing debt. Section 7 considers the security design problem, where we provide conditions under which convertible debt and warrants are the optimal securities. Section 8 discusses robustness issues. Section 9 discusses empirical implications. All the proofs are in the Appendix.

## 2 A simple example

The basic results of our paper, and their intuition, can be shown with a simple numerical example, summarized in Table 1. The more technically inclined reader can refer directly to the details in Section 3. The essence of the pecking order hypothesis is typically illustrated via a pooling equilibrium with two types and a discrete state space. There are two types of firms: good type, \( \theta = G \), and bad type, \( \theta = B \). For reasons that will become apparent below, we will assume that a firm end-of-period asset value, \( Z \), is characterized by a trinomial distribution with possible outcomes \( Z = z \in \{z_1, z_2, z_3\} \). In what follows we assume \( z_1 = 10 \), \( z_2 = 100 \) and \( z_3 = 300 \), and refer to the three states as low/middle/high.

The probability of realization of the different asset values \( Z \) depends on private information held by the firm’s insiders, and is given by \( f_\theta \equiv \{f_{\theta_1}, f_{\theta_2}, f_{\theta_3}\} \) for a firm of type-\( \theta \), with \( \theta \in \{G, B\} \). At the beginning of the period, the firm must raise capital \( I \) to finance the investment project, with \( I = 60 \). When raising capital, the two types of firms pool and issue
the same security, so investors do not change their priors when seeing the security issuance decision. We assume that the two types of firms are equally likely in the eyes of investors.

Our example will make the following assumptions on the two distributions of firm value:
(a) the good type firm will be characterized by \( f_G \equiv \{0.2, 0.4, 0.4\} \), (b) the bad type firm will be characterized by \( f_B \equiv \{0.3, 0.4 - x, 0.3 + x\} \). The parameter \( x \in [0, 0.1] \) affects the amount of probability mass for the type-B firm in the middle versus high states, and it will be the variable of interest in the following discussion.\(^8\) We note that assuming \( x \leq 0.1 \) assures that the the type-B firm probability distribution is dominated in the first-order stochastic sense by the type-G firm. The first three columns of Table 1 give the pooled value of the firm (averaging over \( \theta = G, B \)), and the values for the G-type (independent of \( x \)) and B-type firms (which increases in \( x \)).

Under the given assumptions, the value of standard debt securities with pooling is independent of \( x \). The firm issues debt with a promised face value of \( K = 76.7 \), which pays \{10, 76.7, 76.7\} and is valued at 60. The type-G firm is selling underpriced debt, as under \( f_G \) the debt is worth \( D_G = 63.3 \), since the market pools it with the type-B firm, whose debt is worth \( D_B = 56.7 \). The dilution cost of debt is constant and is equal to \( D_D = 6.7 \) (column 7 in Table 1).

The pricing of the equity security, on the other hand, does depend on the parameter \( x \) that shifts the probability mass for a type-B firm in the right tail. The pooled value of the firm is equal to \( 147.5 + 100x \), so that to finance the investment \( I = 60 \) the firm must sell a fraction \( \lambda \) of the equity given by \( \lambda(147.5 + 100x) = 60 \), which decreases with \( x \). With \( x = 0 \) we have that \( \lambda = 0.407 \), whereas with \( x = 0.1 \) only \( \lambda = 0.381 \) needs to be offered to outside investors. The values of the different equity stakes the firm needs to give up, \( \lambda \), are given in column 5 of Table 1. The mispricing (that is, dilution) generated by the equity securities is given in column 6.

In the case of \( x = 0 \), type-G firms suffers from higher dilution costs under equity than debt: the type-G equity is worth 65.9, whereas the type-G debt is worth only 63.3 (the corresponding values for types-B firms are 54.1 and 56.7). On the other hand, for \( x \geq 0.6 \) the dilution costs of equity for a type-G firm are lower than those of debt. This property is driven by the fact that when \( x \geq 0.6 \) the “right-tail” of the firm distribution is less exposed to asymmetric information than the “middle” of the distribution. This can be seen by noting that \( f_{G2} - f_{B2} = x > f_{G3} - f_{B3} = 1 - x \) for \( x > 0.5 \). As we will argue below, the relative exposure to asymmetric information in three regions of the distribution, namely, the “right tail,” the “middle” and, therefore, the “left tail” will play a crucial role in our analysis.

\(^8\)Note that an equivalent example can be obtained for the type-G distribution by shifting the probability mass from the “right tail” to the “middle” of the distribution.
There are two key ingredients to our basic example. The first is the assumption that the firm is issuing (sufficiently) risky debt. We achieve this in our example assuming $z_1 = 10$. If the level of investment is reduced to $I = 10$, then the firm could issue riskless debt and avoid any dilution costs. Similarly, when debt has little default risk, the potential dilution will be small, whereas for large investment needs the firm will need to issue debt with high default risk, creating the potential for substantial dilution costs. The second ingredient is the fact that extent of asymmetric information is greater in the “middle” of the distribution of firm value than in its “right-tail,” a feature that we will denote as “low-costs-in-the-right-tail,” parametrized in the example by $x$. This condition is novel in literature. The decomposition of three regions in Section 4 makes clear that the trinomial structure of our “simple example” is necessary for our results, but also that it provides its key drivers.

In the rest of the paper we build models that generate a reversal of the pecking order, and we show that a reversal can emerge in many natural situations. One of the main contributions of our paper is to offer cross-sectional predictions that can be used to test when asymmetric information yields a pecking order and when it generates a reversal. For example, the real options model we build in Section 5 is better fitted for calibrations, while sharing many of the same insights as the simple example in this section.

3 The basic model

An all equity-financed firm with no cash has a one-period investment project. The project requires a capital outlay $I$ at the beginning of the period. Conditional on making the investment, the firm’s value at the end of the period is given by a random variable $Z_\theta$. There are two types of firms: “good” firms, $\theta = G$, and “bad” firms, $\theta = B$, which are present in the economy with probabilities $p$ and $1 - p$, respectively. A firm of type $\theta$ is characterized by its density function $f_\theta(z)$ and by the corresponding cumulative distribution function $F_\theta(z)$, with $\theta \in \{G, B\}$. Because of limited liability, we assume that $Z_\theta$ takes values of the positive real line. For ease of exposition, we shall also assume that the density function of $Z_\theta$ satisfies $f_\theta(z) > 0$ for all $z \in \mathbb{R}_+$. In addition, we assume type $G$ firms dominate type $B$ ones by first-order stochastic dominance, defined as follows.

**Definition 1 (FOSD).** We will say that the distribution $F_G$ dominates the distribution $F_B$ by (strong) first-order stochastic dominance if $F_G(z) \leq (<) F_B(z)$ for all $z \in \mathbb{R}_{++}$.

The stronger property of Conditional Stochastic Dominance, CSD, plays a crucial role in the security design problem, as argued in Nachman and Noe (1994).
**Definition 2 (CSD).** We will say that the distribution \( F_G \) dominates the distribution \( F_B \) by conditional stochastic dominance if \( F_G(z|z') \leq F_B(z|z') \) for all \( z' \in \mathbb{R}_+ \), where

\[
F_\theta(z|z') \equiv \frac{F_\theta(z + z') - F_\theta(z')}{1 - F_\theta(z')},
\]

By setting \( z' = 0 \), we see that CSD implies FOSD. We note that CSD can equivalently be defined by requiring that the truncated random variables \([Z_\theta|Z_\theta \geq \bar{z}]\), with distribution functions \( F_{\text{fosd}}(z|z') \equiv (\theta - F_\theta(z')) / (1 - F_\theta(z')) \), satisfy FOSD for all \( \bar{z} \), that is \([Z_G|Z_G \geq \bar{z}] \geq [Z_B|Z_B \geq \bar{z}]\) for all \( \bar{z} > 0 \).\(^9\) In addition, Nachman and Noe (1994) show that CSD is equivalent to the condition that the ratio \((1 - F_G(z))/(1 - F_B(z))\) is non-decreasing in \( z \) for all \( z \in \mathbb{R}_+ \) (see their Proposition 4). Thus, loosely speaking, CSD implies that the set of payoffs in the right tail of the firm-value distribution are progressively more likely to occur for a firm of type \( G \) relatively to a firm of type \( B \). Referring back to the example in Section 2, it is easy to verify that as long as \( x \leq 0.05 \) the type-\( G \) distribution not only dominates the type-\( B \) in the first-order sense, but also in the CSD sense.

Firms raise the amount \( I \) to fund the investment project by seeking financing in capital markets populated by a large number of competitive, risk-neutral investors. Capital markets are characterized by asymmetric information in that a firm’s type \( \theta \in \{G,B\} \) is private information to its insiders. We also assume that the NPV of the project is sufficiently large that firms will always find it optimal to issue securities and invest, rather than not issuing any security and abandon the project.

When insiders have private information, firms will typically issue securities at prices that diverge from their symmetric information values. Under these circumstances, firms will find it desirable to raise capital by issuing securities that reduce the adverse impact of asymmetric information. To fix ideas, let \( S \) be the set of admissible securities that the firm can issue to raise the required capital \( I \). As is common in this literature (see, for example, Nachman and Noe (1994)), we let the set \( S \) be the set of functions satisfying the following conditions:

\[
\begin{align*}
0 \leq s(z) & \leq z, \quad \text{for all } z \geq 0, \quad (1) \\
s(z) & \text{ is non-decreasing in } z \quad \text{ for all } z \geq 0, \quad (2) \\
z - s(z) & \text{ is non-decreasing in } z \quad \text{ for all } z \geq 0. \quad (3)
\end{align*}
\]

Condition (1) ensures limited liability for both the firm and investors, while (2) and (3) are

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\(^9\)The CSD (hazard-rate) ordering is weaker than the Monotone Likelihood Ratio order, which requires \([Z_G|Z_G \in (\bar{z},\bar{z})] \geq [Z_B|Z_B \in (\bar{z},\bar{z})]\) for all \( \bar{z} \) and \( \bar{z} \); see equation (1.B.7) and Theorem 1.C.5 in Shaked and Shanthikumar (2007).
monotonicity conditions that ensure absence of risk-less arbitrage. We define $S \equiv \{ s(z) : \mathbb{R}_+ \rightarrow \mathbb{R}_+ : s(z) \text{ satisfies (1), (2), and (3)} \}$ as the set of admissible securities.

In this paper we will consider the following capital raising game. The firm moves first, and chooses a security $s(z)$ from the set of admissible securities $S$, that is $s \in S$. After observing the security $s(z)$ issued by the firm, investors update their beliefs on firm type $\theta$, and form posterior beliefs $p(s) : S \rightarrow [0, 1]$. Given their posterior beliefs on firm type, investors purchase the security issued by the firm at a price $V(s)$. The value $V(s)$ that investors are willing to pay for the security $s(z)$ issued by the firm is equal to the expected value of the security, conditional on the posterior beliefs $p(s)$, that is

$$V(s) = p(s)E[s(Z_G)] + (1 - p(s))E[s(Z_B)].$$

Condition (4) implies that securities are fairly priced, given investors’ beliefs. If security $s$ is issued, capital $V(s)$ is raised, and the investment project is undertaken, the payoff to the initial shareholders for a firm of a type $\theta$ is given by

$$W(\theta, s, V(s)) \equiv E[Z_0 - s(Z_0)] + V(s) - I.$$  

The firm will choose the security issued to finance the investment project by maximizing its payoff (5), subject to the constraint that the security is admissible and that it raises at least the required funds $I$. Let $s_\theta(z) \in S$ be the security issued by a firm of type $\theta$.

In this paper, following the literature, we will adopt the notion of Perfect Bayesian Equilibrium, PBE, as follows.

**Definition 3 (Equilibrium).** A PBE equilibrium of the capital raising game is a collection $(s^*_G(z), s^*_B(z), p^*(s), V^*(s))$ such that: (i) $s^*_\theta(z)$ maximizes $W(\theta, s, V^*(s))$ subject to the constraint that $s \in S$ and $V^*(s) \geq I$, for $\theta \in \{G, B\}$, (ii) securities are fairly priced, that is $V^*(s) = p^*(s)E[s(Z_G)] + (1 - p^*(s))E[s(Z_B)]$ for all $s \in S$, and (iii) posterior beliefs $p^*(s)$ satisfy Bayes rule whenever possible.

We start with a characterization of the possible equilibrium sets that will be quite useful in simplifying our exposition.\(^{11}\)

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\(^{10}\)See, for example, the discussion in Innes (1990). Note that, as pointed out in Nachman and Noe (1994), condition (2) is critical to obtain debt as an optimal security. In absence of (2), the optimal contract may have a “do or die” component, whereby outside investors obtain all of the firm cash flow when it falls below a certain threshold, and nothing otherwise.

\(^{11}\)We note that the strong form of FOSD is only necessary for Proposition 1. Our main results go through assuming only FOSD. At the same time, all the parametric examples we consider satisfy the strong version of FOSD.
Proposition 1. (Nachman and Noe, 1994) Let $F_\theta$ satisfy strict FOSD. No separating equilibrium exists in the capital raising game. In addition, in any pooling equilibrium, with $s^*_G = s^*_B = s^*$, the capital raising game is uninformative, $p(s^*) = p$, and the financing constraint is met with equality, $E[s^*(Z)] = I$.

Proposition 1 derives from the fact that, with two types of firms only, a type $B$ firm has always the incentive to mimic the behavior of a type $G$ firm (i.e., to issue the same security). This happens because (2) and strict FOSD together imply that securities issued by a type $G$ firms are always priced better by investors than those issued by a type $B$ firm, and type $B$ firm is always better-off by mimicking a type $G$ one. This also implies that, in equilibrium, the type $G$ firm is exposed to dilution due to the pooling with a type $B$ firm, and the corresponding loss of value can be limited by issuing only the securities needed to raise the capital outlay $I$.

Proposition 1 allows us to simplify the exposition as follows. If both type of firms pool and issue the same security $s$ and the capital constraint is met as equality, we have that

$$I = pE[s(Z_G)] + (1 - p)E[s(Z_B)].$$

Combining (5) and (6), it is easy to see that the payoff to the original shareholders of firm type $G$ becomes

$$W(G, s, V(s)) = E[Z_G] - I - (1 - p)D_s,$$

where the term

$$D_s = \mathbb{E}[s(Z_G)] - \mathbb{E}[s(Z_B)]$$

represents the dilution suffered by a firm of type $G$ when security $s \in S$ is used.

Under these circumstances, firms of type $G$ will find it optimal to finance the project by issuing a security that minimizes dilution $D_s$, that is

$$\min_{s \in S} D_s$$

subject to the financing constraint (6). Defining the function $c(z) \equiv f_G(z) - f_B(z)$, the dilution costs of security $s(z)$ can be expressed as:

$$D_s = \int_0^\infty s(z)c(z)dz.$$
the firm is \( z \). Thus, we can interpret the term \( c(z) \) as representing the private “asymmetric information cost” for a firm of type \( G \), relative to a type \( B \) firm, of issuing a security that has a payoff of $1 if the final firm value is \( z \). In particular, if \( c(z) > 0 \) we will say that the information costs for a type \( G \) are “positive,” and that these costs are “negative” if \( c(z) < 0 \). More formally, the asymmetric information costs of a security that pays $1 if and only if the final payoff is in the interval \( z \in [z_L, z_H] \) is equal to \( \int_{z_L}^{z_H} c(z) \, dz \).

In what follows we will be concerned on the asymmetric information costs in the upper tail of the value distribution \( F_\theta(z) \) for a firm of type \( G \) relative to a firm of type \( B \). These asymmetric information costs are related to the function \( H(z) \) defined as:

\[
H(z) \equiv \frac{F_B(z) - F_G(z)}{1 - F(z)},
\]

where \( F(z) \) denotes the mixture of the distributions of the good and bad types, that is,

\[
F(z) = pF_G(z) + (1 - p)F_B(z).
\]

The function \( H(z) \) plays a critical role in our analysis. First note that FOSD implies that \( H(z) > 0 \) for all \( z \in \mathbb{R}_{++} \). In addition, and more importantly, monotonicity of \( H(z) \) is equivalent to CSD, as it is established in the following proposition.

**Proposition 2.** The distribution \( F_G \) dominates \( F_B \) by (strong) conditional stochastic dominance if and only if the function \( H(z) \) is (strictly) increasing in \( z \) for all \( z \in \mathbb{R}_+ \). This is equivalent to requiring that the hazard rates \( h_\theta(z) \equiv f_\theta(z)/(1 - F_\theta(z)) \) satisfy \( h_G(z) \leq (\leq) h_B(z) \) for all \( z \in \mathbb{R}_+ \).

The function \( H(z) \) provides a measure of the extent of asymmetries of information, which for monotonic securities is closely linked to the cost to a type \( G \) firm of promising to investors an extra dollar in state \( z \). In what follows, it will be important to characterize properties the right tail of the firm-value distribution that are stronger than FOSD, but at the same time weaker than CSD. Note first that \( H(0) = 0 \) and that, from FOSD, we have \( H(z) > 0 \) for \( z \) in a right neighborhood of \( z = 0 \), which together imply that \( H'(0) > 0 \). It is important to note that, while the monotonicity properties of \( H(z) \) on the left-tail of the distribution of \( z \) are dictated by FOSD, this is not the case for the right-tail of the distribution. In the simple example of Section 2, the function \( H \) is increasing if \( x \leq 0.5 \).

\footnote{This happens because, for monotonic securities, an extra dollar paid in state \( z \) means that investors will be paid an extra dollar also in all states \( z' > z \). This interpretation will become apparent in Section 7 (see equation (34)).}
To characterize the behavior of the information costs in the right-tail of the distribution, we introduce the following definition, which will play a key role in our analysis.

**Definition 4** (*h*-ICRT). We will say that distribution \( F_G \) has information costs in the right tail of degree \( h \) (*h*-ICRT) over distribution \( F_B \) if \( \lim_{z \to \infty} H(z) \leq h \).

We will use the term NICRT (no-information-costs-in-the-right-tail) to denote the case \( h = 0 \). The relationship between FOSD, CSD and *h*-ICRT may be seen by noting that for two distributions \( \{ F_G, F_B \} \) that satisfy FOSD, there may exist a sufficiently low \( h \in \mathbb{R}_+ \) such that the *h*-ICRT property holds, while conditional stochastic dominance fails. Thus, intuitively, distributions that satisfy the *h*-ICRT condition “fill” part of the space of distributions that satisfy FOSD but do not satisfy the CSD condition. In particular, all distributions that satisfy Definition 4 for \( h = 0 \) (NICRT) will fail to satisfy the CSD condition.

We conclude this section by introducing an additional regularity condition that will simplify the analysis and greatly streamline the presentation of some of the results.

**Definition 5** (SCDP). The distributions \( F_\theta(z) \), for \( \theta = G, B \), satisfy the single-crossing density property (SCDP) if \( F_G \) strictly first-order stochastically dominates \( F_B \), and there exists a unique \( \hat{z} \in \mathbb{R}_+ \) such that \( f_G(\hat{z}) = f_B(\hat{z}) \).

Note that the SCDP condition implies that for all \( z \leq \hat{z} \) we have \( f_B(z) \geq f_G(z) \), and for all \( z \geq \hat{z} \) we have \( f_B(z) \leq f_G(z) \). Intuitively, this means that cash flows above the critical cutoff \( \hat{z} \) have a positive information cost for type \( G \) firms, \( c(z) > 0 \), whereas cash flows below that cutoff have negative information costs, \( c(z) < 0 \). Note that FOSD alone only implies that there exists \( z_1 \) and \( z_2 \) such that \( c(z) < 0 \) for all \( z < z_1 \) and \( c(z) > 0 \) for all \( z > z_2 \), but it does not rule out other interior crossings; in contrast, SCDP ensures that \( z_1 = z_2 \).

### 4 The debt-equity choice

We start the analysis by restricting our attention in this section to two classes of securities, debt and equity. From (7), the dilution costs associated with equity are given by

\[
D_E = \lambda (\mathbb{E}[Z_G] - \mathbb{E}[Z_B]),
\]

with \( \lambda = I/\mathbb{E}[Z] \), whereas those associated with debt

\[
D_D = \mathbb{E}[\min(Z_G, K)] - \mathbb{E}[\min(Z_B, K)],
\]

\footnote{We are assuming SCDP for ease of exposition. The discussion below could be adapted to take into account the presence of multiple crossings.}
where $K$ represents the (smallest) face value of debt that satisfies the financing constraint $I = p\mathbb{E}[\min(Z_G, K)] + (1 - p)\mathbb{E}[\min(Z_B, K)]$. In what follows we will say the pecking order (PO) is satisfied if $D_D > D_E$, and that the model generates a “reverse pecking order” (RPO) if $D_D > D_E$.

We begin our analysis by providing a necessary and sufficient condition for RPO to hold.

**Proposition 3.** The dilution costs of equity will be strictly smaller than those of debt, i.e., $D_E < D_D$, if and only if

$$
\frac{\mathbb{E}[Z_G]}{\mathbb{E}[Z_B]} < \frac{\mathbb{E}[\min(Z_G, K)]}{\mathbb{E}[\min(Z_B, K)]}.
$$

(14)

The proposition implies that when the security choice is restricted between equity and debt, the security that generates lowest dilution in dollar terms is the one with the lowest relative valuation between the good and bad type. This feature supports the traditional intuition that debt dominates equity precisely because debt valuation is less sensitive to the underlying asymmetries in information, limiting dilution.\textsuperscript{14}

We next determine the restrictions on the distribution functions $F_\theta(z)$ for $\theta \in \{G, B\}$ at which (14) holds or fails, that is when PO or RPO obtain. The next proposition shows dominance of debt over equity under the CSD condition of Nachman and Noe (1994).

**Proposition 4.** Condition (14) cannot hold if $F_G$ dominates $F_B$ in the conditional stochastic dominance sense.

The CSD condition is a rather strong requirement that may fail in many economically interesting cases, opening the possibility for equity financing to dominate debt financing. In this paper we will focus on such cases.

To obtain the conditions under which equity financing dominates debt, note RPO holds if and only if the dilution costs of debt, $D_D$, are greater than the dilution costs of equity, $D_E$. From (9), (12) and (13) this happens when:

$$
D_D - D_E = \int_0^\infty (\min(z, K) - \lambda z) c(z) dz > 0.
$$

(15)

Define $\bar{z}(K, \lambda) \equiv K/\lambda$ and note that for $z < \bar{z}(K, \lambda)$ we have that $\min(z, K) > \lambda z$, which implies that the payoffs to debtholders are greater than those to equity holders; the converse holds for $z > \bar{z}(K, \lambda)$.

\textsuperscript{14}A similar condition was obtained, in the context of unit IPOs, in Chakraborty, Gervais, and Yılmaz (2011), see their Propositions 1 and 2.
The factors that drive the relative dilution of debt and equity can be seen by decomposing (15) in more fundamental components. To this aim, it is useful to rank the point where equity payouts are equal to debt payouts, denoted by $\bar{z}(K, \lambda)$, with respect to the critical point in the SCDP condition, given by $\hat{z}$, as follows. As we will show below, this condition is necessary to reverse the pecking order, that is for an “unpecking” order to arise.

**Definition 6 (UNC).** The “unpecking” necessary condition (UNC) is satisfied if $\bar{z} > \hat{z}$.

Under SCDP, the point $\hat{z}$ divides the positive real line into two disjoint sets: a first set at the lower end of the positive real line, $[0, \hat{z})$ where $c(z) < 0$, that is where a type-$G$ firm enjoys “negative information costs” (that is, effectively an information benefit), and a second set $[\hat{z}, \infty)$ where $c(z) \geq 0$, that is where a type-$G$ firm faces “positive information costs.” The point $\bar{z}(K, \lambda)$ divides the positive real line in two other subsets, depending on whether or not equity yield higher payoffs than debt to investors. Thus, SCDP and UNC together divide the positive real line into three regions: (i) a low-value region where $z < \hat{z}$ and $z \leq \bar{z}$; (ii) an intermediate region where $[\hat{z}, \bar{z}]$; and (iii) a high-value region where $z > \bar{z}(K, \lambda)$ (see Figure 1).

The relative dilution costs of equity and debt, $D_D - D_E$, depend on the comparison of the information costs and relative payoffs of debt and equity in each of these different regions, as formalized in the next Proposition.

**Proposition 5.** Assume the SCDP holds. Then a necessary and sufficient condition for the reverse pecking order is that (i) UNC holds, and (ii)

$$D_D - D_E = \int_0^{\hat{z}} (\min(z, K) - \lambda z) c(z) dz - \int_0^{\hat{z}} (\lambda z - \min(z, K)) c(z) dz - \int_{\hat{z}}^{\infty} (\lambda z - K) c(z) dz > 0.$$  

(16)

Under UNC and the maintained assumptions the three integrals in (16) are all positive. The first term of the r.h.s. of (16) measures the dilution cost of debt relative to equity in the intermediate-value region $[\hat{z}, \bar{z}]$, where debt has higher payouts than equity and type-$G$ firms suffer a positive information cost, $c(z) > 0$. In this region dilution costs of equity are lower than those of debt because equity has lower payoff than debt precisely in those states in which type-$G$ firms are exposed to positive information cost (since $c(z) > 0$). Note that existence of this region is guaranteed by UNC. It is the presence of this term that makes equity potentially less dilutive than debt.

The second term of the r.h.s. of (16) measures the benefits of debt financing for low realizations of firm value (i.e., for $z < \hat{z}$). In this low-value region, dilution costs are lower for debt than equity because debt gives a higher payoff than equity, but such payoff has
negative information costs (i.e., \( c(z) < 0 \)). The third and last term measures the dilution costs of equity relative to debt for high realizations of firm value (i.e., for \( z > \hat{z} \)). In this high-value region, equity payoffs are greater than debt in those states that are more likely to occur to a type-\( G \) firm, and thus carry positive information costs (i.e., \( c(z) > 0 \)).

The relative importance of these three regions determines the optimality of debt versus equity choice. In particular, equity financing dominates debt financing when the advantages of equity financing originating from the intermediate region of firm value (for \( z \in [\hat{z}, \bar{z}] \)), that is, the first term on the r.h.s. of (16) dominate the disadvantages in the low (for \( z < \hat{z} \)) and the high (for \( z > \bar{z} \)) regions of firm value, that is, the second and the third term on the r.h.s. of (16). Note that if UNC does not hold (so that \( \bar{z}(K, \lambda) < \hat{z} \)), equity has negative information costs (that is, \( c(z) < 0 \)) precisely in the states where the payouts to equityholders are greater than those to debtholders, making it impossible for the inequality (16) to be satisfied. Thus, UNC is a necessary condition to reverse the pecking order.

5 A real options model

In this section, we present a real options specification of our basic model. This approach presents two main advantages. First, it draws on well established option pricing techniques that provide analytical tractability. Second, it provides sufficient flexibility for modeling asymmetric information between firm owners and outside investors. In particular, this specification allows us to have both first-order stochastic dominance and the right-tail behavior of firm-value distribution that generates a reverse pecking order.

We model the real options problem as follows. By paying the investment cost \( I \) at the beginning of the period, \( t = 0 \), the firm generates a new growth opportunity that is exercisable at date \( T \). We model the growth opportunity as an exchange (or “rainbow”) option. That is, the firm holds an option to exchange the existing assets in place, with a value of \( X_{\theta T} \) at a future date \( T \), for the new assets with value \( Y_{\theta T} \), for \( \theta \in \{G, B\} \). We interpret the new assets \( Y_{\theta T} \) as embedding the incremental firm value of the new investment project; therefore, we refer to \( X_{\theta T} \) as the “assets in place” and to \( Y_{\theta T} \) as the “growth opportunity.” Thus, the firm has a European exchange option on non-dividend paying assets. We adopt the exchange option framework also because it is quite common in the real options literature and it admits a closed form solution (see, for instance, Stulz, 1982).

We assume that, after the initial investment \( I \) is made, at the end of the period \( T \) the value of a firm of type \( \theta \) is given by \( Z_{\theta T} \equiv \max(X_{\theta T}, Y_{\theta T}) = X_{\theta T} + \max(Y_{\theta T} - X_{\theta T}, 0) \), for \( \theta \in \{B, G\} \). We assume that both \( X_{\theta T} \) and \( Y_{\theta T} \) follow a lognormal process, that is, both \( \log(X_{\theta T}) \) and \( \log(Y_{\theta T}) \) are normally distributed with means \( \mu_{\theta x} \) and \( \mu_{\theta y} \) and with variances
Let $\rho_\theta$ be the correlation coefficient between $\log(X_{\theta T})$ and $\log(Y_{\theta T})$. Thus, the real option specification is isomorphic to a model where time flows continuously, that is $t \in [0, T]$, and where asset values $X_{\theta T}$ and $Y_{\theta T}$ follow two geometric Brownian motions with drifts $\mu_{\theta x}$ and $\mu_{\theta y}$, variances $\sigma^2_{\theta x}$ and $\sigma^2_{\theta y}$, respectively, and correlation coefficient $\rho$.

In the spirit of Myers and Majluf (1984), we model asymmetric information by assuming that the firm insiders have private information on the means of the distributions, while their variances are common knowledge. We let $E[X_{\theta T}] = X_\theta$ and $E[Y_{\theta T}] = Y_\theta$, and we assume $X_G \geq X_B$ and $Y_G \geq Y_B$, with at least one strict inequality. We define the average value of the assets in place and of the growth opportunity as $\bar{X}_{\theta} = pX_G + (1 - p)X_B$ and $\bar{Y}_{\theta} = pY_G + (1 - p)Y_B$, respectively, and let $c_x = X_G - X_B$ and $c_y = Y_G - Y_B$. Thus, $c_x$ and $c_y$ measure the exposure to asymmetric information of the assets in place and the growth opportunity. Finally, to ensure FOSD we assume that $\sigma_{Gx} = \sigma_{Bx} = \sigma_x$, $\sigma_{Gy} = \sigma_{By} = \sigma_y$, $\rho_G = \rho_B = \rho$, and, without loss of generality, that $\sigma_y \geq \sigma_x$.\footnote{Recall that we assume that the project’s NPV is sufficiently large for investment to be optimal. Thus, the assumption $\sigma_y \geq \sigma_x$ is without loss of generality.}

We can now proceed to explicitly characterize the choice of financing in our real options model. The value of a firm of type $\theta$ is given by the value of the exchange option, denoted by $A_{\theta} \equiv E[Z_{\theta T}]$. Following Margrabe (1978), we know that, at the beginning of the period, $t = 0$, the value of this option for a firm of type $\theta$ is given by

$$A_{\theta} = X_\theta \hat{\Delta}_{x\theta} + Y_\theta \hat{\Delta}_{y\theta}, \quad (17)$$

where $\hat{\Delta}_{x\theta} \equiv N(a_{x\theta})$, $\hat{\Delta}_{y\theta} \equiv N(a_{y\theta})$, $\Sigma^2 = \sigma^2_x + \sigma^2_y - 2 \sigma_x \sigma_y \rho$, and

$$a_{x\theta} = \frac{\log (X_\theta / Y_\theta)}{\Sigma \sqrt{T}} + \frac{1}{2} \Sigma \sqrt{T}, \quad (18)$$

$$a_{y\theta} = \frac{\log (Y_\theta / X_\theta)}{\Sigma \sqrt{T}} + \frac{1}{2} \Sigma \sqrt{T}, \quad (19)$$

where $N(\cdot)$ denotes the cumulative distribution function of a standard normal random variable. Note that (17) is equal to the value of the replicating portfolio of the exchange option, and the terms $\hat{\Delta}_{x\theta}$ and $\hat{\Delta}_{y\theta}$ represent the deltas of the option, that is, the sensitivity of the value of the exchange option with respect to the value of the underlying assets, $X_\theta$ and $Y_\theta$, respectively.\footnote{In addition, we can interpret the term $X_{\theta} \hat{\Delta}_{x\theta}$ as the expectation of the value of asset in place conditional...}
If the firm raises the required capital by issuing equity, existing shareholders will have to sell to outside investors a fraction $\lambda$ of the firm to satisfy the financing constraint, that is

$$\lambda = \frac{I}{pA_G + (1 - p)A_B}.\quad (20)$$

If the firm raises the required capital by issuing debt, we denote by $V_\theta(K)$ as the value of risky debt with face value of $K$ when issued by a firm of type $\theta$. Note that the debt value $V_\theta$ can be written as $V_\theta(K) = E[min(Z_{\theta T}, K)] = K - P_\theta$, that is, as the value of the default-free debt, $K$, minus the value of the option to default, which is equal to $P_\theta = E[max(K - Z_{\theta T}, 0)]$.

The option to default for a firm of type $\theta$ is given by the compound put option given by $max(K - Z_{\theta T}, 0)$, where in turn $Z_{\theta T}$ is given by the exchange option $max(X_{\theta T}, Y_{\theta T})$.

Following Stulz (1982) and Rubinstein (1991), the value of this put option is given by

$$P_\theta = K \Gamma_\theta - X_{\theta T} \Delta^*_x - Y_{\theta T} \Delta^*_y,\quad (21)$$

where $\Delta^*_x = \Gamma(b_{x\theta}, a_{x\theta}, \rho_x)$, $\Delta^*_y = \Gamma(b_{y\theta}, a_{y\theta}, \rho_y)$, $\Gamma_\theta = \Gamma(b_{x\theta} + \sigma_x \sqrt{T}, b_{y\theta} + \sigma_y \sqrt{T}, \rho)$, $\rho_x = (\sigma_x - \rho \sigma_y)/\Sigma$, and $\rho_y = (\sigma_y - \rho \sigma_x)/\Sigma$. The variables $a_{x\theta}$ and $a_{y\theta}$ are given in (18)–(19), and the variables $b_{x\theta}$ and $b_{y\theta}$ are given by

$$b_{x\theta} = \frac{\log(K/X_{\theta})}{\sigma_x \sqrt{T}} - \frac{1}{2} \sigma_x \sqrt{T},\quad (22)$$

$$b_{y\theta} = \frac{\log(K/Y_{\theta})}{\sigma_y \sqrt{T}} - \frac{1}{2} \sigma_y \sqrt{T},\quad (23)$$

where the function $\Gamma(\cdot)$ denotes the cumulative distribution function of a bivariate standard normal random vector.\(^{17}\) Note that (21) is equal to the value of the replicating portfolio of the put option, where the terms $\Delta^*_x$ and $\Delta^*_y$ in (21) are the deltas of the compound put option with respect to the value of underlying assets $X_{\theta T}$ and $Y_{\theta T}$, respectively, and $K \Gamma_\theta$ represents the investment in the riskless asset in the replicating portfolio.\(^{18}\) Finally, the face

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\(^{17}\)Namely $\Gamma(a, b, c)$ is the area under a bivariate standard normal distribution function with correlation $c$ from $-\infty$ to $a$, $-\infty$ to $b$. Thus, $\Delta_x = N(a_{x\theta})$ can be interpreted (loosely speaking) as the probability that the assets in place are more valuable than the growth opportunity, $Y_{\theta}$, by symmetry, the second-term in (17) has a similar interpretation.

\(^{18}\)Note also that the term $\Gamma_\theta$ can be interpreted as the probability, for a firm of type $\theta$, that the put option in (21) is in the money. In a similar way, the term $\Delta^*_x$ can be interpreted as the probability of the underlying asset $X_{\theta T}$ to be larger than $K$ and $Y_{\theta}$ for a firm of type $\theta$; similarly for the term $\Delta^*_y$.
value of the debt, $K$, has to satisfy the financing constraint, which is given by

$$K - (pP_G + (1 - p)P_B) = I. \quad (24)$$

From (15) we see that equity financing is less dilutive than debt financing, so that the reverse pecking order obtains, if and only if

$$\lambda (A_G - A_B) < P_B - P_G, \quad (25)$$

where $A_\theta$ and $P_\theta$ are given in (17) and (21), respectively. It is important to note that while condition (25) gives a closed-form solution for the preference of equity over debt financing, the left hand side of (25) includes the term $\lambda$, which depends on the model’s primitives via the financing constraint (20), and the right hand side depends on $K$ which is determined by the financing constraint (24). Analytical tractability is further hindered by the presence of the bivariate normal cumulative distribution function $\Gamma$ in the valuation equation (21) for the put option. This means that in what follows we will characterize (25) mainly by numerical means.

We start the analysis by considering a perturbation of the parameter values around the case without asymmetric information, i.e., when $Y_G = Y_B$ and $X_G = X_B$. In the perturbation, only the assets in place are exposed to (a small amount of) asymmetric information: $X_G = \bar{X} + \epsilon$ and $X_B = \bar{X} - \epsilon$. For $\epsilon$ sufficiently close to zero, it is easy to see that condition (25) reduces to

$$\lambda \Delta_x < \Delta^*_x, \quad (26)$$

where we have dropped the type $\theta$ subscript. This “delta” condition implies that equity is less dilutive than debt if the sensitivity to $X$ of the value of the equity sold to outside investors (which depends on the delta of the exchange option, $\Delta_x$) is smaller than the corresponding sensitivity of debt, measured by $\Delta^*_x$ (which depends on the corresponding delta of the compound put option, $\Delta^*_x$). The deltas of the two options measure the sensitivity of the value equity and debt to the value of the underlying asset(s), and therefore their exposure to asymmetric information and potential mispricing around the no asymmetric information case.

Condition (26) can be further simplified in terms of univariate cumulative normal distributions when $\sigma_x = \rho \sigma_y$. In this case, it is easy to see that $\Delta^*_x \equiv \Gamma(b_x, a_x, 0) = N(b_x) \times N(a_x) = \Delta_x \times \Delta_x$, where $\Delta_x$ is the delta of a “plain vanilla” put option written on the assets in place only, namely $\Delta_x = N(b_x)$, and $\Delta^*_x$ is again the delta of the exchange option in (17). This means that the delta of the compound put option (i.e., the default option) that can be
decomposed into the product of the delta of a “simple” put option written only on the assets in place, \(X\), with a strike price equal to the face value of the debt, \(K\), times the delta with respect to the assets in place \(X\) of the underlying exchange option. Substituting \(\Delta^* = \Delta_x \times \hat{\Delta}_x\) into (26) and using (20), we obtain that (25) reduces to

\[
\lambda = \frac{I}{X\Delta_x + Y\Delta_y} < \Delta_x. \tag{27}
\]

The next Proposition allows us to characterize the reverse pecking order under these parametric assumptions.\(^{19}\)

**Proposition 6.** Consider the case where there is no informational asymmetry on \(Y\), \(Y_G = Y_B = \bar{Y}\), but there is on \(X\), namely, \(X_G = \bar{X} + \epsilon\) and \(X_B = \bar{X} - \epsilon\), with \(\bar{X} > 0\). Further assume that \(\rho \sigma_y = \sigma_x\). Then, as we let \(\epsilon \downarrow 0\), we have that: (i) condition (25) holds for sufficiently large values of \(\bar{Y}\), where it can never hold for small values of \(Y\); (ii) as \(\sigma_x \downarrow 0\), condition (25) holds if \(\bar{X} < K\), but cannot hold if \(\bar{X} > K\).

Proposition 6 establishes analytically two of the main results of our paper. Part (i) of Proposition 6 states that a reverse pecking order obtains if the value of the growth opportunity, \(\bar{Y}\), is sufficiently large, while the pecking order prevails when \(\bar{Y}\) is sufficiently small. This can be seen as follows. First, note that (27) is more likely to be satisfied when \(\bar{Y}\) is large compared to \(\bar{X}\). This happens because, in this case, the exchange option is sufficiently in-the-money with respect to \(Y\) to make \(\hat{\Delta}_y\) relatively large. This feature, combined with a large value of the growth opportunity itself, \(\bar{Y}\), leads to a low value of \(\lambda\) on the l.h.s. of (27), while the r.h.s. is independent of \(\bar{Y}\). This implies the firm must issue to outside investors a relatively small equity share \(\lambda\), while the sensitivity of the option to default with respect to \(X\) may still be rather significant. Second, note that for \(\bar{Y} = 0\), (27) can never be satisfied. This happens because, when \(\bar{Y} = 0\), the exchange option is always equal to the value of the assets in place (since the growth opportunity has no value). This means that \(\hat{\Delta}_x = 1\), and (27) requires that \(I < \Delta_x \bar{X}\), which violates the financing constraint (24).\(^{20}\)

Part (ii) of Proposition 6 stresses the role of the option to default and of the initial investment \(I\) (since under debt financing, the face value of the debt \(K\) is an increasing function of the required investment \(I\)) to generate reversals of the pecking order. When the volatility of the assets in place, \(\sigma_x\), is sufficiently small, the variables \(\bar{X}\) and \(K\) identify two separate regions. The first region occurs for \(K < \bar{X}\) (that is, for low levels of the initial investment)

\(^{19}\)We conjecture that the statements in the following Proposition are more general, as we verify in the numerical analysis. Analytical proofs in the general case are much more demanding due to the presence of the bivariate normal cumulative distribution function \(\Gamma\) in the valuation equation (21) for the put option.

\(^{20}\)To see this note that the financing constraint (24) can be written in this case as \(I = (1 - \Gamma)K + \bar{X}\Delta_x\).
and is a “safety region” where the debt is in default with very low probability. In this case, the value of delta of the put option, \( \Delta_x \), is very small, and (27) cannot be verified. Thus, a reversal of the pecking order cannot arise.

The second region occurs for \( K > \bar{X} \) (that is for large levers of the initial investment) and is a “bankruptcy region” where debt has a non-trivial chance of default. This implies that the value of the debt is highly sensitive to changes in value for the assets in place \( X \). Thus, the value of delta of the put option, \( \Delta_x \), in (27) is large (close to one). At the same time, the exchange option still gets a significant value from the growth opportunity component \( Y \). This implies that the l.h.s. of (27), \( \lambda \), is small (i.e., smaller one) and that (27) is always verified, generating a reverse pecking order.

So far we have considered perturbations where only the assets in place (i.e., the assets with the lower volatility) are exposed to a small amount of asymmetric information. The symmetric case occurs when there is no asymmetric information on the assets in place, \( X \), but the growth opportunity \( Y \) is exposed to a small amount of asymmetric information. This corresponds to the case where \( X_G = X_B = \bar{X} \) and \( Y_G = \bar{Y} + \epsilon \) with \( Y_B = \bar{Y} - \epsilon \), for \( \epsilon > 0 \) arbitrarily small.\(^{21}\) We will show in Section 7 that in the case where the asymmetric information loads only on the growth-option \( Y \), debt financing is always optimal, and the standard pecking order holds. More generally, we will show that a reverse pecking order will occur when the assets in place are more exposed to asymmetric information than the growth opportunity. This means that a key feature to generate the reversal of the pecking order is that asymmetric information characterizes the assets with lower volatility.

We conclude this section by conducting a series of numerical examples of the general condition (25), all centered on a base case reported in Table 2. In order to keep the parameters as parsimonious as possible, we will assume that \( Y_G = Y_B = 175 \), \( \sigma_x = 0.3 \), \( \sigma_y = 0.6 \), \( T = 10 \) and \( \rho = 0 \). The asymmetric information corresponds to the assets in place, namely \( X_G = 125 \) and \( X_B = 75 \). We let both types be equally likely, \( p = 0.5 \). The value of the firm post-investment for the two types is given by \( E[Z_{GT}] = 257.6 \) and \( E[Z_{BT}] = 218.3 \), so that \( pE[Z_{GT}] + (1-p)E[Z_{BT}] = 237.9 \). In the base case specification, we let the investment amount \( I = 110 \).

Without the project, the status-quo firm value is the value of asset \( X \), which is equal to \( \bar{X} = 100 \). Since the value of the firm post investment is 237.9, and the investment is \( I = 110 \), the project has an (unconditional) positive NPV of 237.9. Note also that the efficient

\(^{21}\)Note that (26) does not simplify as in the case with \( X \), since the correlation term \( \rho_y = (\sigma_y - \rho \sigma_x) / \Sigma \) defined after (21) satisfies \( \rho_y > 0 \) when \( \sigma_y > \sigma_x \). Its implicit term, the face value of the debt \( K \), makes it analytically challenging. We note how none of the volatility limits in Proposition 6 apply under our stated condition \( \rho \sigma_y = \sigma_x \).
outcome is for both types of firms would be to finance the project, since for a type-\(G\) we have that \(\mathbb{E}[Z_{GT}] - I = 257.6 - 110 = 147.6 > 125 = X_G\), and for a type-\(B\) we have that \(\mathbb{E}[Z_{BT}] - I = 218.3 - 110 = 108.3 > 75 = X_B\).

It is easy to verify that issuing equity will require that the equity holders give up a stake of \(\lambda = 0.462 = 110/237.9\). This means that residual equity value for a type-\(G\) type firm is equal to \((1 - 0.462) \times 257.6 = 138.6 > 125 = X_G\), and for a type-\(B\) type firm is equal to \((1 - 0.462) \times 218.3 = 117.4 > 75 = X_B\). In order to finance the project with debt, the firm needs to promise bondholders a face value of \(K = 198.3\) at maturity. Using (21), one can readily check that the values of debt for the good type and the bad type are \(V_G(K) = \mathbb{E}[\min(Z_{GT}, K)] = 120.6\) and \(V_B(K) = \mathbb{E}[\min(Z_{BT}, K)] = 99.4\), respectively. The dilution costs of equity are \(D_E = 0.462 \times (257.6 - 218.4) = 18.2\), whereas those of debt are \(D_D = 120.6 - 99.4 = 21.2\). Thus, the type-\(G\) firm is exposed to lower dilution by raising capital with equity rather than debt.

For the parameter values in Table 2, Figure 1 displays the plots of the function \(c(z)\) (top panel, solid line) and of the densities of firm value for both type of firms and their average, \(\{f_G(z), f_B(z), f(z)\}\) (bottom panel). Note that in this numerical example the region in which debt has a disadvantage over equity (i.e., the intermediate region of (16)) is large, namely for values of \(z\) that lie in the \([79, 429]\) interval.

Figures 2 and 3 present comparative static exercises around the base case of Table 2. The top graph in Figure 2 displays indifference lines of \(D_D = D_E\), as a function of the exposure to asymmetric information of the assets in place, \(c_x\), and the growth opportunity, \(c_y\), for three levels of the volatility of the growth opportunity, \(\sigma_y \in \{0.6, 0.7, 0.8\}\). In the region above the lines, we have that \(D_D > D_E\) and hence equity is less dilutive than debt and RPO obtains. In the region below the lines, we have that \(D_D < D_E\) and hence equity is more dilutive than debt, and the usual PO obtains. Note that the slope of the indifference lines declines when the volatility of the growth opportunity rises. These graphs reveal that equity is more likely to be less dilutive than debt when less volatile assets in place are more exposed to asymmetric information than more volatile assets.

The bottom graph in Figure 2 sets the exposure to asymmetric information back to the values of the base case of Table 2 (i.e. \(c_x = 25\) and \(c_y = 0\)) and charts indifference lines of \(D_D = D_E\), as a function of the time horizon, \(T\), and the investment cost, \(I\), for three levels of the average value of assets in place \(\bar{X} \in \{95, 100, 105\}\). For pairs of \((I, T)\) below the lines debt is optimal, whereas equity is optimal above the lines. These graphs reveal that equity is more likely to be less dilutive than debt for lower values of assets in place (i.e., smaller firms), higher investment costs, and longer time horizons (i.e., for younger firms).

The top graph of Figure 3 displays the pairs of the average value of assets in place and
the average value of the growth option, \((\bar{X}, \bar{Y})\), for which the dilution costs of equity and debt are the same, i.e. \(D_E = D_D\), for different level of asymmetric information on asset \(c_X \in \{10, 25, 40\}\). For pairs of \((\bar{X}, \bar{Y})\) below the lines debt is optimal, whereas equity is optimal above the lines. These graphs reveal that equity is more likely to be less dilutive than debt when the average value of assets in place is low relative to the average value of growth opportunities, and when the exposure to asymmetric information of assets in place increases.

Finally, the bottom graph of Figure 3 sets \(\bar{X} = 100\) and \(\bar{Y} = 175\), as in the base case of Table 2, and plots the pairs of volatilities, \((\sigma_x, \sigma_y)\), such that the dilution costs of equity and debt are the same, i.e. \(D_E = D_D\), for three levels of the investment cost \(I \in \{100, 110, 120\}\). For pairs of volatilities, \((\sigma_x, \sigma_y)\), below the lines debt is optimal, whereas equity is optimal above the lines. These graphs reveal that equity is more likely to be less dilutive than debt when investment cost is large, when the volatility of assets in place is low, and when the volatility of growth opportunities is large.

In the final set of numerical examples, we consider a small extension of the basic model. In this perturbation of the model we assume that the exercise of the exchange option requires the firm to make at the end of the period \(T\) an additional investment \(I_T\). This means that the end-of-period firm value is now described by \(Z_{\theta T} = \max(X_{\theta T}, Y_{\theta T} - I_T)\). Note that this characterization of firm value represents a dual-strike option (where the first asset, \(X_{\theta T}\), has a zero strike price) and it does not have a closed-form solution. The results for are displayed in Tables 3. We consider similar parameter values as in Table 2, with the difference that now \(\sigma_y = 0.50\), \(I = 100\), and \(Y_G = Y_B = 200\). With this lower value of volatility of the growth opportunity, debt is less dilutive than equity and the pecking order holds if \(I_T = 0\) (see first row in Table 3). Increasing the level of investment \(I_T\) increases the dilution of both debt and equity, but the dilution of debt increases faster than the dilution of equity and a reversal of the pecking order obtains for \(I_T \geq 50\). The intuition for these results is a follows. Increasing the investment level \(I_T\) makes the exchange option to be less in-the-money, which increases the leverage that is embedded in the option. Thus, a larger value of the investment \(I_T\) has a similar effect to a greater value of the volatility of the growth opportunity \(\sigma_y\). The numerical results imply that equity financing is more likely to be less dilutive than debt if the exploitation of the growth opportunity requires a greater investment \(I_T\).

In summary, the examples in Tables 2 and 3, as well as Figures 2 and 3, reveal a very consistent pattern. If growth opportunities are characterized by greater volatility and lower exposure to asymmetric information, equity is more likely to be less dilutive than debt for

\[^{22}\text{See, for example, (Rubinstein, 1991).}\]
small and younger firms that have large investment needs. In addition, equity is more likely to be less dilutive than debt when growth opportunities represent a greater proportion of firm value, when these growth opportunities are riskier, and when assets in place are more exposed to asymmetric information than growth opportunities. These predictions help to explain the stylized fact that small and young firms with large financing needs tend to often prefer equity financing over debt financing in circumstance where asymmetric information is potentially severe.

6 Optimal financing with existing debt

We have considered so far a firm that is all equity-financed ex-ante. In this section, we study the effect of prior financing on the debt-equity choice. In particular, we assume the firm has already issued straight debt with face value $K_0$ prior to the beginning of the period, $t=0$, which is due at the end of the period, $T$. In accordance to anti-dilutive “me-first” rules that may be included in the debt covenants, we assume that this pre-existing debt is senior to all new debt that the firm may issue in order to finance the new project. We maintain the assumption that the new investment is sufficiently profitable that all firms want to raise external capital to finance it.\(^\text{23}\)

As in the previous analysis, we restrict the choice of security to equity or junior debt. This implies that the firm can raise the necessary capital either by sale of junior debt with face value $K$, or by sale of a fraction $\lambda$ of total (levered) equity of the firm to outside investors. Following an argument similar to the one in Section 4, the relative dilution of debt versus equity is now given by:

$$
\mathcal{D}_D - \mathcal{D}_E = \int_{K_0}^{\infty} [(1 - \lambda) \max(z - K_0, 0) - \max(z - (K_0 + K), 0)] c(z) dz. \tag{28}
$$

Note that the main difference of (28) relative to the corresponding expression in (15) is the fact that all payoffs below $K_0$ are allocated to the pre-existing senior debt. This implies that only the probability mass located in the interval $[K_0, \infty)$ is relevant for the determination of the relative dilution costs of debt and equity and, thus, for the choice of financing of the new project. Recall from (16) that the two regions located at the left and the right tails of the probability distribution favor debt financing, while the intermediate region favors equity financing. This means that the presence of pre-existing debt in a firm’s capital structure, by reducing the importance of the left-tail region, makes equity more likely to be the less

\(^{23}\text{This assumption allows us to ignore a possible debt overhang problem in the sense of Myers (1977), whereby the presence of pre-existing debt may induce a firm not to undertake a positive-NPV project.}\)
dilutive source of financing.

The presence of pre-existing debt affects the financing choice in the context of the real options model presented in Section 5 as follows. Similar to the previous case, \( Z_{\theta T} = \max(X_{\theta T}, Y_{\theta T}) \) represents the exchange option between the assets in place and the growth opportunity. At the beginning of the period, \( t = 0 \), the value of levered equity for a firm of type \( \theta \) with a face value of debt \( K_0 \) is given by

\[
C_{\theta}(K_0) \equiv \mathbb{E}[\max(Z_{\theta T} - K_0, 0)]
\]  

(29)

where \( C_{\theta}(K_0) \) represents the value of a call option on the with strike price \( K_0 \), written on the exchange option \( Z_{\theta T} \). Similarly, the value of the junior debt with face value \( K \), \( J_{\theta}(K) \), is given by

\[
J_{\theta}(K) = C_{\theta}(K_0) - C_{\theta}(K_0 + K),
\]  

(30)

From Stulz (1982) and Rubinstein (1991), we know that the value of these call options is given by

\[
C_{\theta}(\hat{K}) = X_{\theta} \Delta^*_{\theta x} + Y_{\theta} \Delta^*_{\theta y} - \hat{K}(1 - \Gamma_{\theta}),
\]  

(31)

where \( \hat{K} \in \{K, K_0 + K\} \) represents the investment in the risk-free asset in the corresponding duplicating portfolios, and \( \Delta^*_{\theta x}, \Delta^*_{\theta y}, \) and \( \Gamma_{\theta} \) are defined in the previous section.

The analysis of the relative dilution costs of debt versus equity follows an argument similar to the one developed in Section 5. Given an investment amount \( I \), the new equity holders require a fraction of the outstanding equity \( \lambda \) such that the financing constraint \( I = \lambda[p C_{G}(K_0) + (1 - p) C_{B}(K_0)] \) holds. Similarly, the new debt holders will ask for a face value \( K \) such that the financing constraint \( I = p J_{G}(K) + (1 - p) J_{B}(K) \) holds. We have the following Proposition.

**Proposition 7.** The capital raising game exhibits a reverse pecking order, in which equity is preferred to debt, if and only if

\[
C_{G}(K_0 + K) - C_{B}(K_0 + K) < (1 - \lambda)(C_{G}(K_0) - C_{B}(K_0)).
\]

The example introduced in Table 4 allows us to illustrate this case. For simplicity, we assume again there is information asymmetry on \( X \), but not \( Y \), namely let \( Y_G = Y_B = 120 \), and \( X_G = 110 \) and \( X_B = 90 \) and we set the volatilities at \( \sigma_x = 0.25 \) and \( \sigma_y = 0.50 \). Furthermore, we assume that \( I = 50 \), and that the project’s payoffs are realized at \( T = 10 \). Finally, we assume that type-\( G \) and type-\( B \) firms are again equally likely, \( p = 1/2 \). We consider the case where the firm has pre-existing debt outstanding maturing in \( T = 10 \) with
a face value of $K_0 = 50$. Equity financing of the project requires setting $\lambda = 0.38$, with associated dilution costs of $D_E = 5.6$. Junior debt financing requires a promised payment of $K = 91.3$, with has associated dilution of $D_D = 6$. Thus, under these parameter values, equity is less dilutive than debt. It is easy to verify that in the absence of pre-existing debt (i.e. for $K_0 = 0$) the project can be financed by selling a fraction $\lambda = 0.28$ of the firm, or promising bond holders a face value $K = 53.7$. It is straightforward to check that in this case the dilution of the new debt is $D_D = 1.2$, while the dilution of new equity is $D_E = 4.4$, which means that debt dominates equity. Thus, due to the existence of senior debt, equity becomes a better financing instrument than junior debt. This happens because when $K_0 = 0$ the firm can finance the project by issuing close to risk-free debt, which makes the dilution costs of debt very small. In contrast, the presence of pre-existing debt forces the firm to issue new debt that is riskier, and thus more information sensitive, creating the potential for greater dilution.

7 Optimal security design

We now consider the general optimal security design problem in a setting where probability distributions of firm value satisfy only FOSD. Following Nachman and Noe (1994), the optimal security design problem in (8) can be expressed as:

$$\min_{s \in S} \int_0^{\infty} s'(z)(F_B(z) - F_G(z))dz, \quad (32)$$

subject to

$$\int_0^{\infty} s'(z)(1 - F(z))dz = I.$$  \quad (33)

The Lagrangian to the above problem is

$$\mathcal{L}(s', \gamma) = \int_0^{\infty} s'(F_B(z) - F_G(z) - \gamma(1 - F(z)))dz.$$  \quad (34)

The determination of the optimal security depends critically on the information costs in the right tail of the payoff distribution. This can be seen by noting that the function $H(z)$ in (10) is a transformation of the Lagrangian expression (34). The following is an immediate consequence of the linearity of the security design problem.

---

24 The payoffs to debt and equity holders are presented in the top part of Figure 4 as dotted lines. The solid line represents again the information costs for a type-$G$ firm. Note that debt yields higher payoffs than equity as long as $z \leq 295$. It is interesting

25 Thus, the only departure from Nachman and Noe (1994) is that we relax CSD.
Proposition 8. (Nachman and Noe, 1994) A solution \( s^* \) must satisfy, for some \( \gamma \in \mathbb{R}_+ \),

\[
(s^*)'(z) = \begin{cases} 
1 & \text{if } H(z) < \gamma; \\
[0, 1] & \text{if } H(z) = \gamma; \\
0 & \text{if } H(z) > \gamma.
\end{cases}
\]  

(35)

Proposition 8 proposes an algorithm to solve the problem: (a) identify the set of \( z \) such that \( H(z) = \gamma \) for a given \( \gamma > 0 \), (b) construct the piecewise linear security \( s^* \), and (c) find the value of \( \gamma \) such a security \( \mathbb{E}[s^*(Z)] = I^* \). The resulting security is the optimal one for the problem for a given \( I^* \). Because of FOSD, Proposition 8 implies that the optimal security must satisfy \( (s^*)'(0) = 1 \), i.e. it must yield maximum payoff to outside investors in a neighborhood of \( z = 0 \). This means that an optimal security will always have a (possibly small) straight debt component.\(^{26}\)

The following Proposition characterizes the optimal security design problem when the firm value distribution satisfies FOSD but not CSD.

Proposition 9. Consider the security design problem (32) - (33) with no pre-existing debt, \( K_0 = 0 \).

(a) (Nachman and Noe, 1994) If the distribution \( F_G \) conditionally stochastically dominates \( F_B \), then \( H'(z) \geq 0 \) for all \( z \), and straight debt is the optimal security.

(b) If the problem satisfies the NICRT condition, and \( H'(z^*) = 0 \) for a unique \( z^* \in \mathbb{R}_+ \), then convertible bonds are optimal for all investment levels \( I \).

(c) If \( \lim_{z \to \infty} H(z) = \bar{h} > 0 \) and there exists a unique \( z^* \in \mathbb{R}_+ \) such that \( H'(z^*) = 0 \), then there exists \( \bar{I} \) such that for all \( I \leq \bar{I} \) straight debt is optimal, whereas for all \( I \geq \bar{I} \) convertible bonds are optimal.

Part (a) of Proposition 9 assumes CSD. As discussed in Nachman and Noe (1994), CSD requires that the ratio of the measure of the upper tails of the probability distribution for the two types, \( (1 - F_G(z))/(1 - F_B(z)) \), is monotonically increasing in \( z \).\(^{27}\) Note also that, for monotonic securities, this ratio can be interpreted as measuring the marginal cost of increasing the payouts to investors by $1 for a type-\( G \) relative to a type-\( B \). The proposition shows that the optimal security design is debt when the relative incremental cost of increasing

\(^{26}\)Note, however, that as Proposition 10 shows, this property hinges critically on the assumption that the firm has no pre-existing debt (i.e. \( K_0 = 0 \)).

\(^{27}\)CSD also requires that the hazard rate of the payoff distribution for a type-\( G \) is smaller than that for a type-\( B \) for all values of \( z \).
a payout for a better type is non-decreasing in the realization of \( z \). This happens because, in this case, firms of better types prefer to increase the payout to investors for low realizations of \( z \) and to limit the payout to investors for high realizations of \( z \), that are relatively more expensive to them. These considerations, together with the requirement that the security is monotonic, lead to the optimality of debt contracts.

The cases considered in parts (b) and (c) of Proposition 9 provide the conditions under which convertible bonds can be the optimal financing instruments. The key driver of the optimal security choice is, as in the debt-equity case discussed in the earlier sections, the size of the informational costs in the right-tail of the payoff distribution. In part (b) we establish that if there are no information costs in the right tail, that is, when the NICRT holds, then convertible bonds will always be the optimal security. This happens because convertible bonds allow type-\( G \) firms to load the payoff to investors in the right tail of the distribution, where information costs are low because of NICRT (in addition to straight debt component, as discussed above). In part (c), neither CSD nor NICRT hold, since we have both a non-monotone function \( H \) and the \( \bar{h} \)-ICRT condition holds for \( \bar{h} > 0 \). The proposition shows that the size of a project affects the financing choices of a firm. Specifically, convertible debt is optimal for large levels of the investment \( I \), while straight debt is optimal for low levels of \( I \). This happens because when investment needs are low, the firm can finance the project by issuing only straight debt, that is a security that loads only in the left-tail of the distribution, where the information costs are the lowest. For greater investment needs, under \( \bar{h} \)-ICRT the firm finds it optimal to increase its payout to investor by issuing a security that also loads on the right-tail of the distribution, that is by issuing convertible debt.

We now turn to the optimal security design problem when the firm has already issued a security to outside investors and we focus on the more empirically relevant case in which the firm has already issued straight debt (as discussed in Section 6). We assume again that pre-existing debt is senior with respect to any of the new securities that the firm may issue in order to finance the project. We also continue to assume that the project is sufficiently profitable, so the firm always seeks external finance to undertake the project (rather than not issuing any security and passing on the new investment opportunity).\(^{28}\)

The presence of pre-existing debt changes the structure of information costs in a non-
trivial way, because cash flows in the left tail of the distribution cannot be pledged any longer to new investors. As we argued earlier, this makes equity-like securities relatively more attractive.

**Proposition 10.** Consider the optimal security design problem in (32)–(33). Assume that $F_\theta(z)$ satisfies the NICRT condition, and that there exists a unique $z^*$ such that $H'(z^*) = 0$.

(a) If $H'(K_0) > 0$, then there exists $\bar{I}$ such that: (i) warrants are optimal for $I < \bar{I}$, and (b) convertible bonds are optimal for $I \geq \bar{I}$.

(b) If $H'(K_0) < 0$, then the optimal securities are warrants.

Proposition 10 provides conditions under which warrants arise as optimal financing instruments, in contrast to the case in which only straight debt or convertible bonds are solution to the optimal security design problem that we discussed in Proposition 9. Intuitively, warrants are optimal securities when pre-existing debt has absorbed the information benefits in the left tail (which generate the optimality of debt when $K_0 = 0$). Note that the value of the warrants will depend on their exercise price. Specifically, when $K_0$ is moderate, so that $H'(K_0) > 0$, NICRT implies that the optimal security design is one that always loads in the right-tail, where information costs are the now the lowest (since now the left tail is already committed). In addition, when the financing needs are low, the firm is able to raise the required capital by issuing only warrants; when the financing needs are high, the firm raises the additional capital by issuing also (junior) debt, that is by using convertible debt. When $K_0$ is large, so that $H'(K_0) < 0$, the firm will always find it optimal to issue only warrants (since the firm now faces decreasing information costs).

We conclude this section by illustrating the characterizations of the optimal securities in Propositions 9 and 10 in the real options model of Section 5. We start by highlighting that the distribution of the random variable $Z_{\theta T}$ does not satisfy CSD, even if the individual random variables $X_{\theta T}$ and $Y_{\theta T}$ satisfy CSD. The next Proposition gives conditions under which the real options model satisfies the NICRT condition, as well as the CSD condition of Nachman and Noe (1994).

**Proposition 11.** The model satisfies the NICRT condition if there is no information asymmetry on $y$, $c_y = 0$, and the volatility of the growth opportunity is higher than that of the assets in place, $\sigma_y > \sigma_x$. In the case the firm’s payoffs are given by a lognormal distribution, $Y_G = Y_B = 0$, the CSD condition holds.

The first part of Proposition 11 gives a sharp parametric example in which CSD fails. Since second moments dominate tail behavior for Gaussian random variables, the assumptions
that $Y$ has no information costs and $\sigma_y > \sigma_x$ are sufficient to generate a non-monotonic $H(z)$ function. Furthermore, Proposition 11 shows the technical limitations of using the standard lognormal specification in models of asymmetric information: in this case the model satisfies the CSD ordering of Nachman and Noe (1994).

Figure 5 plots the $H(z)$ function in the left panels, and the optimal security in the right panels, for three different scenarios, summarized in Table 5. In all cases we assume that $p = 1/2$, $\sigma_x = 0.2$ and $\sigma_y = 0.5$.

The first scenario (Panel A) presents the case where the asymmetric information is concentrated in the high volatility asset, namely we let $X_G = X_B = 120$, $Y_G = 110$, $Y_B = 90$ and we set the investment to be $I = 100$. In this case straight debt will be optimal, as the $H(z)$ function is monotone over its whole domain (see the top left graph in Figure 5). In particular, a standard bond with a face value of $K = 101.6$ is sufficient to finance the project and minimize information costs.

The second case (Panel B) is closer to the examples from Section 5, in that the asymmetric information is concentrated in the low-volatility asset. Namely, we set $Y_G = Y_B = 175$, $X_G = 120$, $X_B = 100$, and the investment amount to $I = 90$. The optimal security in this case is a convertible debt contract with $K = 88.9$, and conversion trigger at $z_c = 309.4$. As in the case of Proposition 11 the intuition is that, under $h$-ICRT, securities should load in the lower end of the payoffs, due to the usual Myers and Majluf (1984) intuition, but also on upper end of the payoff distribution.

The last case (Panel C of Figure 5) provide an illustration of case (b) in Proposition 10. We use the same parameter values as in previous example, but we now assume that the firm has debt outstanding with $K_0 = 100$. Further, let the investment amount be $I = 15$. In this case, Proposition 10 shows that the optimal security is warrants, with an exercise price of $\kappa = 174.8$.

8 Other specifications and robustness

Section 4 and Section 7 of this paper characterize the optimal financing choices under asymmetric information for general distributions that satisfy FOSD but not necessarily CSD. We have linked the possibility of generating a reversal of the pecking order to the non monotonic behavior the function $H(z)$ that describes the information costs of a type-$G$ firm when pooling with a type-$B$ firms. The parametric examples that we use to generate the reverse pecking order have been based on an exchange option specification. In this section, we discuss another specification of firm value which also generates the reverse pecking order.

We consider the following modification of the payoff structure. The firm is now endowed
by two assets, \(X_{\theta T}\) and \(Y_{\theta T}\). For example, we can interpret \(X_{\theta T}\) and \(Y_{\theta T}\) as being the value of the assets of the two divisions of a multi-divisional firm. If the firm makes the capital expenditure \(I\) at the beginning of the period, then the end-of-period value of the firm is given by the random variable \(Z_{\theta T} = X_{\theta T} + Y_{\theta T}\). Again the random variables \(X_{\theta T}\) and \(Y_{\theta T}\) are characterized by lognormal distribution, as in Section 5. Under the lognormality assumption, the model does not admit closed-form solutions, but it is straightforward to be solved numerically.\(^{29}\)

In Table 6 we consider an example that uses parameter values close to those in Table 2. The firm can finance the new project, at a cost of \(I = 110\) by either selling a fraction \(\lambda = 0.40\) of the equity of the firm, or by issuing straight debt with a face value equal to \(K = 213.2\), which carries a credit spread of 451 basis points. As in the base case, the parameter values are such that the NICRT condition is satisfied. It is easy to verify that the dilution costs associated with equity are \(D_E = 20.1\), whereas those associated with debt are \(D_D = 21.2\).

We generate comparative static results around the base case of Table 6 that correspond to Figure 2 and 3. The simulated results for the additive case mimic the results we obtained in the exchange-option case. Equity financing is again more likely to dominate debt financing when the asset with less exposure to asymmetric information has also greater volatility, when the level of investment is greater, when the asset with greater volatility is a greater proportion of firm value and is relatively more volatile.

This specification of our model generates novel predictions on the cross-sectional variation of the capital structures of multidivisional firms. Specifically, we predict that firms are more likely to be equity financed when they consist of relatively heterogenous divisions, and when the relatively more transparent division — that is the one that is less likely to be afflicted by asymmetric information — is also riskier (i.e., it is characterized by greater volatility). In addition, in these situations, firms are more likely to raise new capital by using equity, especially to meet larger investment needs.\(^{30}\)

The additive structure inherits many of the properties of the model solved in closed-form in Section 5. The critical properties of the model are driven, again, by the relative probability mass on the right-tail of the payoff distribution. Other specifications that potentially yield similar results are mixtures of binomial distributions or normal distributions (Leland, 2007). All these specifications can generate situations where the \(h\)-ICRT or the NICRT property are satisfied and, as a consequence, produce a reversal of the pecking order. Thus, while

\(^{29}\)In the analysis that follows we approximate the relevant integrals by simulations, with sample sizes that guarantee accuracy on the order of four significant digits.

\(^{30}\)Note also that multidivisional firms may arise as the outcome of a merger. Thus, our model also generate new predictions on why certain mergers are equity financed while others are financed by cash (where firm raises the cash preemptively by borrowing).
the reason for considering the exchange option specification is its analytical tractability, its modeling flexibility regarding right-tail behavior can be achieved in other specifications too.

Throughout the discussion we have assumed that the payoff to the firm before the investment is $X_{\theta T}$, so that the incremental cash flow from investment is the rainbow option $\max(X_{\theta T}, Y_{\theta T}) - X_{\theta T} = \max(Y_{\theta T} - X_{\theta T}, 0)$. But the model is more general. The analysis, under the positive NPV assumption, goes through as stated if the payoff of the firm without the investment is $w \max(X_{\theta T}, Y_{\theta T})$ for some $w \in (0, 1)$, so the incremental value of the investment at maturity is $(1 - w) \max(X_{\theta T}, Y_{\theta T})$.\footnote{This remark is important in order to formally nest the lognormal specification. In our framework we can take the limit $Y_{\theta T} \downarrow 0$ (a.s.) but still have a non-trivial asymmetric information problem with a positive NPV if $w < 1$ and we do not interpret $Y_{\theta T}$ as the incremental cash flow.}

We conclude this section by noting another implication of the reverse pecking order. There are scenarios in which firms are willing to raise capital and invest in a positive NPV project if the project is financed by equity, but are not willing to do so if the project is financed by debt. This happens when the dilution costs under debt financing are sufficiently large to make the type-$G$ to prefer the no-investment payoff, while the dilution costs under equity financing are sufficiently low to make the firm willing to issue equity and invest in the project.

Consider a modification of the example of Table 2, where $Y_G = Y_B = 130$, $I = 95$, $T = 15$ and all other parameters are the same. The value of the firm post-investment is now 208.2. Equity financing now involves selling a fraction $\lambda = 0.456$, whereas debt financing involves a promised payment of $K = 215$, with a credit spread of 559 basis points. Equity financing is still optimal, as the dilution costs are given by $D_E = 20.2$ and $D_D = 21.9$

The original shareholders of a type-$G$ firm have a status-quo of $X_G = 125$. Under the new set of parameter values, the payoff of the old equity holders is $(1 - 0.456) \times 208.2 = 125.2$. With the new parameter values, debt financing yields a total value for existing shareholders of type-$G$ of 124.3, which is less than the value of their existing shareholders under no investment, 125. Thus, if firms were restricted to issue debt, type-$G$ firms will prefer not issue and invest, preventing the pooling equilibrium we discussed in the paper to exist. This also means that the social optimum, which is to invest in the new project, cannot be achieved under debt financing, while this is possible under equity financing.

\section{Empirical implications}

While the real options modelled based on the rainbow options does deliver analytical expressions, its highly non-linear nature precludes from sharp comparative statics. We discuss next what these comparative statics may look like for “reasonable” set of parameter values. In
We then use the simulated sample to conduct traditional empirical tests with the aim of characterizing the regularities that an econometrician would estimate in our randomly generated economy, as a proxy for comparative statics in a large subset of the relevant state space.\textsuperscript{33}

We simulate the model one million times, solving it numerically, using the closed-form solutions from Section 6 at each iteration, and save only the results for which the relative dilution of equity is within 20\% of that of debt.\textsuperscript{34} We then run sets of standard regressions for models of the form \( Y_i = \beta^T X_i + \epsilon_i \), where \( Y_i \) is either (i) the ratio of the dilution costs of equity over the dilution costs of debt, \( R_i = D_{Ei}/D_{Di} \), or (ii) a dummy that equals to 1 if the firm finds it optimal to issue debt, i.e., \( D_{Ei} > D_{Di} \). In the later case we estimate a logit model, whereas in the former case we shall report ordinary-least-squares (OLS) coefficients. As the set of explanatory variables \( X_i \) we shall include: a constant; two metrics of the information asymmetry faced by investors, \( c_x = X_G - X_B \) and \( c_y = Y_G - Y_B \); the level of the payoffs \( \bar{X} \) and \( \bar{Y} \); the volatilities of each of the components of the assets, \( \sigma_x \) and \( \sigma_y \), as well as the correlation \( \rho \); the probability of a good type \( p \), and the face value of senior debt \( K_0 \), as well as the investment amount \( I \) and the time to maturity \( T \). When giving point estimates of a regression, we normalize all independent variables to zero mean and unit variance, so

\textsuperscript{32}Let \( U_i, i = 1, \ldots, 10 \), denote a set of independent uniformly distributed random variables in \([0, 1]\). We set \( \sigma_x = \min(0.2 + 0.8 U_1, 0.2 + 0.8 U_2) \), and \( \sigma_y = \max(0.2 + 0.8 U_1, 0.2 + 0.8 U_2) \), so that \( Y \) maps into the higher volatility asset component, which we previously referred to as the firm’s “growth opportunity.” Note how the volatilities are bounded in the set \([0.2, 0.8] \). We let \( \rho = -0.5 + 1.5 U_3 \), so that the correlation parameter is uniformly distributed in \([-0.5, 1]\). We set the time to maturity to be \( T = 5 + 25 U_4 \), with support in \([5, 30]\). We further let \( \mu_x = U_5 \) and \( \mu_y = U_6 \). We then set \( \mu_G = \mu_x + kU_7 \) and \( \mu_B = \mu_x - kU_7 \), and similarly \( \mu_G^B = \mu_y + kU_8 \) and \( \mu_B^B = \mu_y - kU_8 \), where we set arbitrarily \( k = 0.3 \). We let \( X_\theta = e^{\mu_G^B} \) and \( Y_\theta = e^{\mu_B^B} \). Note how the information asymmetry is parametrized by a uniformly distributed random variable that spreads the means of the type-G and type-B by at most a log-return of 60\%. We set \( p = 0.2 + 0.6 U_7 \) as the probability of the type-G firm, with support in \([0.2, 0.8]\). We set the value of the existing senior debt at \( K_0 = (0.2 + 0.6 U_9) A \), where \( A \) denotes the value of the (total) assets post-investment. Thus the principal of the old debt will be between 20-80\% of the total firm value. Finally, we let \( \tilde{C} = pC_G(K_0) + (1 - p)C_B(K_0) \) denote the value of the equity of the firm (net of the senior debt), and set the investment amount at \( I = (0.3 + 0.5 U_{10}) \tilde{C} \) (this guarantees that the problem has a solution). Note that the actual generation of the parameter values is rather irrelevant for our purposes, in the sense that we can condition on different subsets of the parameter space in the analysis that follows. In general, the above simulation procedure will generate scenarios where (quasi) risk-less debt is feasible, and thus optimal. But it will also generate parameter values for which the trade-off at the financing choice satisfies conditions that are close to those associated with the \( \lambda \)-ICRT for low values of \( \lambda \).

\textsuperscript{33}See Berk, Green, and Naik (1999) and Carlson, Fisher, and Giammarino (2006) for similar approaches.

\textsuperscript{34}Namely, if we let the relative dilution of equity (over debt) to be defined as \( R = D_{E}/D_{D} \), we only consider those cases where \( R \in (0.8, 1.2) \). About 46.1\% of the simulated parameters satisfy this constraint. For 45.5\% debt’s dilution is less than 20\% that of equity, mostly when debt is (close to) riskfree. For 5.9\% of the cases studied, equity’s dilution is less than 20\% than that of debt. We focus on parameter values for which there is some tension in the debt-equity choice.
the intercept of the OLS regression can be interpreted as an unconditional mean, and the OLS coefficients as the marginal effect of a one standard deviation change in the independent variables. In the logit results, where the point estimates do not have marginal interpretations, the relative size of the estimates do give a sense of the relative importance of each of the explanatory variables.

Panel A of Table 7 gives the estimates of the logit specification, whereas Panel B presents the results where the relative dilution $R_i$ is the dependent variable. Each set of pair of columns contains the estimated coefficients, and the related comparative static. For example, the point estimate on $c_x$ of $-4.8$ (Panel B, second column) means that a one standard deviation increase in $c_x$, decreases the relative dilution of equity, i.e., it makes debt relatively more expensive by 4.8%. The first column presents the OLS estimates over all the cases that comprise the main sample. Columns 3–8 present the results for different sub-samples, depending on whether the observations are in the top or bottom quintiles of the variables that measure existing debt, $K_0$, the information asymmetry on the assets in place, $c_x$, and the information asymmetry on the assets in place, $c_y$. Given the size of the sample, all point estimates are highly significant, so t-statistics are omitted.

The regression suggests that the relative dilution costs of equity increase as $c_y$, $\sigma_x$, and $\mu_x$ increase, but decrease if $c_x$, $\mu_y$, $\sigma_y$, $\rho$, $K_0$, $I$, $p$ and $T$ increase. It is remarkable that across all seven subsets of the parameter space considered, and both the logit and OLS specifications, the comparative statics with respect to ten primitives of the model, out of eleven, do not change signs. Only for the parameter $I$ does the coefficient flip signs in the OLS specification, albeit with economically small magnitudes (0.0), which hints at the non-linearities of the model.

These comparative statics reinforce the intuition behind the reverse pecking order from the previous sections. The information asymmetry needs to be concentrated on the low-volatility asset: the higher the parameter $c_x$ is, the more likely equity will be issued. The information asymmetry on the high-volatility asset, $c_y$, which governs the behavior of the information costs on the right-tail, has the opposite effect. Rather intuitively, if the right-tail cash flows become more expensive in terms of information costs, then the firm is more likely to issue straight debt. The volatility parameters play a dual role — amplifying/reducing the information asymmetry costs. The higher the existing assets volatility ($\sigma_x$) is, the more likely straight debt is optimal, whereas higher volatility ($\sigma_y$) for the new assets favors equity.

Table 7 also shows how the size of the assets (existing and new) favor debt over equity. The mechanism is simple: the higher the asset value, all else equal, the closer the debt contract is to be risk-free. Furthermore, the higher the probability of the good type $p$, the more likely equity becomes optimal. Table 7 further confirms that the presence of existing debt is an
important determinant of the debt/equity choice. In particular, equity is more likely to be optimal if the firm already has some debt in its capital structure. Finally, the larger the investment $I$, the less likely it is that the firm will issue debt.

These regression results for simulated datasets suggest that the critical drivers of the reverse pecking order are low information asymmetry on the right tail of the payoff distribution, large investment needs, and existing debt in the capital structure. Under such conditions, a debt security will be more sensitive to private information than an equity security. As such, equity financing can be less dilutive than debt financing under asymmetric information.

10 Conclusion

In this paper, we revisit the pecking order of Myers and Majluf (1984) and Myers (1984) in the context of a simple real options problem. We model firm value as an exchange option between two risky assets, and show that even if the distribution of each individual assets satisfies the conditional stochastic dominance condition, the distribution of the exchange option may not. This means that, contrary to common intuition, equity financing can dominate debt financing under asymmetric information, even in cases where individual assets would be financed by debt when taken in isolation. We also show that the presence of existing debt makes equity less dilutive than debt. Finally, our model also predicts the optimality of convertible debt and warrants. Taken together, these results suggest that the relationship between asymmetric information and choice of financing is more subtle than previously believed.
Appendix


Proof of Proposition 2. From the definition of $H(z)$ in (10), we have:

\[
\frac{dH(z)}{dz} = \frac{(f_B(z) - f_G(z))(1 - F(z)) + (pf_G(z) + (1 - p)f_B(z))(F_B(z) - F_G(z))}{(1 - F(z))^2} \\
= \frac{f_B(z) - f_G(z) + f_B(z)f_G(z) - F_G(z)f_B(z)}{(1 - F(z))^2} \\
= \frac{f_B(z)(1 - F_G(z)) - f_G(z)(1 - F_B(z))}{(1 - F(z))^2}.
\]

Thus $H'(z) > 0$ if and only if $f_B(z)(1 - F_G(z)) > f_G(z)(1 - F_B(z))$, which reduces to the CSD condition.

Proof of Proposition 3. From the budget constraint for equity and debt securities, one has that

\[
\lambda = \frac{p\mathbb{E}[\min(Z_G, K)] + (1 - p)\mathbb{E}[\min(Z_B, K)]}{p\mathbb{E}[Z_G] + (1 - p)\mathbb{E}[Z_B]} 	ag{36}
\]

Using (36) in (12) and comparing this to (13) one easily arrives at (14).

Proof of Proposition 4. The following result from Shaked and Shanthikumar (2007) is useful.

Lemma 1 (Theorem 1.B.12 from Shaked and Shanthikumar (2007)). Given two distribution functions $F_G$ and $F_B$, the following two statements are equivalent: (a) $F_G$ conditionally stochastic dominates $F_B$; (b) $\mathbb{E}[\alpha(X_B)]\mathbb{E}[\beta(X_G)] \leq \mathbb{E}[\alpha(X_G)]\mathbb{E}[\beta(X_B)]$, for all functions $\alpha$ and $\beta$ such that $\beta$ is non-negative and $\alpha/\beta$ and $\beta$ are non-decreasing.

Let $\alpha(z) = z$ and $\beta(z) = \min(z, K)$ for some $K \geq 0$. Clearly $\beta$ is non-decreasing and non-negative for $x \geq 0$. Furthermore, $\alpha(z)/\beta(z) = z/\min(z, K)$ is non-decreasing. Thus, if $F_G$ conditionally stochastically dominates $F_B$ it must be that

\[
\mathbb{E}[Z_B]\mathbb{E}[\min(Z_G, K)] \leq \mathbb{E}[Z_G]\mathbb{E}[\min(Z_B, K)]
\]

which clearly rules out (14).
Proof of Proposition 5. It is clear than in order for the UPO to hold, it is necessary that \( D_D > D_E \). This condition, if \( \hat{z} > \bar{z} \) (i.e., the UNC does not hold), can be written as

\[
\int_0^{\hat{z}} \left( \min(K, z) - \lambda z \right) c(z) \, dz + \int_0^{\bar{z}} (K - \lambda z) c(z) \, dz + \int_{\hat{z}}^\infty (K - \lambda z) c(z) \, dz > 0 \quad (37)
\]

We note that since \( g \) is the difference of two densities, it must be the case that

\[
\int_0^\infty c(z) \, dz = 0; \quad \Rightarrow - \int_0^{\hat{z}} c(z) \, dz = \int_{\hat{z}}^\infty c(z) \, dz
\]

Further, we have

\[
\int_{\hat{z}}^\infty (\lambda z - K) c(z) \, dz > \int_0^{\hat{z}} (\lambda \hat{z} - K) c(z) \, dz \\
= (\lambda \hat{z} - K) \int_0^{\hat{z}} c(z) \, dz \\
= (K - \lambda \hat{z}) \int_0^{\hat{z}} c(z) \, dz \\
> (K - \lambda \hat{z}) \int_{\hat{z}}^\infty c(z) \, dz \\
> \int_{\hat{z}}^\infty (K - \lambda z) c(z) \, dz
\]

Therefore, the sum of the last two terms in (37) are negative, and since the first one is negative as well it is clear that \( D_D - D_E < 0 \), i.e., UPO cannot hold if UNC is not true.

The statement in the Proposition is immediate from (15), the definitions of \( \hat{z} \) and \( \bar{z} \), and the discussion in the text.

Proof of Proposition 6. We first note that letting \( \epsilon \downarrow 0 \) the statements in the Proposition boil down to the delta condition given in (27). Equation (27) can be expressed more explicitly as

\[
\frac{I}{X N \left( \frac{\log(X/Y)}{\Sigma \sqrt{T}} + \frac{1}{2} \Sigma \sqrt{T} \right)} + \frac{Y}{X} N \left( \frac{\log(Y/X)}{\Sigma \sqrt{T}} + \frac{1}{2} \Sigma \sqrt{T} \right) < N \left( \frac{\log(K/X)}{\sigma_x \sqrt{T}} - \frac{1}{2} \sigma_x \sqrt{T} \right) \quad (38)
\]

We note that the right-hand side is independent of \( \bar{Y} \). The left-hand side of this condition tends to zero for \( I \) sufficiently large, so (27) holds in this case. For \( \bar{Y} \) sufficiently small, the
financing constraint for debt reduces to

\[ K(1 - N(b_x + \sigma_x \sqrt{T})) + XN(b_x) = I \]

so that

\[ \frac{I}{X} = N(b_x) + \frac{K}{X}(1 - N(b_x + \sigma_x \sqrt{T})). \]  

(39)

As \( \hat{Y} \) goes to zero, (27) reduces to \( I/\hat{X} < N(b_x) \), which is impossible from the financing constraint (39). This proves (i).

In order to prove (ii), we note that the limit of the left-hand side of (38) as \( \sigma_x^2 \downarrow 0 \) is finite, and strictly greater than zero. On the other hand, the argument of \( N(\cdot) \) in the right-hand side of the condition tends to either positive or negative infinity, depending on whether \( X < K \) (or \( X > K \)). In the former case (38) always holds, whereas if \( X > K \) it can never hold. This completes the proof.

Proof of Proposition 7. Immediate from the discussion in the text.


Proof of Proposition 9. Since \( H \) is increasing in (a), there is a single crossing point \( z \) such that \( H(z) = \gamma \), for any \( \gamma \in \mathbb{R}_+ \). The claim in (a) follows immediately from Proposition 8. Assuming the NICRT, and that \( H'(z^*) = 0 \) at most once, it is immediate that there are two crossing points for \( H(z^*) = \gamma \), for any \( \gamma \in \mathbb{R}_+ \). The claim is immediate from Proposition 8. Case (c) is analogous, but noting that for \( \gamma \leq \bar{\gamma} \) there is a single point satisfying \( H(z^*) = \gamma \), but two such points for \( \gamma \) sufficiently large.

Proof of Proposition 10. The proof is analogous to that of Proposition 8. The first-order conditions require \( s'(z) \) to be either one (or zero) at points for which \( H(z) < \gamma \) (or \( H(z) > \gamma \)). Under the conditions in (b), and the initial assumptions, there is only one crossing, and all mass of the security is concentrated in the right tail. This occurs for low values of \( \gamma \), or equivalently of the investment \( I \). The claim in (a) mirrors case (b) from Proposition 9.

Proof of Proposition 11. Using l’Hopital’s rule, one has

\[
\lim_{z \uparrow \infty} H(z) = \lim_{z \uparrow \infty} \frac{F_B(z) - F_G(z)}{1 - F(z)} = \lim_{z \uparrow \infty} \frac{f_G(z) - f_B(z)}{pf_G(z) + (1 - p)f_B(z)}. \]  

(40)

(41)
From basic principles it is clear that:

$$\mathbb{P}(Z_{\theta T} = z) \equiv f_\theta(z) = f_{x\theta}(z) + f_{y\theta}(z)$$

with

$$f_{x\theta}(z) = \frac{1}{z\sigma_x\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\log(z) - \mu_{x\theta}}{\sigma_x} \right)^2} \left( \frac{\log(z) - \mu_{y\theta}}{\sigma_y\sqrt{1 - \rho^2}} - \frac{\rho (\log(z) - \mu_{y\theta})}{\sigma_y\sqrt{1 - \rho^2}} \right)$$

$$f_{y\theta}(z) = \frac{1}{z\sigma_y\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\log(z) - \mu_{y\theta}}{\sigma_y} \right)^2} \left( \frac{\log(z) - \mu_{x\theta}}{\sigma_x\sqrt{1 - \rho^2}} - \frac{\rho (\log(z) - \mu_{x\theta})}{\sigma_x\sqrt{1 - \rho^2}} \right)$$

where \( \mu_{x\theta} = \log(X_\theta) \) and \( \mu_{y\theta} = \log(Y_\theta) \).

The limit in (41) is easy to compute by factoring out leading terms. We note that when \( \sigma_y > \sigma_x \) the right-tail behavior is determined by the piece of the densities \( f_\theta(z) \) that corresponds to the density of \( Y \). When \( c_y = 0 \), the limit of these densities is zero.

Next consider the case where \( Y_G = Y_B = 0 \). The good type distribution is then given by a lognormal distribution with log-mean \( \mu_{xG} \) and variance \( \sigma_x^2 \), whereas the bad type follows a lognormal law with log-mean \( \mu_{xB} \) and variance \( \sigma_x^2 \). We argue next that in this case the distribution of the good type dominates the distribution of the bad type in the likelihood ratio sense, namely \( f_G(z)/f_B(z) \) is monotonically non-decreasing for all \( z \in \mathbb{R}_+ \). From basic principles we have:

$$\frac{f_G(z)}{f_B(z)} = \frac{\frac{1}{z\sigma_x\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\log(z) - \mu_{xG}}{\sigma_x} \right)^2}}{\frac{1}{z\sigma_x\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\log(z) - \mu_{xB}}{\sigma_x} \right)^2}} = e^{\frac{1}{2} \left( \frac{\log(z) - \mu_{xG}}{\sigma_x} \right)^2 - \frac{1}{2} \left( \frac{\log(z) - \mu_{xB}}{\sigma_x} \right)^2 + \frac{1}{2} \left( \frac{\sigma_x^2 - \sigma_{xG}^2}{\sigma_x^2} \right)^2 \log(z) + \frac{\mu_{xG} - \mu_{xB}}{\sigma_x^2}} = e^{\frac{1}{2} \left( \frac{\sigma_x^2 - \sigma_{xG}^2}{\sigma_x^2} \right)^2 \log(z) + \frac{\mu_{xG} - \mu_{xB}}{\sigma_x^2}};$$

which is monotonically increasing in \( z \) when \( \mu_{xG} > \mu_{xB} \), as we set to prove. Since the likelihood ratio order implies conditional stochastic dominance (Shaked and Shanthikumar, 2007), we conclude that the lognormal specification yields debt financing as the optimal security. This completes the proof.
References


### Table 1: A simple example

The table presents the parameter values and equilibrium outcomes of the capital raising problem discussed in Section 2. The payoff of the firm is given by a trinomial random variable \( Z \in \{10, 100, 300\} \). The probabilities of each state depend on the type of firm, \( \theta = G, B \), where \( f_G = \{0.2, 0.4, 0.4\} \), and \( f_B = \{0.3, 0.4 - x, 0.3 + x\} \), for \( x \in [0, 0.1] \). The column labelled “Pooled value” computes the expected value of the firm, \( \mathbb{E}[Z] \), where each type is assumed equally likely. The variable \( \lambda \) denotes the fraction of equity the firm needs to issue to finance the investment of \( I = 60 \). The column labelled \( D_E \) denotes the dilution costs of equity, namely \( \lambda(\mathbb{E}[Z_G] - \mathbb{E}[Z_B]) \). For all values of \( x \), the firm can also finance the project with a debt security with a face value \( K = 76.7 \), for which the dilution costs are 6.7 (last column).

<table>
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<tr>
<th>( x )</th>
<th>Pooled value</th>
<th>( \mathbb{E}[Z_G] )</th>
<th>( \mathbb{E}[Z_B] )</th>
<th>( \lambda )</th>
<th>( D_E )</th>
<th>( D_D )</th>
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Table 2: Optimal debt-equity choice

The table presents the parameter values and equilibrium outcomes of the capital raising problem discussed in Section 5. The payoff of the firm for type $\theta$ is given by $Z_{\theta T} = \max(X_{\theta T}, Y_{\theta T})$, where both $X_{\theta T}$ and $Y_{\theta T}$ are lognormal, with $\mathbb{E}[X_{\theta T}] = X_\theta$, $\mathbb{E}[Y_{\theta T}] = Y_\theta$. We further denote $\text{var}(\log(X_{\theta T})) = \sigma_x^2 T$, $\text{var}(\log(Y_{\theta T})) = \sigma_y^2 T$, and $\text{cov}(\log(X_{\theta T}), \log(Y_{\theta T})) = \rho \sigma_x \sigma_y T$. Figure 1 plots the equilibrium debt and equity securities, as well as the densities of the good and bad types.

<table>
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<th>Value</th>
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<td>$Y_G$</td>
<td>175</td>
</tr>
<tr>
<td>$Y_B$</td>
<td>175</td>
</tr>
<tr>
<td>$T$</td>
<td>10</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>0.30</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>0.60</td>
</tr>
<tr>
<td>$p$</td>
<td>0.50</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0</td>
</tr>
<tr>
<td>$I$</td>
<td>110</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p\mathbb{E}[Z_{GT}] + (1 - p)\mathbb{E}[Z_{BT}]$</td>
<td>237.9</td>
</tr>
<tr>
<td>$\mathbb{E}[Z_{GT}]$</td>
<td>257.6</td>
</tr>
<tr>
<td>$\mathbb{E}[Z_{BT}]$</td>
<td>218.3</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.462</td>
</tr>
<tr>
<td>$K$</td>
<td>198.3</td>
</tr>
<tr>
<td>$r_D = (K/D)^{1/T} - 1$</td>
<td>6.06%</td>
</tr>
<tr>
<td>$\mathbb{E}[\min(Z_{GT}, K)]$</td>
<td>120.6</td>
</tr>
<tr>
<td>$\mathbb{E}[\min(Z_{BT}, K)]$</td>
<td>99.4</td>
</tr>
<tr>
<td>$\mathbb{E}[\min(Z_{GT}, K)] - \mathbb{E}[\min(Z_{BT}, K)]$</td>
<td>18.2</td>
</tr>
<tr>
<td>$\mathbb{E}[\min(Z_{GT}, K)] - \mathbb{E}[\min(Z_{BT}, K)]$</td>
<td>21.1</td>
</tr>
</tbody>
</table>
The table presents the parameter values and equilibrium outcomes of the capital raising problem discussed in Section 8. The payoff of the firm for type $\theta$ is given by $Z_{\theta T} = \max(X_{\theta T}, Y_{\theta T} - I_T)$, where both $X_{\theta T}$ and $Y_{\theta T}$ are lognormal, with $\mathbb{E}[X_{\theta T}] = X_\theta$, $\mathbb{E}[Y_{\theta T}] = Y_\theta$. We further denote $\text{var}(\log(X_{\theta T})) = \sigma^2_x T$, $\text{var}(\log(Y_{\theta T})) = \sigma^2_y T$, and $\text{cov}(\log(X_{\theta T}), \log(Y_{\theta T})) = \rho \sigma_x \sigma_y T$.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_G$</td>
<td>125</td>
</tr>
<tr>
<td>$X_B$</td>
<td>75</td>
</tr>
<tr>
<td>$Y_G$</td>
<td>200</td>
</tr>
<tr>
<td>$Y_B$</td>
<td>200</td>
</tr>
<tr>
<td>$T$</td>
<td>10</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>0.30</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>0.50</td>
</tr>
<tr>
<td>$p$</td>
<td>0.50</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0</td>
</tr>
<tr>
<td>$I$</td>
<td>100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Investment at exercise ($I_T$)</th>
<th>Equity offered ($\lambda$)</th>
<th>Face value ($K$)</th>
<th>Debt dilution $D_D$</th>
<th>Equity dilution $D_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.397</td>
<td>140.0</td>
<td>12.4</td>
<td>14.0</td>
</tr>
<tr>
<td>25</td>
<td>0.415</td>
<td>151.6</td>
<td>15.5</td>
<td>15.6</td>
</tr>
<tr>
<td>50</td>
<td>0.430</td>
<td>161.7</td>
<td>17.9</td>
<td>16.9</td>
</tr>
<tr>
<td>75</td>
<td>0.445</td>
<td>170.9</td>
<td>19.9</td>
<td>18.0</td>
</tr>
<tr>
<td>100</td>
<td>0.458</td>
<td>179.4</td>
<td>21.6</td>
<td>19.0</td>
</tr>
</tbody>
</table>
Table 4: Optimal debt-equity choice with existing debt

The table presents the parameter values and equilibrium outcomes of the capital raising problem discussed in Section 6. The payoff of the firm for type θ is given by Zθ = max(Xθ, Yθ), where both Xθ and Yθ are lognormal, with E[Xθ] = X, E[Yθ] = Y. We further denote var(log(Xθ)) = σ²x, var(log(Yθ)) = σ²y, and cov(log(Xθ), log(Yθ)) = ρσxσy. The firm already has debt outstanding with principal payment of K₀.

The dilution costs of equity are defined as

\[ D_E = \lambda (E[\max(Z_{GT} - K_0, 0)] - E[\max(Z_{BT} - K_0, 0)]) \]

where λ satisfies \( I = \lambda (pE[\max(Z_{GT} - K_0)] + (1 - p)E[\max(Z_{BT} - K_0)]) \). The dilution costs of equity are defined as

\[ D_D = E[\max(\min(K, Z_{GT} - K_0), 0)] - E[\max(\min(K, Z_{BT} - K_0), 0)] \]

where K satisfies \( I = pE[\max(\min(K, Z_{GT} - K_0), 0)] + (1 - p)E[\max(\min(K, Z_{BT} - K_0), 0)] \). Figure 4 plots the equilibrium debt and equity securities, as well as the densities of the good and bad types.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primitives</td>
<td></td>
</tr>
<tr>
<td>Value of assets in place for the good type</td>
<td>X_G</td>
</tr>
<tr>
<td>Value of assets in place for the bad type</td>
<td>X_B</td>
</tr>
<tr>
<td>Value of new assets for the good type</td>
<td>Y_G</td>
</tr>
<tr>
<td>Value of new assets for the bad type</td>
<td>Y_B</td>
</tr>
<tr>
<td>Time to maturity</td>
<td>T</td>
</tr>
<tr>
<td>Volatility of assets in place</td>
<td>σ_x</td>
</tr>
<tr>
<td>Volatility of new assets</td>
<td>σ_y</td>
</tr>
<tr>
<td>Probability of the good type</td>
<td>p</td>
</tr>
<tr>
<td>Correlation between assets</td>
<td>ρ</td>
</tr>
<tr>
<td>Investment amount</td>
<td>I</td>
</tr>
<tr>
<td>Existing debt</td>
<td>K</td>
</tr>
<tr>
<td>Equilibrium outcomes</td>
<td></td>
</tr>
<tr>
<td>Value of firm post-investment</td>
<td>pE[Z_{GT}] + (1 - p)E[Z_{BT}]</td>
</tr>
<tr>
<td>Value of good-type firm</td>
<td>E[Z_{GT}]</td>
</tr>
<tr>
<td>Value of bad-type firm</td>
<td>E[Z_{BT}]</td>
</tr>
<tr>
<td>Equity fraction issued</td>
<td>( \lambda )</td>
</tr>
<tr>
<td>Face value of debt</td>
<td>K</td>
</tr>
<tr>
<td>Credit spread</td>
<td>( r_D = (K/D)^{1/T} - 1 )</td>
</tr>
<tr>
<td>Value of good-type new equity</td>
<td>( \lambda E[\max(Z_{GT} - K_0, 0)] )</td>
</tr>
<tr>
<td>Value of bad-type new equity</td>
<td>( \lambda E[\max(Z_{BT} - K_0, 0)] )</td>
</tr>
<tr>
<td>Value of good-type new debt</td>
<td>E[\max(\min(K, Z_{GT} - K_0), 0)]</td>
</tr>
<tr>
<td>Value of bad-type new debt</td>
<td>E[\max(\min(K, Z_{BT} - K_0), 0)]</td>
</tr>
<tr>
<td>Dilution costs of equity</td>
<td>( D_E )</td>
</tr>
<tr>
<td>Dilution costs of debt</td>
<td>( D_D )</td>
</tr>
</tbody>
</table>
Table 5: Optimal security design problem

The table presents the parameter values and equilibrium outcomes of the security design problem discussed in Section 7. The payoff of the firm for type $\theta$ is given by $Z_{\theta T} = \max(X_{\theta T}, Y_{\theta T})$, where both $X_{\theta T}$ and $Y_{\theta T}$ are lognormal, with $\mathbb{E}[X_{\theta T}] = X_\theta$, $\mathbb{E}[Y_{\theta T}] = Y_\theta$. We further denote $\text{var}(\log(X_{\theta T})) = \sigma_x^2 T$, $\text{var}(\log(Y_{\theta T})) = \sigma_y^2 T$, and $\text{cov}(\log(X_{\theta T}), \log(Y_{\theta T})) = \rho \sigma_x \sigma_y T$. The labels “Straight debt,” “Convertibles,” and “Warrants” refer to the functions $s(z) = \min(K, z)$, $s(z) = \min(K, z) + \max(z - \kappa, 0)$, and $s(z) = \max(z - \kappa, 0)$ respectively.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Primitives</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value of assets in place type $G$</td>
<td>$X_G$</td>
<td>150</td>
<td>120</td>
</tr>
<tr>
<td>Value of assets in place type $B$</td>
<td>$X_B$</td>
<td>150</td>
<td>100</td>
</tr>
<tr>
<td>Value of new assets type $G$</td>
<td>$Y_G$</td>
<td>120</td>
<td>175</td>
</tr>
<tr>
<td>Value of new assets type $B$</td>
<td>$Y_B$</td>
<td>100</td>
<td>175</td>
</tr>
<tr>
<td>Time to maturity</td>
<td>$T$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Volatility of assets in place</td>
<td>$\sigma_x$</td>
<td>0.20</td>
<td>0.25</td>
</tr>
<tr>
<td>Volatility of new assets</td>
<td>$\sigma_y$</td>
<td>0.50</td>
<td>0.30</td>
</tr>
<tr>
<td>Probability of the good type</td>
<td>$p$</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>Correlation between assets</td>
<td>$\rho$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Investment amount</td>
<td>$I$</td>
<td>100</td>
<td>90</td>
</tr>
<tr>
<td>Existing debt face value</td>
<td>$K_0$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Equilibrium outcomes</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value of firm post-investment</td>
<td>$p\mathbb{E}[Z_{GT}] + (1 - p)\mathbb{E}[Z_{BT}]$</td>
<td>218.9</td>
<td>186.3</td>
</tr>
<tr>
<td>Optimal security</td>
<td>$s(z)$</td>
<td>Straight debt</td>
<td>Convertibles</td>
</tr>
<tr>
<td>Face value</td>
<td>$K$</td>
<td>101.9</td>
<td>88.9</td>
</tr>
<tr>
<td>Conversion trigger/exercise price</td>
<td>$\kappa$</td>
<td>–</td>
<td>309.4</td>
</tr>
</tbody>
</table>
Table 6: Robustness, additive cash-flows and the optimal debt-equity choice

The table presents the parameter values and equilibrium outcomes of the capital raising problem discussed in Section 8. The payoff of the firm for type $\theta$ is given by $Z_{\theta T} = X_{\theta T} + Y_{\theta T}$, where both $X_{\theta T}$ and $Y_{\theta T}$ are lognormal, with $E[X_{\theta T}] = X_{\theta}$, $E[Y_{\theta T}] = Y_{\theta}$. We further denote $\text{var}(\log(X_{\theta T})) = \sigma_x^2 T$, $\text{var}(\log(Y_{\theta T})) = \sigma_y^2 T$, and $\text{cov}(\log(X_{\theta T}), \log(Y_{\theta T})) = \rho \sigma_x \sigma_y T$.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primitives</td>
<td></td>
</tr>
<tr>
<td>Value of assets in place for the good type</td>
<td>$X_G$</td>
</tr>
<tr>
<td>Value of assets in place for the bad type</td>
<td>$X_B$</td>
</tr>
<tr>
<td>Value of new assets for the good type</td>
<td>$Y_G$</td>
</tr>
<tr>
<td>Value of new assets for the bad type</td>
<td>$Y_B$</td>
</tr>
<tr>
<td>Time to maturity</td>
<td>$T$</td>
</tr>
<tr>
<td>Volatility of assets in place</td>
<td>$\sigma_x$</td>
</tr>
<tr>
<td>Volatility of new assets</td>
<td>$\sigma_y$</td>
</tr>
<tr>
<td>Probability of the good type</td>
<td>$p$</td>
</tr>
<tr>
<td>Correlation between assets</td>
<td>$\rho$</td>
</tr>
<tr>
<td>Investment amount</td>
<td>$I$</td>
</tr>
<tr>
<td>Equilibrium outcomes</td>
<td></td>
</tr>
<tr>
<td>Value of firm post-investment</td>
<td>$E[Z_T]$</td>
</tr>
<tr>
<td>Equity fraction issued</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>Face value of debt</td>
<td>$K$</td>
</tr>
<tr>
<td>Credit spread</td>
<td>$\tau_D = (K/D)^{1/T} - 1$</td>
</tr>
<tr>
<td>Dilution costs of equity</td>
<td>$D_E = \lambda (E[Z_{GT}] - E[Z_{BT}])$</td>
</tr>
<tr>
<td>Dilution costs of debt</td>
<td>$D_D = E[\min(Z_{GT}, K)] - E[\min(Z_{BT}, K)]$</td>
</tr>
</tbody>
</table>
Table 7: Comparative statics via regression

The table presents estimates of: (a) a logit regression model where the dependent variable is a dummy that equals to one if the firm optimal chooses debt, zero if the firm prefers equity, as a function of a set of explanatory variables from the model (Panel A); (b) a classical regression model of the form $R_t = \beta^\top X_t + \epsilon_i$ in Panel B, where $R_t$ denotes the relative dilution of debt versus equity, $R_d = \frac{D_{Ei}}{D_{Di}}$, and $X_t$ denotes a the set of explanatory variables (Panel B). The set of explanatory variables include: measures of asymmetric information on the assets in place and the new assets ($c_x$ and $c_y$), the two parameters on volatility ($\sigma_x$ and $\sigma_y$), the level of the cash flows ($\mu_x = \log(\bar{X})$ and $\mu_y = \log(\bar{Y})$), the probability of the good type ($p$), the amount of existing debt ($K_0$), as well as the investment amount ($I$). Details on the construction of the simulated dataset are given in Section 9.

### A. Logit regressions (success if straight debt issued)

<table>
<thead>
<tr>
<th></th>
<th>Base case</th>
<th>Existing debt</th>
<th>Info. asy. $X$</th>
<th>Info. asy. $Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low $K_0$</td>
<td>High $K_0$</td>
<td>Low $c_x$</td>
<td>High $c_x$</td>
</tr>
<tr>
<td>Asy. info on existing assets $c_x$</td>
<td>-1.8</td>
<td>-2.0</td>
<td>-1.8</td>
<td>-6.3</td>
</tr>
<tr>
<td>Asy. info on new assets $c_y$</td>
<td>3.2</td>
<td>3.7</td>
<td>2.9</td>
<td>8.7</td>
</tr>
<tr>
<td>Size existing assets $\mu_x$</td>
<td>1.5</td>
<td>2.1</td>
<td>1.0</td>
<td>1.4</td>
</tr>
<tr>
<td>Size new assets $\mu_y$</td>
<td>-1.1</td>
<td>-1.1</td>
<td>-1.2</td>
<td>-1.1</td>
</tr>
<tr>
<td>Volatility existing assets $\sigma_x$</td>
<td>2.5</td>
<td>2.5</td>
<td>2.7</td>
<td>2.5</td>
</tr>
<tr>
<td>Volatility new assets $\sigma_y$</td>
<td>-1.9</td>
<td>-2.0</td>
<td>-2.0</td>
<td>-2.0</td>
</tr>
<tr>
<td>Correlation $\rho$</td>
<td>-0.2</td>
<td>-0.4</td>
<td>-0.0</td>
<td>-0.1</td>
</tr>
<tr>
<td>Prob. high type $p$</td>
<td>-0.2</td>
<td>-0.2</td>
<td>-0.1</td>
<td>-0.0</td>
</tr>
<tr>
<td>Debt’s principal $K_0$</td>
<td>-0.3</td>
<td>-0.7</td>
<td>-0.0</td>
<td>-0.2</td>
</tr>
<tr>
<td>Investment $I$</td>
<td>-0.2</td>
<td>-0.5</td>
<td>-0.1</td>
<td>0.0</td>
</tr>
<tr>
<td>Time to maturity $T$</td>
<td>-0.5</td>
<td>-0.6</td>
<td>-0.5</td>
<td>-0.6</td>
</tr>
</tbody>
</table>

### B. Relative dilution regressions (dilution equity/dilution debt)

<table>
<thead>
<tr>
<th></th>
<th>Base case</th>
<th>Existing debt</th>
<th>Info. asy. $X$</th>
<th>Info. asy. $Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low $K_0$</td>
<td>High $K_0$</td>
<td>Low $c_x$</td>
<td>High $c_x$</td>
</tr>
<tr>
<td>Asy. info on existing assets $c_x$</td>
<td>-4.8</td>
<td>-5.3</td>
<td>-4.4</td>
<td>-7.3</td>
</tr>
<tr>
<td>Asy. info on new assets $c_y$</td>
<td>7.4</td>
<td>8.9</td>
<td>6.4</td>
<td>7.5</td>
</tr>
<tr>
<td>Size existing assets $\mu_x$</td>
<td>4.3</td>
<td>5.9</td>
<td>3.1</td>
<td>1.9</td>
</tr>
<tr>
<td>Size new assets $\mu_y$</td>
<td>-2.4</td>
<td>-2.6</td>
<td>-2.4</td>
<td>-1.5</td>
</tr>
<tr>
<td>Volatility existing assets $\sigma_x$</td>
<td>6.7</td>
<td>6.7</td>
<td>6.9</td>
<td>3.4</td>
</tr>
<tr>
<td>Volatility new assets $\sigma_y$</td>
<td>-5.0</td>
<td>-5.6</td>
<td>-4.7</td>
<td>-2.7</td>
</tr>
<tr>
<td>Correlation $\rho$</td>
<td>-0.5</td>
<td>-1.1</td>
<td>-0.2</td>
<td>-0.1</td>
</tr>
<tr>
<td>Prob. high type $p$</td>
<td>-0.5</td>
<td>-0.6</td>
<td>-0.3</td>
<td>-0.1</td>
</tr>
<tr>
<td>Debt’s principal $K_0$</td>
<td>-1.0</td>
<td>-2.3</td>
<td>-0.3</td>
<td>-0.4</td>
</tr>
<tr>
<td>Investment $I$</td>
<td>-1.8</td>
<td>-2.6</td>
<td>-1.6</td>
<td>-0.9</td>
</tr>
<tr>
<td>Time to maturity $T$</td>
<td>-0.9</td>
<td>-1.4</td>
<td>-0.5</td>
<td>-0.6</td>
</tr>
</tbody>
</table>
Figure 1: The top graph plots on the $x$-axis the payoffs from the firm at maturity, and in the $y$-axis it plots as a solid line the difference in the densities of the good and bad type firms, $f_G(z) - f_B(z)$ ($y$-axis labels on the left), and as dotted lines the payoffs from debt and equity ($y$-axis labels on the right). The left-most vertical dashed line is the point $\hat{z}$ for which $f_G(\hat{z}) = f_B(\hat{z})$, so points to the right of that line have positive information costs. The right-most vertical dashed line is the point $\bar{z}$ for which $K = \lambda \bar{z}$, so for payoffs to the right of that line equityholders receive more than debtholders. The bottom graph plots the densities of the good and bad types (dotted lines), as well as the joint density (integrated over types). The parameter values correspond to the case summarized in Table 2.
Figure 2: The top graph plots the set of points \((c_x, c_y)\) for which the dilution costs of equity and debt are the same, i.e. \(D_E = D_D\). We consider the following parameter values: \(\bar{X} = 100, \bar{Y} = 175, \sigma_x = 0.3, \sigma_y = 0.6, I = 110, T = 10, \rho = 0\) and \(p = 0.5\). Recall we set \(X_G = \bar{X} + c_x\) and \(X_B = \bar{X} - c_x\), and similarly for \(Y_G\) and \(Y_B\). The dashed line corresponds to the base case parameters from Table 2, namely sets \(\sigma_y = 0.6\), whereas the other two lines correspond to \(\sigma_y = 0.7\) and \(\sigma_y = 0.8\). For pairs of \((c_x, c_y)\) below the lines debt is optimal, whereas equity is optimal above the lines. The bottom graph fixes \(c_x = 25\) and \(c_y = 0\), as in Table 2, and plots the set of points \((I, T)\) for which the dilution costs of equity and debt are the same, i.e. \(D_E = D_D\). The solid line corresponds to the base case of Table 2, where \(\bar{X} = 100\), whereas the dashed and dotted lines consider the cases \(\bar{X} = 105\) and \(\bar{X} = 95\). For pairs of \((I, T)\) below the lines debt is optimal, whereas equity is optimal above the lines.
Figure 3: The top graph plots the set of points $(\bar{X}, \bar{Y})$ for which the dilution costs of equity and debt are the same, i.e. $D_E = D_D$. We consider the following parameter values: $c_x = 25$, $c_y = 0$, $\sigma_x = 0.3$, $\sigma_y = 0.6$, $I = 110$, $T = 10$, $\rho = 0$ and $p = 0.5$. Recall we set $X_G = \bar{X} + c_x$ and $X_B = \bar{X} - c_x$, and similarly for $Y_G$ and $Y_B$. The solid line corresponds to the base case parameters from Table 2, namely sets $c_x = 25$, whereas the other two lines correspond to $c_x = 10$ and $c_x = 40$. For pairs of $(\bar{X}, \bar{Y})$ below the lines debt is optimal, whereas equity is optimal above the lines. The bottom graph fixes $\bar{X} = 100$ and $\bar{Y} = 175$, as in Table 2, and plots the set of points $(\sigma_x, \sigma_y)$ for which the dilution costs of equity and debt are the same, i.e. $D_E = D_D$. The solid line corresponds to the base case of 2, where $I = 110$, whereas the dashed and dotted lines consider the cases $I = 100$ and $I = 120$. For pairs of $(\sigma_x, \sigma_y)$ below the lines debt is optimal, whereas equity is optimal above the lines.
Figure 4: The top graph plots on the x-axis the payoffs from the firm at maturity, and in the y-axis it plots as a solid line the difference in the densities of the good and bad type firms, $f_G(z) - f_B(z)$ (y-axis labels on the left), and as dotted lines the payoffs from debt and equity (y-axis labels on the right). The left-most vertical dashed line is the point $\hat{z}$ for which $f_G(\hat{z}) = f_B(\hat{z})$, so points to the right of that line have positive information costs. The right-most vertical dashed line is the point $\bar{z}$ for which $K = \lambda(\bar{z} - K_0)$, so for payoffs to the right of that line equityholders receive more than the new debtholders. The bottom graph plots the densities of the good and bad types (dotted lines), as well as the joint density (integrated over types). The parameter values correspond to the case summarized in Table 4.
Figure 5: The left panels plot the function $H(z) = (F_B(z) - F_G(z))/(1 - F(z))$, whereas the right panels plot the optimal securities. The parameter values correspond to the cases listed in Table 5.