Demographic Trends, Asset Prices and Capital Accumulation

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Abstract

Most economies are currently experiencing simultaneous declines in fertility rates and increases in life expectancy. We study the implications of these demographic trends for asset prices and capital accumulation. We show that the conclusions depend crucially on how both defined benefit (DB) pension schemes and social security (SS) will adjust to these changes. If we assume that both DB and SS retirement replacement ratios remain unchanged, the riskfree rate, equity premium and capital stock all decrease. However, if the DB schemes are replaced by defined contribution systems, a trend observed in several countries, the riskfree rate will actually increase while the reductions in the capital stock and equity premium are twice as large. Increasing the retirement age, by contrast, leads to a more modest reduction in the capital stock and the equity premium, and only a small increase in the risk-free rate. Our results highlight that the macroeconomic and asset pricing implications of demographic changes will be potentially very different across countries, depending both on their current pension and social security systems, and on how those two adjust to the new demographic patterns.

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1 Introduction

Most economies are currently experiencing declines in fertility rates (e.g., Geruso and Spears (2026), Kearney and Levine (2026)) and increases in life expectancy (e.g. Acemoglu and Johnson (2007)), leading to a substantial shift in the demographic structure of their populations. As populations age, the average life-span during retirement increases, changing households' wealth accumulation and asset allocation decisions. At the same time the ratio of retirees to workers increases and this leads to changes in aggregate asset demands due to the differences in policy functions across age groups.

These demographic shifts also contribute to a deterioration in the financial position of both social security and defined benefit pension schemes. These pressures are forcing governments to either increase social security contribution rates for workers, or decrease benefits for future retirees (or even for current ones). Similarly, they are one of the main driving forces behind a shift towards defined contribution pension plans that we observe in several countries.

In this paper we study the implications of these structural changes for asset prices, risk premia and capital accumulation. We explicitly consider alternative scenarios for how both social security and defined benefit pension schemes will respond to those demographic shifts. We show that the conclusions regarding movements in asset prices and capital stock depend crucially on those alternative adjustments.

Our baseline model considers a production economy with incomplete markets and overlapping generations of households. Markets are incomplete because there is aggregate risk and because households face borrowing constraints and have uninsurable idiosyncratic labor income shocks. We also consider preference heterogeneity, thus capturing the savers-spenders dichotomy that is important for delivering a realistic wealth distribution and realistic household asset demands. Importantly, in our setting the "spenders" are not at binding corner solutions, so that their savings and investment decisions can also respond to the changing macro-economic environment.

Crucially, the model also includes a pay-as-you-go social security, and a defined benefit (DB) pension scheme, and both are allowed to change in the new equilibrium. As in reality, both social security transfers and DB pensions are financed by contributions from current workers and firms. In addition, the DB system also has a (representative) pension fund with an accumulated

endowment of wealth with its (stochastic) return constituting an additional source of financing. As a result, the asset demands of the pension fund are an important channel determining asset prices (as in Coimbra et al. (2026)), namely its strong demand for bonds which we calibrate from historical data.[1]

We first calibrate a version of the model with an average historical demographic structure to match a range of macro-finance moments. The asset pricing moments include the mean and volatility of both the riskless rate and the equity risk premium. On the macro side, we target the ratios of capital, government debt and consumption relative to output, and the volatility of aggregate consumption growth. Having established a well-calibrated baseline benchmark, we then use it as a laboratory to study the impact of changes in both fertility rates and mortality rates.

We first consider changes in the fertility rates, while keeping life expectancy constant. A reduction in labor supply reduces the marginal productivity of capital and, therefore, the return on capital. This leads to a reduction in household savings and thus aggregate capital. If we assume that the social security and the DB pension retirement replacement ratios remain unchanged, then the contribution rates of both workers and firms must increase. This reduces household savings during working life, increasing the risk-free rate and further decreasing the aggregate capital stock.

However if, in addition, the pension fund accumulates a larger endowment of wealth to finance the increase in (expected) liabilities, then its increased demand for capital offsets the lower demand from households such that the total capital stock remains largely unchanged.

The previous conclusions are significantly altered if the reduction in fertility rates is instead accompanied by a shift from DB to DC pension plans, as has been observed in several countries. As the DB pension fund is shut down, there is a large reduction in the demand for bonds. In the new equilibrium, the riskfree rate is now higher than before, and the equity premium is lower. Without the DB pension scheme, households must now save more for retirement, but they will never fully replace (or surpass) the wealth accumulation of the DB fund, since they could have done that before already if it was optimal for them. Therefore, the total capital stock falls relative to the value in the baseline economy.

[1] Scharfstein (2018) emphasizes the role of pension policies in different countries on financial markets.

We next consider a scenario where the retirement replacement ratio of social security is decreased just enough to keep the contribution rates of workers and firms unchanged. This is likely to be an upper bound on the potential adjustment, since it imposes the full cost on retirees. Most likely, the political response will be a division of the cost between retirees and workers, thus an intermediate case between the previous scenario and this one.

The reduction in the replacement ratio forces households to save more for retirement and this is reflected in a higher value of the capital stock. This can offset the change from DB to DC such that its equilibrium value is now only moderately lower. As households save more, the returns on both assets decrease, but the riskfree rate is still higher than in the baseline economy.

In a third and final scenario we consider that the adjustment to social security and defined benefit pension comes from an increase in the retirement age, from 65 to 67. We choose 67 since it keeps the contributions rates from both workers and firms roughly at their original values. An increase in the retirement age reduces retirement savings, unlike the previous two scenarios. However, with households working longer their human capital increases, shifting their portfolios more towards equities and away from bonds. In equilibrium we have a higher capital stock, higher risk-free rate and lower equity premium.

Having studied the economic impact of changes in fertility we proceed to consider an economy with both lower fertility rates and higher life expectancy. Consistent with recent trends, our new equilibrium is also characterized by an increase in the supply of government debt to match the current values in the U.S..

In addition to reporting results for the new steady-state, we also consider an economy where the survival probabilities have changed but the demographic weights are still constant. This can be interpreted as a first stage in the transition to the new steady-state and it isolates the impact of the changes in individual optimization, from the compositional effects due to the change in demographic weights.

As life expectancy increases, individuals have to save more for retirement. In equilibrium the capital stock increases and the rates of return on both assets (capital and bonds) fall. However, once we have converged to the new steady-state we have an older population. This compositional effect more than offsets the individual savings responses and the capital stock actually falls, as the

gradual change in demographic structure reverse the initial increase over time. Since the fraction of the working age population is now smaller, the capital output ratio still increases, leading to a further decrease in the return on capital.

As discussed, the demographic changes create budgetary pressures on DB pensions and social security which will require adjustments in both. Keeping the retirement replacement ratio constant and letting the burden fall completely on current workers and firms is just one option. As before, we also consider a second scenario where the DB pension scheme is shut down and replaced by a DC system, and a third one where we also reduce the social security replacement ratio such that the contribution rates remain largely unchanged relative to the baseline economy.

When we consider these alternative responses, the real riskfree rate more than doubles instead of falling, while the decreases in the equity premium and aggregate capital stock are both twice as large. These results highlight that the macroeconomic and asset pricing implications of demographic changes are highly dependent on the corresponding adjustments in pension schemes and the social security system, and therefore it is very important to take those into account.

Our paper is part of a literature studying implications of demographic changes on savings, capital accumulation and asset returns, see for example, Hansen (1939), Conesa and Krueger (1999), Abel (2003), Krueger and Ludwig (2007), Doepke et al. (2015), Curtis et al. (2015), Carvalho et al. (2016), Curtis et al. (2017), Eggertsson et al. (2019), Gagnon et al. (2021), Carvalho et al. (2025), Auclert et al. (2026) and Weil (2026).[2] Most of the previous work studying implications for equilibrium returns is in the context of one-asset models. Therefore, they have no differential implications for the risk-free rate and return on capital, and no implications for the risk-premium.[3] Abel (2003), Carvalho et al. (2016), Auclert et al. (2026), Gagnon et al. (2021), Kopecky and Taylor (2022) and Carvalho et al. (2025) find these demographic shifts can account for a significant fall in long-run real interest rates and economic growth.

Our analysis is closest to the one in Kopecky and Taylor (2022), who also have a two-asset model with aggregate uncertainty, and in Auclert et al. (2026), who also consider different social

[2]For the determinants of recent trends in fertility rates and their cross-country differences see, for example, De Silva and Tenreyro (2017), Manuelli and Seshadri (2009), and Geruso and Spears (2026).

[3]Since they don't consider aggregate uncertainty, these papers are able to study transition dynamics and thus try to match recent observed patterns, while we focus on steady-state equilibria.

security adjustments.[4] In addition to combining both of those features simultaneously, we explicitly model defined benefit pension funds and their endowments, and explore potential adjustments to the pension scheme, including a switch to a DC pension system. Finally, we include household preference heterogeneity and endogenous limited stock market participation, which allow us to capture important features of the cross-sectional distributions of wealth and asset holdings. Those are particularly relevant for quantifying the impact of different pension and social security responses.

On the empirical side Poterba (2001) finds no robust evidence for asset pricing effects resulting from demographic shifts, but cautions that "this is partly due to the limited power of statistical tests based on the few 'effective degrees of freedom'." By contrast, Kopecky and Taylor (2022) and Auclert et al. (2026) find large effect of aging on different asset returns, namely a decreasing risk-free rate. Kopecky and Taylor (2022) further document an increasing risk premium.[5]

Our paper is also part of the literature on asset pricing with incomplete risk sharing, such as the production economy models of Krueger and Kubler (2006), Storesletten et al. (2007), Gomes and Michaelides (2008), Guvenen (2009), Croce et al. (2012), Favilukis (2013), Gomes et al. (2013), Kung and Schmid (2015), Favilukis et al. (2017), Elenev et al. (2021), and Coimbra et al. (2026) and the exchange economy models of Telmer (1993), Lucas (1994), Constantinides and Duffie (1996), Heaton and Lucas (1996), Constantinides et al. (2002), Schmidt (2016), and Constantinides and Ghosh (2017).

The paper is organized as follows. In section 2 we discuss recent demographic trends and how we incorporate them in our analysis. In section 4 we present the baseline model and calibration. Section 5 discusses the different economic mechanisms while in sections 6 and 7 we present results for the new steady-states with lower fertility and higher life expectancy. We provide conclusion remarks in section 8

[4] Auclert et al. (2026) extend their baseline model to include two assets, with a reduced form parameterization for risky asset demand as a function of the steady-state risk premium and age.

[5] Acemoglu and Johnson (2007), Liang et al. (2018), Peters and Walsh (2021) and Hopenhayn et al. (2022) document consequences of changes in population growth on economic growth, new business formation, and industry and firm dynamics, respectively.

2 Demographic Data and Trends

Global demographic dynamics have been characterized by a significant and ongoing decline in fertility rates and an increase in life expectancy.

The decrease in fertility rates has moved most high-income and many middle-income nations below the replacement level of 2.1 children per woman. This has reshaped the traditional age pyramid, leading to an important acceleration of population aging. This demographic transition carries substantial macroeconomic consequences, as the shrinking ratio of working-age individuals to retirees can place considerable pressure on pension systems and social security financing, with long-term implications for aggregate savings, capital allocation, and asset prices.

Concurrently, the world has also experienced a steady decline in age-specific mortality rates, propelling life expectancy to unprecedented highs. While in the 1930s the US had a life expectancy of around 61 years, this had risen to around 78 in the 2000s. This upward shift in survival probabilities compounds the effects of falling fertility by shifting forward the peak of the age structure and significantly extending the average duration of the retirement period. Social security and pension systems therefore face a shrinking share of contributing workers, and are required to accumulate more wealth to pay for longer life spans.

In the left panel of Figure 1 we show the recent trends in fertility rates for the US and the world. Using World Bank data, we see a strong decreasing trend in fertility rates in the world, including the US which will be the focus of our calibration. Between 1970 and 2008, US fertility rates were consistently close to the replacement level of 2.1. This is the rate that ensures that cohorts remain of the same size given infant mortality and the ratio of male to female births. However, rates have fallen well below since (1.62 latest data point) and are currently at their lowest in the sample. Moreover, using data from the Human Fertility Database,[6] we plot on the middle panel the mean age at birth in the United States, which has also been steadily increasing. Finally, in the right panel, we show the recent evolution of US life expectancy.[7] As can be seen in the chart, life expectancy has increased steadily since the 1960s, despite the persistent Covid impact at the end of the sample.

[6]Produced and maintained by the Max Planck Institute for Demographic Research the and Vienna Institute of Demography.

[7]Calculated using survival probabilities from the Human Mortality Database.

The demographic weights of each age group are then a function of age-specific mortality rates and the relative share of births between cohorts. To determine these weights, we will require four sets of parameters: fertility rates, the mean age at birth, the ratio of female births and age-specific mortality rates.

To determine the ratio of cohort size *at birth* as a function of fertility, we first calculate the Gross Reproduction Rates (GRR) implied by a given fertility rate. This is a demographic measure that represents the average number of female births each mother is expected to have, and can be calculated simply as total fertility rates (TFR) multiplied by the share of female births (approximately 48.78%):

$$GRR = TFR * \frac{\text{Female Births}}{\text{Total Births}} \tag{1}$$

This gives us a ratio of the expected births in cohort $t+h$ relative to cohort t where h is the average mother's age at birth (MAB). Let w^F as the ratio of births between cohort t and $t+1$. Under constant parameters, this ratio will also be constant over time:

$$w^F = GRR^{1/MAB} \tag{2}$$

Finally, we combine w^F with age-specific survival rates. Let w_c be the population weight of cohort c in proportion to the youngest cohort ($w_1 = 1$). Defining w_c^M as the survival rate at age c, the cohort weight is the (normalized) product of the ratio of births and survival rates[8]

$$w_c = \frac{\left(w_c^F\right) w_c^M}{\sum_c \left(w_c^F\right) w_c^M} \tag{3}$$

In the model below, we will consider three main scenarios for the population weights based on U.S. data. The baseline one calibrates demographic weights using fertility and mortality data to their long run averages (1930-2025). We then consider a scenario with lower fertility, calibrated to post-2000 average fertility. Finally, we consider a scenario where additionally mortality is also lower, and also calibrated to post-2000 averages. We plot the population weights in each of the three cases in Figure 2. Note that the weights are normalized to one, so some of the important differences between fertility and mortality are masked in this static normalization.

[8]We consider a closed economy, so there are no immigration flows.

3 Model

3.1 Outline

We model an overlapping generations production economy with stochastic mortality and a demographic structure which we will be varying in our counterfactuals. Crucially for our analysis, we incorporate a retirement phase, during which households receive income from both social security and a defined benefit pension. Both social security and the defined benefit pension fund (DBPF) have their own budget constraints. Transfers to current retirees are financed by taxes/contributions on both current workers and on firms. The DBPF also has an endowment of wealth and its (stochastic) return provides an additional source of income.

We consider two groups of households with heterogeneous preferences. This heterogeneity generates significant cross-sectional wealth inequality and endogenous limited stock market participation with realistic participation costs (as in Gomes and Michaelides (2008), for example). Both features are important for studying the impact of different counterfactuals. Households also face uninsurable labor income risk and borrowing constraints. Investing in equity requires paying participation costs, both a first-time entry cost and a per-period cost. Equity is a claim on the capital stock of firms. We assume that firms are identical and perfectly competitive.

Finally, there is a government sector, which issues riskless bonds and has exogenous expenditures. Those expenditures and the interest on the bonds are both financed by taxes (on capital gains, bequests, and wages).

3.2 Firms

Firms are perfectly competitive and atomistic, such that they can be aggregated into a representative firm that hires aggregate labor (L) and rents the aggregate capital stock (K) from households while taking both factor prices (wages (W_t) and the return on capital (R_t^K)), as given.

3.2.1 Production technology

The firm makes decisions after observing the aggregate shocks and combines capital and labor supply in a Cobb-Douglas technology, such that total output at time t given by

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha} \tag{4}$$

where Z is a stochastic productivity shock, which follows the process

$$Z_t = G_t U_t \tag{5}$$

where $G_t = (1+g)^t$ and g captures aggregate growth, while the productivity shocks (U_t) follow a two-state Markov chain capturing business cycle fluctuations.

Capital accumulation is frictionless, and given by

$$K_{t+1} = (1-\delta_t)K_t + I_t. \tag{6}$$

Agents in the economy can trade claims on the capital stock ("equity"), subject to paying participation costs which are described when presenting the households' optimization problem.

Standard frictionless production economies allow for excessive consumption smoothing and thus cannot generate sufficient return volatility (see Jermann (1998)). Therefore, we follow a common practice in the literature (see, for example Krueger and Kubler (2006), Storesletten et al. (2007), Gomes and Michaelides (2008), Gottardi and Kubler (2011), Gomes et al. (2013), and Coimbra et al. (2026)) and consider a stochastic depreciation rate:

$$\delta_t = \bar{\delta}(U_t) + \sigma^{\delta}(U_t)\eta_t \tag{7}$$

where η_t is an i.i.d. standard normal shock. We allow for correlation between both the conditional mean ($\bar{\delta}$) and the standard deviation (σ^{δ}) of depreciation, as discussed in the calibration section.

3.2.2 Social security and DB pension contributions

In reality social security and DB pensions are both financed by payments from current workers and from firms. Regardless of who makes the payment, the ultimate incidence of those taxes/contributions will depend on the specific equilibrium in the labor market. In our economy, with perfectly competitive firms and exogenous labor supply, we have a clean separation: the incidence of taxes on wages falls fully on workers, while taxes on capital are ultimately faced by the shareholders.[9] Therefore we consider both of those in the model, and they should be viewed as capturing the differential incidence of the different real-life taxes.

We let $\tau^K_{ss,t}$ and $\tau^K_{db,t}$ denote the contribution rates from firms to social security and to the defined benefit pension fund, respectively.[10] As previously discussed, these rates are specified as a proportion of capital at time t, so that $\tau^K_{ss,t}K_t$ is the total value of employer payments to social security and $\tau^K_{db,t}K_t$ is the total value employer contributions to the DB pension fund.[11]

3.2.3 Maximization problem

Since firms are perfectly competitive and face no frictions (e.g., no adjustment costs of capital) they solve a sequence of static maximization problems,

$$\max_{K_t,L_t} Z_t K_t^{\alpha} L_t^{1-\alpha} - W_t L_t - (R_t^K + \tau^K_{db,t} + \tau^K_{ss,t})K_t \tag{8}$$

where the capital stock evolves according to

$$K_{t+1} = (1-\delta_t)K_t + I_t. \tag{9}$$

The first-order conditions of this optimization problem are

$$W_t = (1-\alpha)Z_t(K_t/L_t)^{\alpha} \tag{10}$$

[9] Taxes on capital appear in the first-order condition for capital while taxes on workers will appear in the household's optimization problem. In equilibrium firms pay workers a gross wage that equals their marginal productivity, since that is implied by their first-order-condition. It is workers that must then pay the taxes and receive a net wage.

[10] We discuss how they are determined in section 3.5.

[11] Alternatively, we can write down the contribution rate as applying to the return on capital. The two formulations are equivalent as there is a one-to-one mapping between them.

$$R_t^K = \alpha Z_t (L_t/K_t)^{1-\alpha} - \delta_t - \tau_{db,t}^k - \tau_{ss,t}^K \tag{11}$$

As the productive capital stock K_t is pre-determined by the investment decisions of households and pension funds (as described later), equation (11) pins down the rate of return on capital, while equation (10) pins down the aggregate wage.

3.3 Government Sector

The government's budget constraint is given by

$$C_t^G + (1 + R_t^B)B_t = B_{t+1} + T_t \tag{12}$$

where C^G is government consumption, B is the stock of (riskless) government bonds, R^B is the interest rate on those bonds, and T are tax revenues.

Government debt is exogenously set as a fixed fraction of GDP. The government collects taxes on capital income (tax rate τ^K), bond interest payments (tax rate τ^B), wages (tax rate τ^W), and bequests (tax rate τ^E). Government expenditures do not enter the agents' utility function, and are determined by Equation (12).

3.4 Households

3.4.1 Preferences

Agents have a stochastic life-span and live for a maximum of 81 periods.[12] The conditional survival probability from age a to age $(a+1)$ is denoted by p_a (with $p_0 = 1$). They have Epstein-Zin-Weil preferences over consumption of a single non-durable good ($C_{a,t}^i$):

$$V_{a,t} = \left\{ (1-\beta)(C_{a,t}^i)^{1-1/\psi} + \beta (E_t(p_a V_{a+1,t+1}^{1-\gamma}))^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}} \tag{13}$$

where β is the discount factor, γ is the coefficient of relative risk aversion, and ψ is the elasticity of intertemporal substitution.

[12] For calibration purposes, agents are "born" into the economy at age 20.

For simplicity, we do not include bequest motives or consumption equivalent scales in the baseline version of the model.[13] In Appendix H and Appendix I we consider extensions where we include these two features respectively, and the conclusions are very similar.

In order to match the participation rates in the data and to deliver more realistic wealth inequality, we consider two types of households in the model (A and B). Both types of households have the same degree of risk aversion ($\gamma^A = \gamma^B$), but they have a different EIS and different discount factors. Type-B households have preferences that imply high wealth accumulation, while type-A households have preferences that lead them to consume most (but not all) of their labor income. In equilibrium, most type-B households will then find it optimal to pay the participation costs and invest in stocks, whereas most type-A households will invest only in government bonds.

3.4.2 Labor income

Before retirement, all households supply labor inelastically and face individual-specific productivity shocks. Individual labor income ($H^i_{a,t}$) is the product of individual productivity ($L^i_{a,t}$) and the aggregate wage per unit of productivity (W_t):

$$H^i_{a,t} = W_t L^i_{a,t} \tag{14}$$

The aggregate wage is determined in equilibrium by equation (10), while the stochastic process for individual productivity is given by a permanent component $P^i_{a,t}$ and a transitory shock ε^i_t:

$$L^i_{a,t} = P^i_{a,t}\varepsilon^i_t \tag{15}$$

$$P^i_{a,t} = \exp(f(a))P^i_{a-1,t-1}\xi^i_t \tag{16}$$

where $f(a)$ is a deterministic function of age, capturing the typical hump-shaped profile in life-cycle earnings and ξ^i_t are shocks to the permanent component. We assume that $\ln \varepsilon^i$ is independent and identically distributed with mean $\{-.5 * \sigma^2_\varepsilon\}$ and variance σ^2_ε.

[13] The ratio of (accidental) bequests to GDP in the model already closely matches the one in the data, hence adding a bequest motive would overshoot that. In addition, including bequests would require (young) agents to form expectations about the future bequest they might receive. Instead, we assume that accidental bequests are fully taxed. Another alternative would be to assume perfect annuity markets, but that would remove longevity risk from the model.

Conditional on the state of the economy (U_t), the innovation $\ln \xi_t^i$ is drawn from one distribution with probability q_1 and with probability $(1 - q_1)$ from a second distribution:

$$\ln \xi_t^i = \begin{cases} \ln \xi_{t,1}^i \sim N(\mu_{1,U_t}, \sigma_{1,U_t}^2) & \text{with prob } q_1 \\ \ln \xi_{t,2}^i \sim N(\mu_{2,U_t}, \sigma_{2,U_t}^2) & \text{with prob } 1 - q_1 \end{cases} \tag{17}$$

Both the expected growth rates and the conditional volatility are functions of the aggregate productivity state (U_t), such that this formulation captures countercyclical earnings risk as in Guvenen et al. (2014).

3.4.3 Retirement income

After retirement, households receive income from both social security and defined benefit pension payments. These transfers are proportional to the product of permanent income at retirement age and the aggregate wage, each with its own replacement ratio (λ^{ss} and λ^{db}, respectively). Total retirement income is then given by

$$H_{a,t}^i = (\lambda^{ss} + \lambda^{db}) P_{a^R,t^R}^i W_t, \; a > a^R \tag{18}$$

The notation t^R refers to the calendar year in which the individual has retired (i.e., the year with age $a = a^R$).

Consumption at retirement is also financed by private savings, which partially reflect defined contribution pension schemes as in the data. Our baseline economy should therefore be viewed as an economy where both systems (DB and DC) co-exist, as is currently the case in the U.S..

3.4.4 Wealth accumulation

Agents earn returns on their investments (bonds ($B_{a,t}^i$) and stocks ($K_{a,t}^i$)), and labor income ($L_{a,t}^i W_t$), which is subject to the social security tax (τ_{ss}^L) and the DB pension contribution ($\tau_{db,t}^L$).[14]

Before investing in stocks for the first time, households must pay a fixed cost: $F^0 P_{a,t}^i W_t$. This entry fee captures both explicit pecuniary costs (e.g., a transaction cost from opening a brokerage

[14]We describe both the social security tax rate and the DB contribution rate in Section 3.5.

account or hiring a financial advisor) and the (opportunity) cost of acquiring information about the stock market. In addition, every period in which they have positive stockholdings, they must pay a (lower) per-period participation cost, $F^1 P^i_{a,t} W_t$, which reflects the (opportunity) cost of managing the portfolio and (again) acquiring information about the stock market.[15]

We define the dummy variable I^i_E as equal to one in the period in which the individual enters the stock market for the first time, and zero otherwise. Similarly the dummy variable I^i_S is equal to one if the household has a positive holding of stocks that year, and zero otherwise. Total participation costs paid by agent i at time t are then

$$PC^i_{a,t} = I^i_E F^0 P^i_{a,t} W_t + I^i_S F^1 P^i_{a,t} W_t. \tag{19}$$

Wealth (cash-on-hand) at time t is given by

$$X^i_{a,t} = K^i_{a,t}(1 + (1-\tau^K)R^K_t) + B^i_{a,t}(1 + (1-\tau^B)R^B_t) + L^i_{a,t}(1 - \tau^L_{ss,t} - \tau^W - \tau^L_{db,t})W_t - PC^i_{a,t} \tag{20}$$

before retirement ($a < a^R$), and by

$$X^i_{a,t} = K^i_{a,t}(1 + (1-\tau^K)R^K_t) + B^i_{a,t}(1 + (1-\tau^B)R^B_t) + (\lambda^{db} + \lambda^{ss}) P^i_{a^R t^R}(1-\tau^W)W_t - PC^i_{a,t} \tag{21}$$

during retirement ($a \geqslant a^R$).

Households cannot borrow against their future labor income and cannot short either asset, so both their bond holdings, $B^i_{a,t}$, and their stock holdings, $K^i_{a,t}$, must be non-negative:

$$B^i_{a,t} \geq 0 \tag{22}$$

$$K^i_{a,t} \geq 0\ . \tag{23}$$

[15] As standard in the literature we scale the participation costs by the current value of the permanent component of labor income ($P^i_{a,t}$) and by the aggregate wage (W_t) because it simplifies the solution of the model and because this is consistent with the opportunity cost interpretation for these variables.

3.4.5 Expectations

Solving the households' optimization problem requires expectations about future equity returns, future aggregate wages and the future tax/contribution rates for social security and the DB pension fund. In equilibrium equity returns and wages are determined by equations (10) and (11), respectively, and are therefore functions of the equilibrium level of the capital stock and the exogenous aggregate shocks.

As is standard in the literature, we assume rational expectations, such that agents use equations (10) and (11) to forecast wages and returns, and we follow the approach proposed by Krusell and Smith (1998) to approximate the equilibrium value of capital stock state-by-state.[16] More precisely, conditional on the shocks (productivity, U_t , and stochastic depreciation, η_t), the future normalized aggregate capital is very well predicted by its current value:

$$k_{t+1} = \Gamma_K(k_t, U_t, \eta_t) \ . \tag{24}$$

Since bonds are only riskless over one period, households must also forecast future bond prices (P^B_{t+1}). Following the literature (e.g., Krusell and Smith (1997), Storesletten et al. (2007) and Gomes and Michaelides (2008)), we consider a similar forecasting model, augmented to include the current bond price:

$$P^B_{t+1} = \Gamma_P(P^B_t, k_t, U_t, \eta_t) \ . \tag{25}$$

Details are given in Online Appendix C. This procedure introduces four aggregate state variables in the individual's maximization problem (P^B_t, k_t, U_t, and η_t).

As described below, in Section 3.5 below, the future social security tax rate, and the future contribution rate for the DB pension are fully determined by the aggregate capital stock and the aggregate shocks. Therefore, conditional of a forecast of k_{t+1}, the agent already has a forecast of both $\tau^L_{ss,t}$ and $\tau^L_{db,t}$, and we don't need separate forecasting equations for those two variables.

[16] For the equations below we consider the model in its stationary form with lower-case letters denoting variables scaled by aggregate productivity growth. For example, $k_t = K_t/G_t^{\frac{1}{1-\alpha}}$ and $w_t = W_t/G_t^{\frac{1}{1-\alpha}}$.

3.5 Social security and pension system

3.5.1 Social security

We model social security as a fully funded pay-as-you-go system, financed by proportional tax rates on both labor income (τ_{ss}^{L}) and capital (τ_{ss}^{K}).[17] As previously discussed, given our set-up, the incidence of taxes on wages falls fully on workers, and therefore the tax on capital captures the incidence of social security taxes on firm profits and thus on the return on capital.

The two tax rates are determined by the relative demographic weights of workers and retirees, such that the social security budget constraint is satisfied at all times:

$$\sum_{a=20}^{65} \int_{i \in I^a} \tau_{ss,t}^{L} L_{a,t}^{i} w_t di + \tau_{ss,t}^{K} k_t = \sum_{a=66}^{100} \int_{i \in I^a} [\lambda^{ss} \exp(f(a^R)) w_t P_{a^R,t^R}^{i}] di \ , \tag{26}$$

where the notation I^a refers to the set of individuals with age a.

Equation (26) has two unknowns ($\tau_{ss,t}^{L}$ and $\tau_{ss,t}^{K}$). We solve for both by making the additional assumption that the tax burden is equally divided between workers and firms, i.e. that the two terms on the left-hand-side of the equation are identical. Further details on this calculation are provided in Online Appendix D.

Since the value of the capital stock is endogenous and time-varying, for equation (26) holds period-by-period, the social security tax rate on capital must also be a function of time. On the other hand, given our assumption about an equal split of the tax burden, the tax rate on wages is constant over time, and therefore we will henceforth refer to it as τ_{ss}^{L}.

3.5.2 Defined Benefit Pension Fund

As in reality, the defined benefit pension fund (hereafter DBPF) is also financed by contributions from both employers and employees (respectively, $\tau_{db,t}^{K}$ and $\tau_{db,t}^{L}$).[18] Just as for social security, we assume that the total value of these two sets of contributions is the same every period. In addition, the fund has an accumulated endowment (denoted by $\bar{\omega}^{P}$ in the scaled version of the

[17] In the U.S., social security also has a trust fund, but it is almost depleted and it is projected to disappear in the near future.

[18] Public DB pension funds can be financed by taxes. Labor taxes impact workers' net wages, whereas corporate taxes decrease firms' after-tax profits. In the context of our model, they are therefore equivalent to the employer and employee contributions of private funds.

model), and therefore its return constitutes another important source of income.[19]

The period-by-period budget constraint for the DBPF is similar to the one for social security, with the additional inclusion of the return on the endowment as an additional source of income:

$$\sum_{a=20}^{65} \int_{i \in I^a} \tau_{db,t}^{L} L_{a,t}^{i} w_t di + \tau_{db,t}^{K} k_t + R_t^P \bar{\omega}^P = \sum_{a=66}^{100} \int_{i \in I^a} [\lambda^{db} \exp(f(a^R)) w_t P_{a^R,t^R}^{i}] di \ , \tag{27}$$

where R_t^P is the return on the endowment at time t which is given by

$$R_t^P = \alpha_t^P R_t^K + (1 - \alpha_t^P) R_t^B, \tag{28}$$

For simplicity, we assume that the pension fund's asset allocation is constant over time, so $\alpha_t^P = \alpha^P$ at all times.[20]

Relative to the corresponding equation for social security, the stochastic return on the endowment provides an additional source of time-variation in the budget constraint. As a result, both contribution rates ($\tau_{db,t}^K$ and $\tau_{db,t}^L$) have to be time-varying for equation (46) to hold period-by-period. More precisely, when returns are low/low, contribution rates (for both workers and firms) must increase/decrease. Details are provided in Online Appendix E.

For simplicity, and to avoid adding an additional state variable in the model, we assume that the endowment remains constant over time, such the adjustment is fully captured by those changes in the contributions rates. The value of endowment ($\bar{\omega}^P$) is endogenously determined such that, if the system is suddenly closed and additional generations are enrolled in it, this endowment (plus its return) is exactly enough to pay off existing (net) liabilities in expectation. Details are provided in Online Appendix F.[21]

[19] Munnell and Soto (2007) document that this was by far the most important source of financing for state and local DB pension funds between 1996 and 2006 (70% of their total income). Coimbra et al. (2026) solve a similar model to the one in this paper and explore the asset pricing and risk sharing implications of incorporating the pension fund's endowment.

[20] In the context of a similar model, Coimbra et al. (2026) explore different portfolio allocation rules for the pension fund and find that the steady-state moments are almost unchanged.

[21] This also ensures that when we later consider a scenario where the DB pension plan has been closed, it will have exactly enough resources to pay off its existing liabilities and therefore, in the new steady-state, it will be left with exactly zero wealth in expectation.

3.6 Calibration

We now discuss the baseline calibration of the model. The corresponding parameter values are reported in Table 1.

3.6.1 Aggregate variables

The productivity shock follows a two-state first-order Markov. We calibrate the transition matrix to match NBER data on expansions and recessions. The probability of remaining in recessions (π_r) is 16/37, and the probability of remaining in expansions (π_e) is 60/81, We fit the expansion and recession values of productivity to match the conditional growth rate of labor income, as described in labor income calibration. This implies a standard deviation of TFP of 2.6%. The capital's share of output (α) is set to 34%, and the average annual depreciation rate (δ) is 10% with a standard deviation of also 10%. Firms in the model are unlevered, so we obtain an implied levered equity return by assuming a leverage ratio of 0.44, taken from Rajan and Zingales (1995).

The aggregate supply of government bonds is calibrated to deliver an endogenous ratio to GDP of 42%, the average value of U.S. Treasury securities held by the U.S. public.[22] The tax rate on interest payments (τ^B) is 20% and on stock returns (τ^K) its 40%, capturing a 20% personal tax rate on equity income (dividends and capital gains) and a 20% corporate tax rate. Bequests are fully taxed to avoid having to re-distribute wealth of deceased households, and forming expectations over such transfers. The tax rate on labor income (τ^W) is taken from Gomes et al. (2013) to match the share of labor taxes to GDP in the aggregate.

3.6.2 Household variables

The coefficient of risk aversion coefficient is set to 6 for both groups ($\gamma^A = \gamma^B = 6$), to generate a realistic equity premium in our economy. The other preference parameters (discount factor and EIS) are calibrated to match the standard deviation of consumption growth and the level of the risk-free rate. For the type-A households, their preference parameters are also chosen to deliver low wealth accumulation, and thus allow us to match the observed stock market participation

[22]Data from the Congressional Budget Office. We exclude debt held by foreigners since ours is a closed-economy model.

with reasonable values of the participation costs. More precisely, we set β^A equal to 0.83 and ψ^A equal to 0.25. For the type-B households, we set β^B at 0.965 and ψ^B at 0.65.

The stock market participation entry cost (F^0) is set to 6% and the per-period (F^1) to 1% in line with the values in the literature (see, for example, Gomes and Michaelides (2008), Favilukis (2013), and Fagereng et al. (2017)).

We use the deterministic labor income profile from Cocco et al. (2005). The standard deviation of transitory shocks (σ_ε) is set to 10%. For the calibration of the permanent income shocks we follow Guvenen et al. (2014) and first set the probability of the mixture q_1 to 0.49, and then discretize the underlying normal distributions. We calibrate the eight additional parameters from equation (17)) so that the mixture distributions match the time variation in the third and fourth moments across recessions and expansions, as in Guvenen et al. (2014).

3.6.3 Social Security and DB Pension fund

We first take the total replacement ratio of age-65 income ($\lambda^{db} + \lambda^{ss}$) 0.68212, from Cocco et al. (2005). We then decompose this into λ^{ss} and λ^{db} equal to 0.4596 and 0.2225 respectively, using data from the Social Security Administration, Federal Reserve Economic Data (FRED), and Public Plans Data, on the relative distributions of DB pension schemes and social security. We use Flow-of-Funds data in the U.S. to calibrate the portfolio allocation of the defined benefit pension fund's endowment (α^P). We assign a risky weight of one to corporate equities and mutual fund shares, and a weight of 0.15 to debt securities such as Treasury securities, municipal securities, repurchase agreements, commercial paper, mortgages, and foreign corporate debt. This yields an average risky share of 52% for the period 1970 to 2023.

4 Baseline Results

In this Section, we present results for a version of the model with historical demographic weights. This provides a baseline for the counterfactual experiments that we consider in subsequent sections.

Table 2 reports the model-implied moments and their empirical counterparts. The stock return data is taken from CRSP, while the mean and standard deviation of the real risk-free rate are

from Croce et al. (2012). Stock market participation data is taken from the Survey of Consumer Finances, while the values for the consumption and capital output ratios are from the NIPA tables provided by the Federal Reserve Bank of St. Louis. The data on bond supply to GDP is from Congressional Budget Office, and we exclude debt held by foreigners since ours is a closed-economy model.

Both the asset pricing and the macro moments in the model are very close to their corresponding values in the data. The model delivers a low riskfree rate (1.26%) and high equity premium (6.28%) while closely matching the second moment of both return series. The volatility of consumption growth is 3.03%, and the ratios of bond supply, consumption and capital stock to GDP are all very close to their empirical counterparts.

Finally, the stock market participation rate in the model is also very close to the one in the data (55.6% versus 51.1%). Naturally, almost all type-B agents are stockholders since they accumulate significant wealth (as shown in Figure 1). By contrast, very few type-A agents invest in stocks, typically only those close to retirement age, when their wealth increases enough to justify paying the entry cost.

Having established that the model closely replicates relevant macro and asset pricing moments, in the next sections we proceed to our counterfactual analysis, where we compare the current equilibrium with the one implied by the demographic projections discussed in Section 2.[23]

5 Economic Mechanisms

We can trace the impact of demographic shocks through five core mechanisms present in our full quantitative model. The first two are mechanical consequences of shifting population weights, even with constant policy functions and equilibrium prices. The third channel represents the partial equilibrium response, i.e for constant prices and a fixed tax and transfer system. The final two arise from market clearing and imposing aggregate level constraints on both social security and defined benefit pension funds.

[23] We do not solve for the transition dynamics as this would be computationally challenging in a model with aggregate uncertainty and heterogeneous agents. We would have to introduce additional state variables and solve for the forecasting equations over the transition path. In addition, we would have to make further assumptions about the speed of adjustment of both DB pension and social security.

5.1 Composition effects

1. Demand composition channel: A decrease in mortality or fertility shifts the demographic weight towards older cohorts, as shown in figure 2. Those cohorts have higher savings rates and accumulate more wealth. Therefore, for the same policy functions and aggregate conditions, this will increase total wealth in the economy.

Figure 3 reports wealth accumulation in the baseline economy for different age groups and the change in population weights in each of the two alternative scenarios. Wealth is aggregated over both agent types and into 5 age groups to facilitate visual interpretation. Moreover, we report differences from the mean to illustrate which age groups have above or below average wealth. The change in population weights is computed from the series reported in Figure 2, namely the difference in the weights reported for each alternative scenario ("Lower Fertility" and "Lower Fertility and Mortality" relative to the "Baseline" weights).

As shown in Figure 3, in the lower fertility scenario the population weights decrease significantly for the first age group which has the lowest wealth accumulation and increase particularly for those close to retirement (ages 50-65) and the first retiree group (ages 66-80), which are the ones with the most wealth in the economy.[24] As a result, from a partial equilibrium analysis, aggregate wealth accumulation would be expected to increase. Later we will discuss how general equilibrium effects can dominate this effect.

Multiplying the wealth in each group by its relative demographic weight and then computing the change due to the shift in those weights we can compute the aggregate change in wealth for each scenario implied by the demand composition channel alone. We obtain an increase in total wealth of 2.0% for the lower fertility scenario and an increase of 2.3% for the lower fertility and lower mortality case.

2. Labor-capital composition channel: An increase in the dependency ratio mechanically increases the capital-to-labor ratio in the economy. This relative labor scarcity lowers the marginal product of capital reducing the demand for capital by firms.

Figure 4 illustrates the impact of these two channels in the market for capital (left panel) and

[24]There is also an increase for the group of those age 81 and above but only a small fraction of the population reaches those ages.

the bond market (right panel). The demand composition channel shifts to the right both the supply of capital by households (from K^S to $K^{S'}$) and their demand for bonds (from B^D to $B^{D'}$). This implies a reduction in the market clearing return (in both markets).

As discussed, the labor-capital composition channel implies a reduction in the demand for capital (the curve shifts from K^D to $K^{D'}$), which creates additional downward pressure on the equilibrium return in this market. This suggests that the two returns shouldn't move one for one and, in particular, we should expect a reduction in the equity premium. Furthermore, the new equilibrium capital stock could be higher of lower, depending on the relative shift of the supply and demand curves in this market.

5.2 Individual re-optimization

3. Partial equilibrium demand channel: When demographic aging is driven by a decrease in mortality the expected duration of retirement increases forcing individuals to save more.[25] This channel is illustrated in figure 5. As household save more for retirement both their supply of capital and their demand for bonds shift further to the right, leading to an additional downward pressure on returns in both markets.

In summary, from these first three channels we should expect lower returns, as generally predicted in this literature, but particularly in the market for capital. The impact on the level of capital itself is uncertain.

5.3 Social Security, Defined Benefit Pension and Market clearing

4. Social security and DB contributions channel: As the age distribution changes, the budget constraints of both social security and defined benefit pension funds are impacted. In particular, as the ratio of retirees to workers increases, both institutions face a cash-flow shortfall. Covering such a shortfall requires either increasing the contributions of current workers and firms, and/or decreasing the payments to retirees, or some other adjustment.

[25]Note that this is not the case if we only have a change in fertility rates. In such a scenario, once an individual is born, her optimization problem is unchanged.

Figure 6 illustrates the impact of covering the shortfall with higher contributions from workers and firms. As the disposable income of working-age households falls their supply of capital and their demand for bonds both fall. This has a counter-acting effect on the downward return pressure from the first three channels. On the firm side, however, the higher contributions imply a further shift to the left of the demand for capital.

As an alternative, consider a scenario in which the shortfall is covered by decreasing the payments to retirees. In this case we have exactly the opposite impact on rates of return. Under this scenario, workers need to save more for retirement and their ability to save is not impacted since their disposable income is not affected. As figure 7 illustrates, this shifts to the right both the supply of capital and the demand for bonds, placing additional downward pressure on returns. Furthermore, in this case, there is no impact on the demand for capital from firms.

Figures 6 and 7 highlight how the equilibrium in the two markets will depend crucially on the margin(s) of adjustment that social security and defined benefit pensions decide to utilize. And they only consider two possible adjustments, while in the paper we also explore extending retirement age or shifting from a DB to a DC system. In addition, for simplicity, the previous illustrations ignored the role of the pension fund's endowment, which can be significant as we show later in the calibrated model.

5. General equilibrium channel: Finally, once we impose general equilibrium, the price changes in either market will lead to additional shifts in the curves in the other market since the policy functions of households are solved simultaneously as a function of the (current and expected) prices in both market.

6 Equilibrium with lower fertility rate

In our first steady-state comparison we change the fertility rate as discussed in Section 2, while keeping life expectancy constant. As mentioned in Section 5, a change in fertility rates has no direct impact on the individual optimization problem, shutting down the partial equilibrium demand channel and thus facilitating the discussion of the results. We solve the model for new steady-states which explore alternative scenarios with respect to the potential adjustments of

social security and pension schemes to the new demographics.

6.1 Social security and DB retirement income unchanged

In our first comparison, we assume that both social security and the DB pension scheme continue to provide the same retirement replacement ratios as before, as commonly assumed in the literature. Given the new demographic weights, this implies higher tax/contribution rates for both workers and firms. The results are reported in Table 3, with the baseline results reproduced in column (i) to facilitate comparison.

We start by comparing the results in columns (i) with those in (ii), which report the new equilibrium with lower fertility. As discussed in section 5 the combination of the two composition channels (demand and labor-capital) and partial equilibrium re-optimization would imply lower rates of return, particularly on capital, with an ambiguous impact on the level of capital itself. As also shown in section 5, if we hold the retirement replacement ratio fixed, the impact on returns is attenuated. Consistent with this, columns (i) and (ii) of Table 3 reveal modest changes in returns, with a larger reduction on the return of capital leading to a lower equity premium. As illustrated in figure 6, this scenario drives down the equilibrium level of capital, and that is indeed what we obtain. Capital directly held by households falls from 4.02 to 3.96. This is reflected almost one for one in the reduction in aggregate capital since, in this first experiment, we are assuming that the endowment of the DB pension fund remains unchanged.

In the other columns of Table 3 we explore the role of the asset demands of the pension fund by reporting different scenarios for how those might respond to the new equilibrium. While in column (ii) we have assumed that the endowment of the DB pension fund remains constant at its previous value, in column (iii) we scale it up proportionally to the increase in its (expected) liabilities.

In a scenario where the pension fund scales up its holdings of both bonds and capital, the latter offsets the lower demand from households such that the total capital stock remains almost unchanged. The pension fund's significant allocation to bonds creates additional downward pressure on the riskfree rate. As a result, although in our model the risk-free rate decreases, consistent with previous results in the literature (e.g. Gagnon et al. (2021), Carvalho et al. (2016), Auclert

et al. (2026) and Kopecky and Taylor (2022)),[26] the quantitative effect is much larger and the new equilibrium risk-free rate is even negative.

With a lower percentage of working age population in the economy, but unchanged retirement benefits, both social security taxes and contribution rates to the DB pension fund must increase. For example, in case (ii) social security taxes increase from 4.84% and 1.35% in the baseline economy to 5.25% and 1.47%, respectively. Interestingly, the additional downward pressure on equilibrium returns when the pension fund's endowment is increased (economy (iii)) leads to a significant reduction in the risk free rate. As a result, the expected return of the fund's endowment falls and the new equilibrium contribution rates for the defined benefit pension are actually higher than in economy (ii), when the endowment is kept constant.

Given these results, the pension fund is either unlikely to scale up its endowment that much or, it might change its asset allocation, increasing the fraction of equities in its portfolio.[27] We find that, if the DB fund increases its equity share (from 52.0%) to 55% when it scales up its endowment. Therefore, any value above 55% would be enough to avoid the large reduction in the riskless rate, such that the total (expected) return on the endowment is indeed higher. In column (iv) of Table 3 we report results for a scenario the DB's risky share increases to 60.0%. The required DB contribution rates are much lower than in case (iii) and, crucially, they are now lower than in case (ii) (when the endowment is held constant).

In economy (iv) the interest rate increases relative to the baseline economy, as the additional bond demand from the pension fund is more moderate than in economy (iii), just not as much as in economy (ii). With a higher equity portfolio share, the fund almost fully replaces the reduction in capital directly held by households, such that the total capital stock is almost unchanged. When the pension fund's endowment remains constant (economy (ii)) the return on capital falls, from 4.77% to 4.58%, reflecting the lower marginal productivity of capital in an economy with a reduced labor supply. In scenario (iv) this is further magnified by the increased demand for capital from the DB fund. As a result, in both cases we observe a lower levered equity return and equity risk premium. In economy (iii), however, the large reduction in the risk-free dominates such that both

[26] Abel (2003) obtains an increase in the equilibrium rate of return, but for an increase in fertility.

[27] This would also be consistent with a reaching for yield behavior, as documented in the literature, e.g. Ioannidou et al. (2022).

the levered equity return, and the equity risk premium are higher than in the baseline economy.

Across the different equilibria the macro-economic quantities (namely total capital, capital to GDP ratio, and consumption to GDP ratio) remain fairly close to their baseline values, particularly in economies (iii) and (iv) where the reduction in capital directly held by households is largely offset by the increase in stock holdings by the DB pension fund.

In these comparisons we have kept the retirement replacement ratios of social security and DB pensions constant, which must therefore be financed by higher contribution rates. This drives important political debates on how these costs should be shared across generations and motivates our next experiments where we consider alternative responses.

6.2 Defined benefit pension scheme is replaced by a DC system

The demographic changes that we are considering in this paper have led to substantial funding problems for defined benefit (hereafter DB) pension funds. As a result, several DB schemes have gradually closed down and replaced by defined contribution schemes. In this section we consider such a scenario, i.e. the lower fertility equilibrium is associated with a pure DC system where each individual saves for herself in a private account. The DB pension scheme has been shut down: there is no pension fund anymore, no contribution rates and its retirement replacement ratio is now zero:[28]

$$\lambda^{db} = 0 \tag{29}$$

$$\omega^{P} = 0 \tag{30}$$

$$\tau^{L}_{db} = 0 \text{ and } \tau^{K}_{db} = 0 \tag{31}$$

The results for this economy are reported in column (iii) of Table 4. To facilitate the comparison, in columns (i) and (ii) we repeat the same columns from Table 3, more precisely the moments for the baseline economy and for the economy with lower fertility rate and the DB pension scheme.

When the reduction in fertility rates is accompanied by a shift from DB to DC pension plans,

[28]Importantly, given our calibration of the pension fund's endowment (Section 3.5.2), after the DB system is closed and no new contributions are made, the fund has exactly the necessary funds to pay off its existing liabilities in expectation. Therefore, in the new steady-state the expected wealth of the DB pension fund is indeed zero, thus providing for a clean comparison of the different economies.

as has been observed in several countries, we obtain very different movements in asset prices. There is a large reduction in the demand for bonds such that the new equilibrium risk-free rate is substantially higher than in the baseline economy. Similarly, the decrease in the equity premium is now almost one order of magnitude larger (159 bp versus 19bp).

In the absence of the DB pension scheme, households must now save more for retirement, and this is reflected in a higher capital stock directly held by them, relative to the steady-state in (ii). However, households do not fully replace the total wealth accumulation of the DB pension fund, as the type-A agents in particular don't want to save much for retirement. Therefore, the total capital stock falls by 6.3%, compared with a 1.5% decrease in scenario (ii). The increase in household savings also induces a higher stock market participation rate, but this is largely offset by the lower equity premium, so that the share of stockholders in the population remains fairly constant.

6.3 Defined benefit pension scheme is replaced by a DC system and the social security replacement ratio is reduced

Social security faces the same demographic pressures as DB pension schemes, but political motives make it less clear that there will be a significant change in replacement ratios, and it is extremely unlikely that it will be completely eliminated. In this section, we consider a scenario in which the retirement replacement ratio of social security is decreased just enough to keep the contribution rates of workers and firms unchanged. We view this as a likely upper bound on the potential change, since it imposes the full cost on retirees. The corresponding results are shown in column (iv) of Table 4.

As illustrated in figure 7, the reduction in the retirement replacement ratio provided by social security forces households to save (even) more for retirement relative to the previous case (column (iii)). This is reflected in a higher value of the capital stock, now much closer to the one in the baseline economy. As households save more for retirement, the returns on both assets decrease relative to economy (iii), but the riskfree rate is still higher than in the baseline economy.

Comparing economies (iv) and (ii) we see that the total capital stock is very similar in both. However, in economy (iv) this is now being fully held by households, and as a result we have very

different asset prices: higher riskfree rate and lower equity premium. The lower equity premium is driven by two separate mechanisms. First, as in the data, in the model the DB pension fund had a stronger demand for bonds than the average household. Second, with the disappearance of the DB and the reduction in social security replacement ratio, the DB contribution rates are naturally set to zero, and social security taxes are lower. As a result, households are now better able to smooth income shocks early in life and this is reflected in a lower consumption volatility.

6.4 Increased retirement age

In the final scenario the adjustment to Social Security and Defined Benefit pension comes from an increase in the retirement age (to 67), such that the retirement replacement ratio is kept constant and the contributions rates from both workers and firms are also kept roughly at their original values. The corresponding results are shown in column (v) of Table 4.

While the adjustments considered in scenarios (iii) and (iv) forced households to save more for retirement, an increase in the retirement age does exactly the opposite as they have a smaller expected number of retirement years.[29] However, as the ratio of retirees to workers decreases, the social security tax rates and the contributions for the defined benefit pension scheme both fall, which has the opposite effect of savings. Households can afford to save more and have a higher incentive to do so, since the ratio of their working life income to their retirement income is now higher. In addition, as households work longer, their human capital increases and this tilts their portfolios more towards equities and away from bonds. As a result, in equilibrium we observe an increase in the total capital stock. The increase in capital stock drives down the return on equity but this is counter-balanced by the higher marginal productivity of capital due to the increase in labor supply, so that we observe a more modest reduction in the equity premium.

7 Equilibrium with lower fertility and higher life expectancy

We now consider changes in both fertility rates and life expectancy simultaneously. As detailed in the calibration section, we incorporate an increase in survival probabilities in order to match

[29] For now we are still considering a constant life-expectancy.

post-2000 averages. These survival probabilities imply a large increase in life expectancy of 5.7 years (from 71.7 to 77.4).

Our previous results also highlighted the importance of bond supply. As the DB endowment grows, its high portfolio share in bonds makes them scarce for private investors in equilibrium, putting downward pressure on risk-free rates (see Table 3). To account for this dynamic, as well as the substantial increase in government debt in most advanced economies, we now recalibrate the supply of government debt to 64% of GDP to also match its post-2000 average in the US.

7.1 Results for fixed retirement replacement ratios

The new equilibrium is reported in column (iv) of Table 5, where we also report the results for the baseline economy (column (i)) and for an economy with lower fertility rates and higher government debt but the same mortality rates (column (ii)).[30]

Even for fixed equilibrium quantities and prices, a higher life expectancy changes individual optimization behavior as they must now save more for retirement (partial equilibrium demand channel). To isolate this channel, in column (iii) we report results for an economy where the survival probabilities have changed but the demographic weights have not yet adjusted to this shock.[31] This economy can be interpreted as a first stage in the transition to the new steady-state, and it helps to isolate the impact of the changes in individual optimization. Since the demographic weights are unchanged relative to column (ii), the contribution rates (to both social security and DB pension) remain almost constant, only indirectly impacted by the changes in aggregate moments.

As life expectancy increases, individuals have to save more for retirement, and this is reflected in a higher capital stock (4.56 versus 4.45), and lower rates of return on both assets (capital and bonds). However, when we have converged to the new steady-state (column (iv)) the demographic weights have shifted due to the lower mortality rates, and we have an older population, with the increase in (relative) population particularly concentrated in later ages. This implies very

[30] We have also solved for equilibria where we only change government debt and keep mortality rates constant but, since the focus of our paper is the impact of demographic changes, we relegate those results to appendix G to minimize repetition.

[31] In this case we model households expectations assuming that the current demographic weights will remain constant, so that we can use the Krussell-Smith algorithm to solve for a fixed point.

substantial increases in both social security taxes and DB contribution rates, leading to a reduction in total capital accumulation, from 4.56 to 4.41. Therefore, although capital initially increases (economy (iii)), it eventually falls such that in the new steady-state (economy (iv)) it is even lower than the one in economy (ii). However the capital output ratio increases (labor-capital composition channel), and this leads to a further reduction of the return on capital. Comparing economies (iv) and (i) we obtain a 14.9% reduction in the capital stock held by households but, due to the presence of the DB pension fund, the reduction in aggregate capital stock is much smaller at 7.2%. So, the reduction in capital stock is less than half of what we would have obtained in a model where we failed to include the DB pension fund and its endowment.

Interestingly, the risk-free rate also falls. This is because the DB pension fund must accumulate a larger endowment of wealth to finance payments to an increased pool of retirees, and this leads to a higher demand for bonds.[32]. As previously discussed, this is an unlikely equilibrium, as those pressures on DB pensions and social security will likely lead to adjustments in both. We consider those adjustments in our next set of results.

7.2 Results with alternative adjustments for both Social Security and Defined Benefit pensions

In the previous results we assumed that workers and firms would pay more social security taxes and pension contributions to finance constant retirement replacement ratios. In Table 6 we consider three new economies, where we explore alternate responses of both the pension system and social security to the new equilibrium. In column (iii) we lower the social security replacement ratio so that the contribution rates remain at the same level as in the baseline economy (column i)), while in column (iv) the DB pension scheme is shut down and replaced by a DC system. In column (v) we consider both changes simultaneously.

Both when the social security replacement ratio falls and when the DB system is replaced by a DC, individuals must save more for retirement and the private capital stock increases relative to economy ii), where neither of those adjustments is taking place. In scenario iii) this also leads

[32]Naturally the fund also increases its holdings of capital, which is why the decrease in total capital stock (4.56 to 4.41) is smaller than the decrease in the capital held directly by individuals (3.87 to 3.42).

to a larger decrease in the riskfree rate (to -0.94% instead of 1.15%). Naturally, with a negative riskfree rate few households are holding bonds. Mostly those that don't find it worthwhile to pay the stock market participation cost but still want to save some wealth for precautionary reasons or retirement. The majority of the bond supply is being held by the DB pension fund.[33]

However, when we shut down the DB system, the endowment of the pension fund is also depleted.[34] Households don't fully substitute for this since the type-A agents in particularly don't have a strong incentive to save for retirement. Therefore, there is less total wealth accumulation and in this scenario the capital stock falls and the return on equity increases. Given that the DB pension fund had a particular high demand for bonds, its disappearance has a stronger impact on the riskfree rate and equity premium. Relative to the baseline economy we now observe a significant increase in the riskfree rate (to 2.85%) and the equity premium falls by more than 2 percentage points (to 4.06%).

In column (v) we consider both changes together. With both a lower social security replacement ratio and no DB plan, households must save much more for retirement and therefore the private capital stock is even higher than when considering each change in isolation. It is now close to the value of the total capital stock in the baseline economy, although still lower. In terms of asset prices, the dominant effect comes from shutting down the pension fund, with a higher riskfree rate and substantially lower equity premium.

Finally, in column (vi), we consider the alternative policy response of increasing the retirement age. In the lower fertility scenario, keeping the contribution rates of workers close to the baseline values required increasing retirement age to 67 (Table 4). In this case, since we also have higher life expectancy, we must increase it further, to age 70.

A reduction in the expected number of years at retirement reduces the demand for retirement savings. However, by extending the working life, the dependency ratio improves relative to scenario (ii) and therefore smaller increases in social security and DB labor contribution rates are required. As discussed in Section 5 this increases savings both because individuals have more disposable

[33]With a negative riskfree the DB pension fund would likely shift its portfolio allocation away from bonds, as in the scenarios discussed in Table 3. We don't repeat the same analysis here to avoid repetition.

[34]The wealth of the pension fund does not disappear, it is exhausted by paying accrued retirement benefits to those that had previously contributed to the fund. As discussed in the appendix we compute the equilibrium value of the endowment by determining the level of wealth required to make those payments in expectation.

income and because, in an after-tax basis, the ratio of their working life income to their retirement income is now higher. As shown in Table 6, the contributions from workers to both social security and the DB pension are now 50% lower than in economy (ii). As a result we observe substantially higher household savings in the scenario with higher retirement age. Both total capital and directly held private capital now even higher than in the baseline economy. As before, asset demands shift from bonds to equity, increasing the risk-free rate to 2.32%, while the equity premium compresses to 4.75%.

The results in Table 6 therefore highlight the importance of explicitly considering the different alternative responses of social security and pension schemes to changes in fertility and mortality rates.

8 Conclusion

Many economies around the world are experiencing simultaneous declines in fertility rates and increases in life expectancy. In this paper, we study the implications of these trends for asset prices and capital accumulation. We show that movements in the risk-free rate, the equity premium and the capital stock depend crucially on the pension and social security systems in place and how these respond (or not) to these demographic trends.

If retirement replacement ratios remain unchanged, the contribution rates of workers and firms must increase and the DB pension fund will accumulate a larger endowment of wealth. This reduces risk-free rates, the equity premium and the capital stock. We also find that capital accumulation initially increases as agents save more for a longer retirement, but eventually falls once we converge to the new demographic structure due to a decreased share of the employed population. If the pension system is reformed such that retirement age increases, these dynamics are significantly muted with much smaller adjustments to asset prices and capital accumulation.

If instead DB plans are replaced by defined contribution schemes, the implication for risk free rates is the opposite. The lack of pension fund bond demand causes the risk-free rate to increase, and this effect is compounded by households not replacing institutional savings one for one. The combination of these two effects generates even larger falls in the capital stock and equity premia than the one observed under the DB system.

Our results highlight that the macroeconomic and asset pricing implications of demographic changes are highly contingent on the type of pension system but also the reforms put in place to address these shifts. From a policy perspective, the long-run impact of demographic trends on financial markets and aggregate investment will be largely determined by the institutional choices available to policy-makers. This also indicates that the consequences of demographic shifts will likely be heterogeneous across countries, depending on their pension and social security schemes and on how they respond.

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Table 1: Calibrated Parameters for the Baseline Models

Aggregate Variables		Household Variables	
Technology		Preferences	
π_r	16/37	γ^A	6
α	34%	γ^B	6
Mean(δ)	10%	β^A	0.83
Vol(δ)	10%	β^B	0.965
		ψ^A	0.25
Debt and Taxes		ψ^B	0.65
b/y	42%		
τ^B	20%	Participation Costs	
τ^K	40%	F^0	6%
τ^W	6.8%	F^1	1%
τ^E	100%		
Pension Fund		Retirement Income	
α^P	52%	λ^{ss}	0.4596
θ^P	0.5	λ^{db}	0.2225

Table 1 reports the baseline calibration of the different parameters of the model. The household-level income processes are given by a combination of values from Cocco et al. (2005), Guvenen et al. (2014), and authors' own estimations, as described in the calibration section.

Table 2: Baseline Model

Variable	Moment	Model	Data
r^B	Mean	1.26%	0.86%
r^B	St. Dev.	1.21%	1.35%
r^M	Mean	7.53%	8.17%
r^M	St. Dev.	19.41%	19.81%
$r^M - r^B$	Mean	6.28%	7.55%
$\frac{Mean(r^M - r^B)}{Std.Dev.(r^M - r^B)}$		0.32	0.36
Output growth	St. Dev.	3.56%	4.78%
Cons. growth (all)	St. Dev.	3.03%	2.90%
Cons. growth (A)	St. Dev.	2.62%	(-)
Cons. growth (B)	St. Dev.	3.58%	(-)
k	Mean	4.75	(-)
k_{hh}	Mean	4.02	(-)
b/y	Mean	0.39	0.42
c/y	Mean	0.65	0.64
k/y	Mean	2.28	2.50
Participation (all)	Mean	55.6%	51.1%
Participation (A)	Mean	17.9%	(-)
Participation (B)	Mean	93.4%	(-)

Table 2 reports asset pricing and macro moments for the baseline model (where r^M denotes levered r^K), and the data. k_{hh} denotes the capital stock held directly by households. The asset pricing data are taken from CRSP, with the risk-free data from Croce et al. (2012). Stock market participation is from the Survey of Consumer Finances. Real consumption, capital and GDP are from the NIPA Tables.

Table 3: Comparison with lower fertility steady-state

Variable	Moment	Models			
		(i)	(ii)	(iii)	(iv)
Fertility		Baseline	Lower	Lower	Lower
DB Endowment		Baseline	Baseline	Higher	Higher
DB Risky Share		Baseline	Baseline	Baseline	Higher
r^B	Mean	1.26%	1.18%	-4.85%	1.45%
r^B	St. Dev.	1.21%	1.23%	0.42%	1.14%
r^K	Mean	4.77%	4.58%	4.21%	4.57%
r^M	Mean	7.53%	7.26%	11.36%	7.02%
r^M	St. Dev.	19.41%	19.44%	20.10%	19.60%
$r^M - r^B$	Mean	6.28%	6.09%	16.20%	5.57%
$\frac{Mean(r^M - r^B)}{Std.Dev.(r^M - r^B)}$		0.32	0.31	0.81	0.28
Output growth	St. Dev.	3.56%	3.56%	3.57%	3.56%
Cons. growth (all)	St. Dev.	3.03%	3.03%	3.31%	3.08%
Cons. growth (A)	St. Dev.	2.62%	2.63%	3.22%	2.66%
Cons. growth (B)	St. Dev.	3.58%	3.58%	3.59%	3.62%
k	Mean	4.75	4.68	4.79	4.72
k_{hh}	Mean	4.02	3.96	3.84	3.97
b/y	Mean	0.39	0.40	0.40	0.40
c/y	Mean	0.65	0.66	0.64	0.66
k/y	Mean	2.28	2.28	2.27	2.29
Participation (all)	Mean	55.6%	53.8%	69.8%	53.2%
Participation (A)	Mean	17.9%	14.3%	42.7%	13.9%
Participation (B)	Mean	93.4%	93.3%	96.8%	92.5%
τ^L_{ss}	Mean	4.84%	5.25%	5.25%	5.25%
τ^K_{ss}	Mean	1.35%	1.47%	1.47%	1.45%
τ^L_{db}	Mean	2.25%	2.55%	4.69%	2.28%
τ^K_{db}	Mean	0.19%	0.23%	0.72%	0.18%

Table 3 reports asset pricing and macro moments for the baseline economy (column (i)), and an economies with a lower fertility rate (columns (ii) to (iv)). In columns (iii) and (iv) the endowment of the DB fund increases proportionally to the increase in its expected liabilities, while in column (ii) it remains at its original value. In column (iv) we further consider that the DB pension fund's equity share increases to 60%. The asset pricing data are taken from CRSP, and r^M denotes levered r^K in the model. k_{hh} denotes the capital stock held directly by households. The mean and volatility of the real risk-free rate are taken from Croce et al. (2012). Stock market participation is computed from the Survey of Consumer Finances. Real consumption, capital and GDP are from the NIPA Tables.

Table 4: Lower fertility steady-state with adjustments in Social Security and DB pensions

Variable	Moment	Models				
		(i)	(ii)	(iii)	(iv)	(v)
Fertility		Baseline	Lower	Lower	Lower	Lower
Def. Benefit		Baseline	Baseline	Zero	Zero	Baseline
Social Sec.		Baseline	Baseline	Baseline	Lower	Baseline
Ret. Age		65	65	65	65	67
r^B	Mean	1.26%	1.18%	2.62%	2.40%	1.72%
r^B	St. Dev.	1.21%	1.23%	1.02%	0.92%	1.02%
r^K	Mean	4.77%	4.58%	5.25%	5.00%	4.85%
r^M	Mean	7.53%	7.26%	7.32%	7.06%	7.32%
r^M	St. Dev.	19.41%	19.44%	18.33%	17.77%	19.17%
$r^M - r^B$	Mean	6.28%	6.09%	4.69%	4.66%	5.60%
$\frac{Mean(r^M - r^B)}{Std.Dev.(r^M - r^B)}$		0.32	0.31	0.26	0.26	0.29
Output growth	St. Dev.	3.56%	3.56%	3.59%	3.52%	3.56%
Cons. growth (all)	St. Dev.	3.03%	3.03%	2.61%	2.45%	2.94%
Cons. growth (A)	St. Dev.	2.62%	2.63%	1.93%	1.79%	2.45%
Cons. growth (B)	St. Dev.	3.58%	3.58%	3.43%	3.25%	3.55%
k	Mean	4.75	4.68	4.45	4.65	4.86
k_{hh}	Mean	4.02	3.96	4.45	4.65	4.21
b/y	Mean	0.39	0.40	0.41	0.40	0.39
c/y	Mean	0.65	0.66	0.64	0.64	0.65
k/y	Mean	2.28	2.28	2.21	2.27	2.29
Participation (all)	Mean	55.6%	53.8%	56.7%	59.7%	52.9%
Participation (A)	Mean	17.9%	14.3%	23.4%	29.9%	13.4%
Participation (B)	Mean	93.4%	93.3%	90.0%	89.5%	92.4%
τ_{ss}^L	Mean	4.84%	5.25%	5.54%	4.88%	4.53%
τ_{ss}^K	Mean	1.35%	1.47%	1.59%	1.36%	1.26%
τ_{db}^L	Mean	2.25%	2.55%	0.0%	0.0%	1.74%
τ_{db}^K	Mean	0.19%	0.23%	0.0%	0.0%	0.14%

Table 4 reports asset pricing and macro moments for the baseline economy (column (i)), and four versions of the economy with lower fertility rate. In column (ii) the replacement ratios for DB pension and social security are kept constant (same columns as in table 3). In column (iii) we consider an economy without a DB system. In column (iv) we further reduce the replacement ratio of social security to keep the respective contribution rates at the same level as in the baseline economy. In column (v) we instead consider an increase in the retirement age to 67. The asset pricing data is from CRSP, and r^M denotes levered r^K in the model. k_{hh} denotes the capital stock held by households. The real risk-free rate moments are from Croce et al. (2012). Stock market participation is computed from the Survey of Consumer Finances. Real consumption, capital and GDP are from the NIPA Tables.

Table 5: Lower fertility, higher government debt and higher life expectancy

Variable	Moment	Models			
		(i)	(ii)	(iii)	(iv)
Debt		Baseline	Higher	Higher	Higher
Mortality		Baseline	Baseline	Lower	Lower
Dem Weights		Baseline	LowFert	LowFert	New
Fertility		Baseline	Lower	Lower	Lower
r^B	Mean	1.26%	2.32%	2.14%	1.15%
r^B	St. Dev.	1.21%	1.03%	1.03%	1.04%
r^K	Mean	4.77%	5.14%	4.90%	3.98%
r^M	Mean	7.53%	7.37%	7.09%	6.22%
r^M	St. Dev.	19.41%	19.40%	19.41%	20.40%
$r^M - r^B$	Mean	6.28%	5.05%	4.95%	5.07%
$\frac{Mean(r^M - r^B)}{Std.Dev.(r^M - r^B)}$		0.32	0.26	0.25	0.25
Output growth	St. Dev.	3.56%	3.55%	3.56%	3.58%
Cons. growth (all)	St. Dev.	3.03%	2.91%	2.90%	3.24%
Cons. growth (A)	St. Dev.	2.62%	2.49%	2.48%	2.87%
Cons. growth (B)	St. Dev.	3.58%	3.45%	3.44%	3.75%
k	Mean	4.75	4.45	4.56	4.41
k_{hh}	Mean	4.02	3.78	3.87	3.42
b/y	Mean	0.39	0.61	0.60	0.64
c/y	Mean	0.65	0.67	0.67	0.69
k/y	Mean	2.28	2.21	2.25	2.30
Participation (all)	Mean	55.6%	52.7%	52.5%	52.4%
Participation (A)	Mean	17.9%	14.3%	13.5%	12.5%
Participation (B)	Mean	93.4%	91.1%	91.5%	92.3%
τ^L_{ss}	Mean	4.84%	5.25%	5.25%	6.76%
τ^K_{ss}	Mean	1.35%	1.51%	1.48%	1.86%
τ^L_{db}	Mean	2.25%	1.94%	2.04%	3.59%
τ^K_{db}	Mean	0.19%	0.15%	0.16%	0.29%

Table 5 reports asset pricing and macro moments for the baseline economy (column (i)), and the economy with lower fertility rate and higher government debt (column (ii)). In columns (iii) and (iv) we further consider a reduction in survival probabilities. The economy in column (iii) uses the same demographic weights as in (ii), while the one in column (iv) considers the demographic weights implied by the new mortality rates as well. The asset pricing data are is from CRSP, and r^M denotes levered r^K in the model. k_{hh} denotes the capital stock held by households. The moments for the real risk-free rate are from Croce et al. (2012). Stock market participation is from the Survey of Consumer Finances. Real consumption, capital and GDP are from the NIPA Tables.

Table 6: Lower fertility, higher government debt and life expectancy: alternative scenarios

Variable	Moment	Models					
		(i)	(ii)	(iii)	(iv)	(v)	(vi)
Debt		Baseline	Higher	Higher	Higher	Higher	Higher
Mortality		Baseline	Lower	Lower	Lower	Lower	Lower
Fertility		Baseline	Lower	Lower	Lower	Lower	Lower
Def. Benefit		Baseline	Baseline	Baseline	Zero	Zero	Baseline
Social Sec.		Baseline	Baseline	Lower	Baseline	Lower	Baseline
Ret. Age		65	65	65	65	65	70
r^B	Mean	1.26%	1.15%	-0.94%	2.85%	2.00%	2.32%
r^B	St. Dev.	1.21%	1.04%	1.15%	1.12%	1.00%	1.01%
r^K	Mean	4.77%	3.98%	3.11%	5.12%	4.26%	4.97%
r^M	Mean	7.53%	6.22%	6.32%	6.91%	6.05%	7.07%
r^M	St. Dev.	19.41%	20.40%	20.73%	18.33%	17.77%	19.19%
$r^M - r^B$	Mean	6.28%	5.07%	7.25%	4.06%	4.05%	4.75%
$\frac{Mean(r^M - r^B)}{Std.Dev.(r^M - r^B)}$		0.32	0.25	0.35	0.22	0.23	0.25
Output growth	St. Dev.	3.56%	3.58%	3.60%	3.59%	3.53%	3.56%
Cons. growth (all)	St. Dev.	3.03%	3.24%	3.43%	2.49%	2.05%	2.87%
Cons. growth (A)	St. Dev.	2.62%	2.87%	3.13%	1.91%	1.41%	2.39%
Cons. growth (B)	St. Dev.	3.58%	3.75%	3.85%	3.22%	2.80%	3.45%
k	Mean	4.75	4.41	4.94	3.97	4.71	4.80
k_{hh}	Mean	4.02	3.42	3.73	3.97	4.71	4.16
b/y	Mean	0.39	0.64	0.61	0.66	0.62	0.58
c/y	Mean	0.65	0.69	0.65	0.67	0.64	0.67
k/y	Mean	2.28	2.30	2.48	2.16	2.40	2.27
Participation (all)	Mean	55.6%	52.4%	64.0%	57.0%	62.3%	50.3%
Participation (A)	Mean	17.9%	12.5%	33.7%	24.6%	37.5%	10.2%
Participation (B)	Mean	93.4%	92.3%	94.4%	89.3%	87.2%	90.3%
τ^L_{ss}	Mean	4.84%	6.76%	5.10%	7.24%	4.73%	4.57%
τ^K_{ss}	Mean	1.35%	1.86%	1.31%	2.13%	1.25%	1.28%
τ^L_{db}	Mean	2.25%	3.59%	4.84%	0.0%	0.0%	1.51%
τ^K_{db}	Mean	0.19%	0.29%	0.58%	0.0%	0.0%	0.10%

Table 6 reports asset pricing and macro moments for the baseline economy (column (i)), and different versions of the economy with lower fertility, higher government debt and lower mortality (columns (ii) to (iv)). In Column (ii) social security and defined benefit replacement ratios are unchanged. In column (iii) we decrease the social security replacement ratio, while in column (iv) the DB pension has been shut down. In column (v) we consider both simultaneously. Column (vi) considers an increase in retirement age. The asset pricing data is from CRSP, and r^M denotes levered r^K in the model. k_{hh} denotes the capital stock held by households. The moments for the real risk-free rate are from Croce et al. (2012). Stock market participation is from the Survey of Consumer Finances. Real consumption, capital and GDP are from the NIPA Tables.

Figure 1: Demographic data

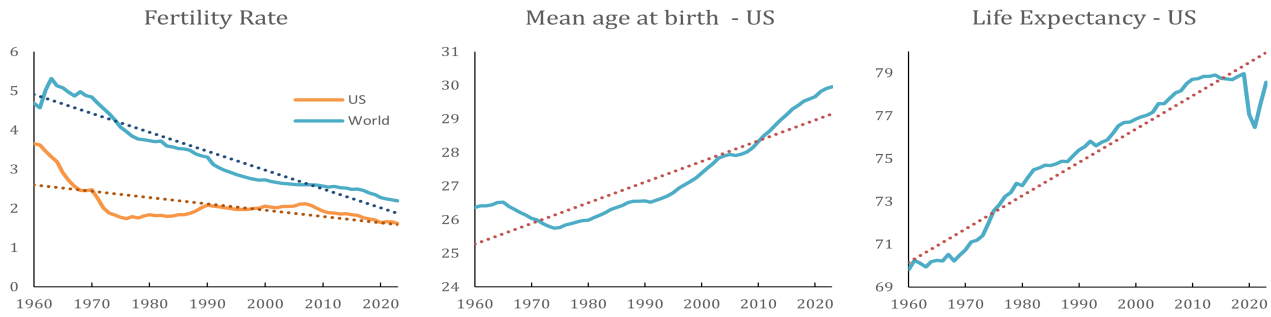


Figure 1 reports historical demographic data and projections.

Figure 2: Population Weights

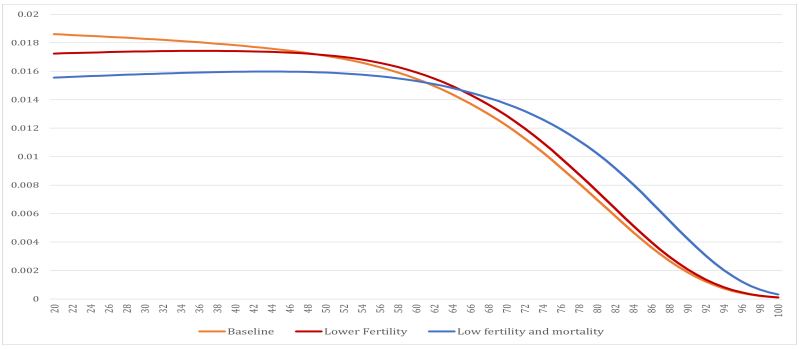


Figure 2 reports the population weights for the different demographic scenarios.

Figure 3: Wealth Accumulation and Change in Population Weights

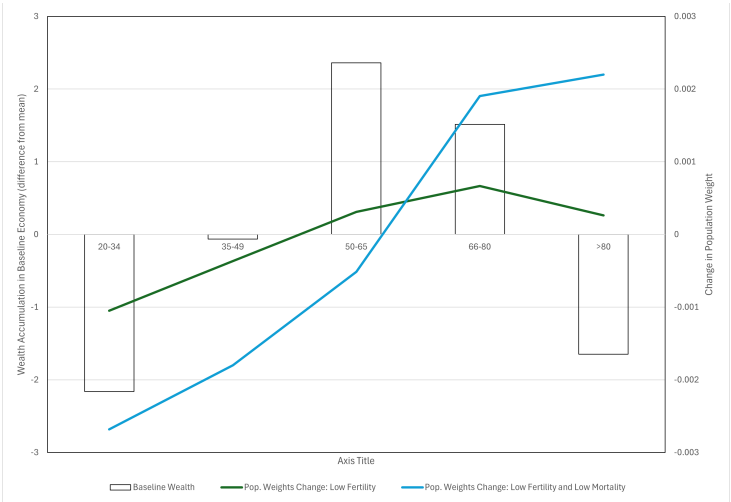


Figure 3 reports the average wealth accumulation in the baseline economy for different age groups and the change in population weights for different demographic scenarios.

Figure 4: Impact of Compositional Channels

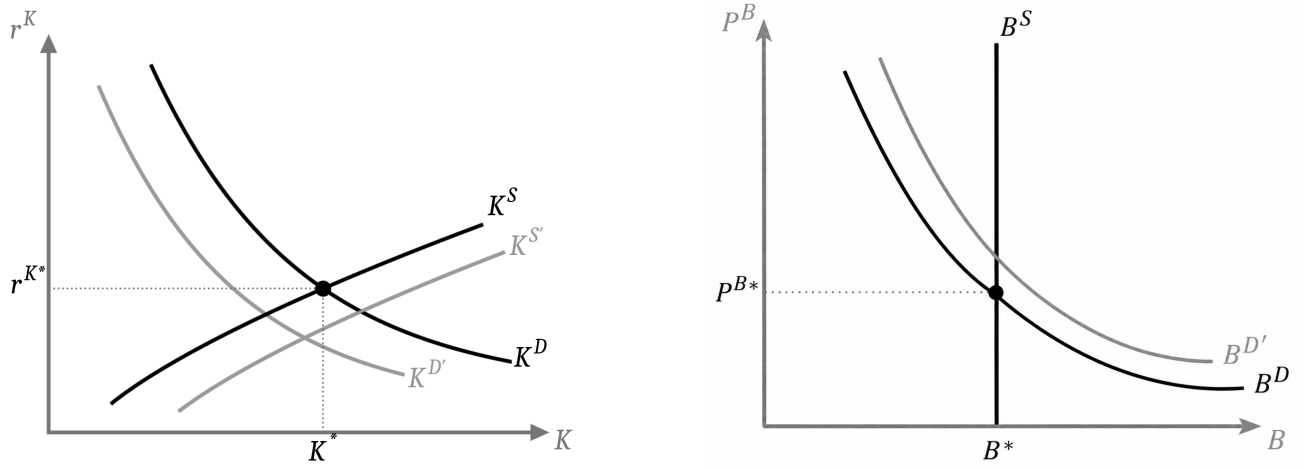


Figure 4 illustrates the impact of the two composition channels on the market for capital (left panel) and the bond market (right panel).

Figure 5: Impact of Partial Equilibrium Channel

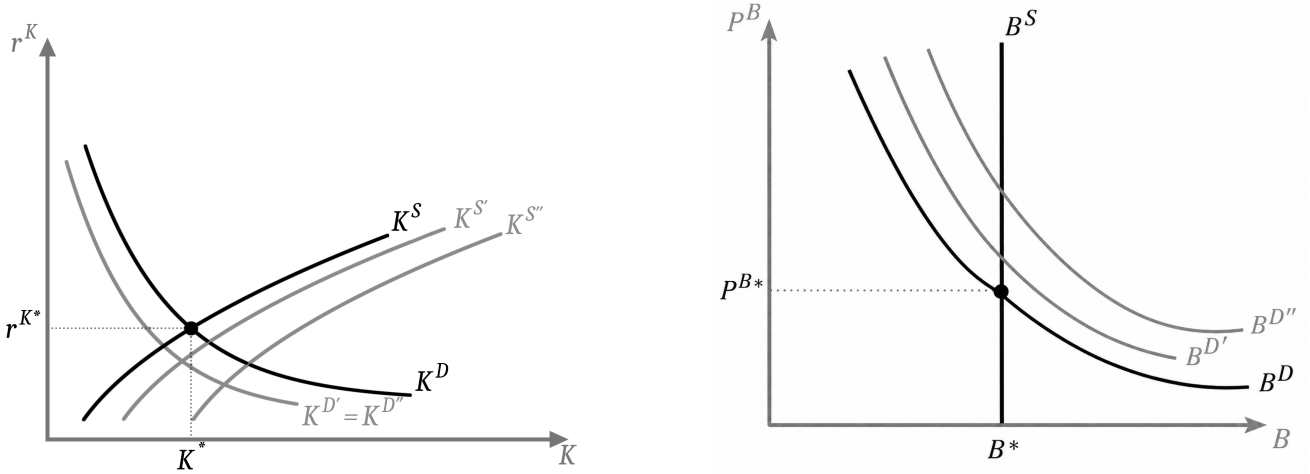


Figure 5 illustrates the impact of the partial equilibrium channel on the market for capital (left panel) and the bond market (right panel).

Figure 6: Higher social security and defined benefit contributions rates

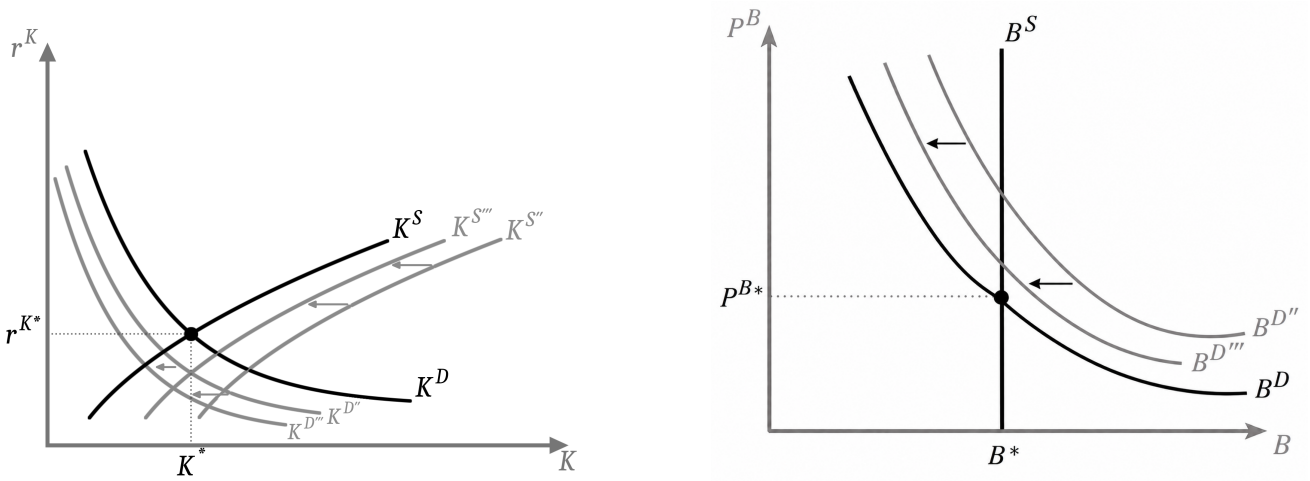


Figure 6 illustrates the impact of increasing social security and defined benefit contributions rates on the market for capital (left panel) and the bond market (right panel).

Figure 7: Lower social security and defined benefit retirement replacement ratios

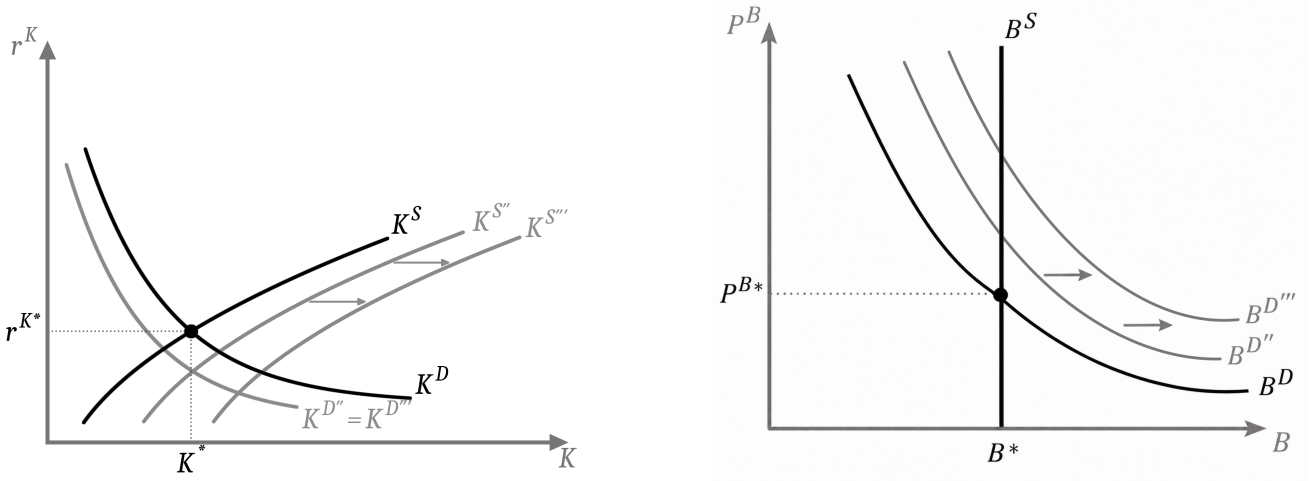


Figure 7 illustrates the impact of dereasing social security and defined benefit retirement replacement ratios on the market for capital (left panel) and the bond market (right panel).

ONLINE APPENDIX FOR

"Demographic Trends, Asset Prices and Capital Accumulation"

Nuno Coimbra (Banque de France), Francisco Gomes (London Business School),
Alexander Michaelides (Imperial College) and Jialu Shen (Fudan University)

Online Appendix A: The dynamic programming problem

As described in the main text we normalize aggregate state by the growth rate of technology (G_t) such that we have stationary economy. We further normalize individual variables by the current level of permanent labor income ($P^i_{a,t}$) to reduce the dimensionality of the state vector by one.

In the normalized equilibrium, the household-specific variables are

$$x^i_{a,t} \equiv \frac{X^i_a}{P^i_{a,t} G_t^{\frac{1}{1-\alpha}}} \tag{32}$$

and similarly for $c^i_{a,t}$, $b^i_{a+1,t+1}$ and $k_{a+1,t+1}$.

The normalized individual maximization problem has seven state variables: age (a), normalized cash on hand ($x^i_{a,t}$), a zero-one variable indicating whether the entry cost has been paid or not (E^i_a), and the four aggregate variables from the forecasting equations ((24) and (25)).

The full optimization problem is

$$V_a(x^i_{a,t}, E^i_a; k_t, U_t, \eta_t, P^B_t) = \underset{\{k^i_{a+1,t+1}, b^i_{a+1,t+1}\}_{a=1}^{A}}{Max} \{(1-\beta)(c^i_{a,t})^{1-1/\psi} \tag{33}$$

$$+\beta(E_t[(\frac{P^i_{a+1,t+1}}{P^i_{a,t}}(1+g)^{\frac{1}{1-\alpha}})^{1-\rho} p_a V^{1-\rho}_{a+1}(x^i_{a+1,t+1}, E^i_{a+1}; k_{t+1}, U_{t+1}, \eta_{t+1}, P^B_{t+1})])^{\frac{1-1/\psi}{1-\rho}}\}^{\frac{1}{1-1/\psi}},$$

subject to the constraints

$$k^i_{a+1,t+1} \geq 0 \quad , \quad b^i_{a+1,t+1} \geq 0 \tag{34}$$

$$c^i_{a,t} + b^i_{a+1,t+1} + k^i_{a+1,t+1} = x^i_{a,t} \tag{35}$$

and

$$x^i_{a+1,t+1} = \begin{cases} \frac{[k^i_{a+1,t+1}(1+(1-\tau^K)R^K_{t+1}) + b^i_{a+1,t+1}(1+(1-\tau^K)R^B_{t+1})]}{[(P^i_{a+1,t+1}/P^i_{a,t})(1+g)^{\frac{1}{1-\alpha}}]} \\ +\varepsilon^i(1-\tau^L_{ss}-\tau^W-\tau^L_{db,t})w_{t+1} - I^i_E F^0 w_{t+1} - I^i_S F^1 w_{t+1} \quad a < a^R \\ \frac{[k^i_{a+1,t+1}(1+(1-\tau^K)R^K_{t+1}) + b^i_{a+1,t+1}(1+(1-\tau^K)R^B_{t+1})]}{[(P^i_{a+1,t+1}/P^i_{a,t})(1+g)^{\frac{1}{1-\alpha}}]} \\ +(\lambda^{db}+\lambda^{ss})w_{t+1} - I^i_E F^0 w_{t+1} - I^i_S F^1 w_{t+1} \quad a > a^R \end{cases}, \tag{36}$$

and the stochastic process for individual labor productivity (equations (14) to (17)). Households form expectations about future asset returns as before, using equations (24) and (25).

The individual takes as given all aggregate variables, that is, capital stock, returns, bond price, wages, tax rates, and the other government variables.

Online Appendix B: Equilibrium

Equilibrium prices and quantities are determined by the following conditions:

1. Firms hire capital and labor to maximize profits (equations (10) and (11)).

2. Individuals choose their consumption and asset allocation to maximize their expected life-time utility, (i.e., they maximize equation (33) subject to the constraints described in the previous subsection).

3. The social security system is balanced at all times, as given by equation (42).

4. The defined benefit pension contribution rates ($\tau^L_{db,t}$ and $\tau^K_{db,t}$) are set such that the fund has a balanced budget at all times.

5. The government budget (Equation (12)) is balanced every period.

6. All markets clear, specifically the markets for capital, bonds, and the consumption good:[35]

$$k_{t+1} = \int_i \int_a P^i_{a,t} k^i_{a,t+1} da di \tag{37}$$

$$b_{t+1} = \int_i \int_a P^i_{a,t} b^i_{a,t+1} da di \tag{38}$$

$$U_t k_t^{\alpha} L_t^{1-\alpha} = \frac{C^G_t}{G_t^{\frac{1}{1-\alpha}}} + (1+g)^{\frac{1}{1-\alpha}} k_{t+1} - (1-\delta_t) k_t + \int_i \int_a P^i_{a,t} c^i_{a,t} da di \tag{39}$$

By Walras' law, once two of these equations are verified, the third is also automatically satisfied.

7. Household expectations for market prices (equations (24) and (25)) are verified in equilibrium.

We describe the numerical solution of the model in Online Appendix C.

[35] The market for labor is trivial since there is no labor-leisure choice.

Online Appendix C: Solution Method

Since this is a constrained optimization we use grid search to solve for the optimal policy functions of individuals: capital and bond holdings (k^i_{at} and b^i_{at}). The problem has seven state variables: age (a), normalized cash on hand (x^i_{at}), stock market participation status (E^i_a, zero-one variable indicating whether the entry cost has been paid or not), and the four aggregate variables from the forecasting equations. We discretize the state space for the continuous state variables, with a log grid for cash-on-hand and linear grids for capital and the bond price. We use cubic spline interpolations for wealth and linear interpolation for the aggregate states.

For the forecasting rules we follow the approach in Krusell and Smith (1998) where households predict the next period capital stock using its lagged value and the realizations of the aggregate shocks.

$$log(k_{t+1}) = a_k + b_k \cdot log(k_t) \tag{40}$$

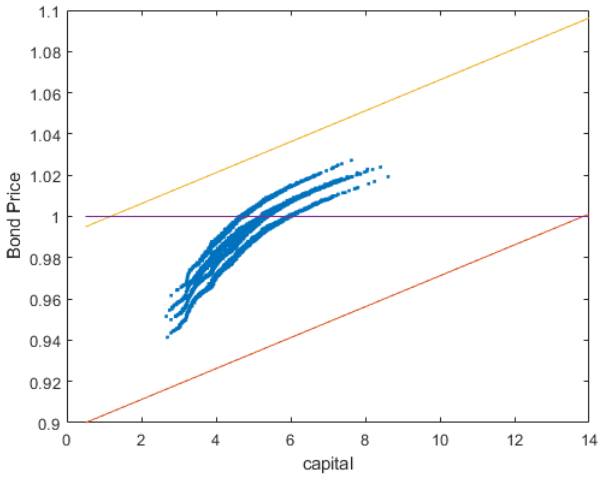
where each coefficient in the equation depends on the current realization of the aggregate productivity shock and the stochastic depreciation shock.

In our case we have to forecast the evolution of the bond price as well. The guess-and-verify equation for the log-bond price is similarly given by

$$log(P^B_{t+1}) = a_P + b_P \cdot log(k_t) + c_P \cdot log(P^B_t) \tag{41}$$

where again each coefficient in the bond pricing function depends on the current realization of the aggregate productivity shock and the stochastic depreciation shock.

Once the coefficients in the forecasting equations have converged in an outer loop, we verify that (i) the conditional R-2 are very high and (ii) all simulated state variables are within the range assumed when setting the parameters of the model. We use a parallel grid (rather than a rectangular grid) for the bond pricing function as a function of the capital stock, as shown in the graph below, together with the simulation results:



Online Appendix D: Social Security Tax Rates

In this appendix we discuss how we compute the values of the two social security tax rates (τ^L_{ss} and τ^K_{ss}).

Let us first define τ^*_{ss} as a hypothetical social security tax rate on wages if the (social security) tax rate on capital was zero.

$$\sum_{a=20}^{65} \int_{i \in I^a} \tau^*_{ss} L^i_{a,t} w_t di = \sum_{a=66}^{100} \int_{i \in I^a} [\lambda^{ss} \exp(f(a^R)) w_t P^i_{a^R,t^R}] di \ , \tag{42}$$

The only endogenous variable in this equation is w_t, which appears on both sides and thus cancels out. We can therefore compute τ^*_{ss} as a function of exogenous variables only. For the same reason, this tax rate would be a constant, which is why we have not included a subscript t in the notation.

Since the RHS in equations (26) and (42) is the same, we can combine both equations and express the actual tax rates as a function of the hypothetical τ^*_{ss}.

$$\sum_{a=20}^{65} \int_{i \in I^a} \tau^L_{ss,t} L^i_{a,t} w_t di + \tau^K_{ss,t} k_t = \sum_{a=20}^{65} \int_{i \in I^a} \tau^*_{ss} L^i_{a,t} w_t di \tag{43}$$

Given with our assumption that social security costs are split equally between workers and firms, we obtain:

$$\tau^L_{ss,t} w_t \bar{L} = 0.5 \tau^*_{ss} \bar{L} w_t \Rightarrow \tau^L_{ss} = 0.5 \tau^*_{ss} \tag{44}$$

$$\tau^K_{ss,t} k_t = 0.5 \tau^*_{ss} \bar{L} w_t \tag{45}$$

where $\bar{L} = \sum_{a=20}^{65} \int_{i \in I^a} L^i_{a,t} di$.

From these equations we can see that the tax rate on wages is a constant, hence we can write it as τ^L_{ss}. On the other hand, the tax rate on capital must be time-varying since both capital and the wage rate are endogenous variables. The exact value of $\tau^K_{ss,t}$ is determined by the equation above, such that $\tau^K_{ss,t} k_t / w_t$ is constant (and equal to $0.5 \tau^*_{ss} \bar{L}$).

Online Appendix E: DB Pension Fund Contribution Rates

In this appendix we discuss how we compute the values of the two contribution rates to the defined benefit pension plan (τ^L_{db} and τ^K_{db}). Repeating the same sequence as for the social security tax rates, we first write down the budget constraint of the pension fund in a hypothetical scenario where the only contributions come from wages.

$$\sum_{a=20}^{65} \int_{i \in I^a} \tau^*_{db,t} L^i_{a,t} w_t di + R^P_t \bar{\omega}^P = \sum_{a=66}^{100} \int_{i \in I^a} [\lambda^{db} \exp(f(a^R)) w_t P^i_{a^R,t^R}] di \tag{46}$$

Comparing with equation (42) we see that there is an additional term on the LHS, the return on the endowment ($R^P_t \bar{\omega}^P$) which is an endogenous and time-varying quantity in the model. As a result, $\tau^*_{db,t}$ must also be time-varying.

Under the assumption of equal split between contributions from workers and from firms, the corresponding contribution rates for each are:

$$\tau^L_{db,t} = 0.5 * \tau^*_{db,t} \tag{47}$$

$$\tau^K_{db,t} k_t = 0.5 \tau^*_{db,t} \bar{L} w_t \tag{48}$$

Since the hypothetical contribution rate from (46) is time-varying, now both actual contribution rates ($\tau^L_{db,t}$ and $\tau^K_{db,t}$) are also functions of time.

Online Appendix F: DB Pension Fund Endowment

The endowment of the pension fund is kept constant at a steady-state value ω^P. This value is chosen such, that if the DB pension is shut down and no new generations are enrolled, the endowment (plus its return) is exactly enough to pay off the fund's existing (net) liabilities, in expectation. This makes the comparison with other economies cleaner, since there is no remaining DB wealth or shortfall from the corresponding transitions.

More specifically, assume that the fund is closed to new members, in an arbitrary year t^*. Then it must still pay retirement benefits to current retirees while they are still alive. It will continue to receive contributions from current workers (the last cohorts to do so), but it will also have to pay them pension benefits once they retire. Therefore, the pension payments in year t ($>= t^*$) are given by:

$$Payments_t = \begin{cases} \sum_{a=66}^{100} \int_{i \in I^a} [\lambda^{db} \exp(f(a^R)) w_t P^i_{a^R t^R}] di & \text{t} \in [t^*, t^* + 45] \\ \sum_{a=66+t-(t^*+45)}^{100} \int_{i \in I^a} [\lambda^{db} \exp(f(a^R)) w_t P^i_{a^R t^R}] di & \text{t} \in]t^* + 45, t^* + 80] \\ 0 & t > t^* + 80 \end{cases} \tag{49}$$

Until year $t = t^* + 45$ the pension payments remain as before: the fund must make payments to all agents that are retired, i.e. all those between ages 66 and 100. At $t = t^* + 45$, the last cohort of DB members retires. So, for each following year, the pension payments shrink, as there is one less cohort of retired individuals to whom they are due. This process continues until t exceeds $t^* + 80$, when there are no more remaining members of the plan still alive, and thus the payments become zero.

On the contribution side, each year there is one less cohort contributing until year $t = t^* + 45$, when the last cohort of DB members retires, and the contributions become zero. Denoting by $Contributions_t$ the value of the contributions at time t, we first write the required endowment of the fund as

$$\bar{\omega}^P = \sum_{t=1}^{80} (Payments_t - Contributions_t)/(1 + E[R^P])^t \tag{50}$$

where $E[R^P]$ is the expected return on the pension fund's endowment. The equation above only holds for a deterministic steady-state, where the return on the portfolio is equal to its expected value. For the purpose of the calibration, we use a geometric mean of the return to correct for Jensen's inequality.[36]

[36]This equation is not to be interpreted as a fair value/economic calculation of the fund's funding ratio. Such valuation would assign appropriate discount rates to future contributions and liabilities.

Online Appendix G: Comparative Statics for Higher Government Debt

In this appendix we report results for a steady-state with higher fertility, higher government debt but unchanged mortality rates so that we can visualize the impact of a ceteris paribus change in government debt

An increase in the supply of government debt, similarly to the change in fertility rates, "only" impacts individual optimization indirectly through equilibrium prices and expectations. Therefore, we first solve for a new steady-state with lower fertility and a higher level of government debt (64% of GDP to match the more recent period, starting in 2000).

The results are reported in Table A1 where, as before, the first two columns are identical to the ones in Table 3, and report the steady-states in the baseline economy and the economy with lower fertility rate for the same pension and social security replacement ratios. In equilibrium, an increase in government debt (column (iii) of A1) implies a higher risk-free rate to induce households to hold the additional supply. This reduces the equilibrium equity premium and crowds-out capital.

Similarly to the analysis in the previous sections, in columns (iv) to (vi) we consider different adjustments for both social security and pensions. A shift from DB to DC creates further pressure in the riskless rate and a further reduction in the equity premium, while a reduction in the social security replacement ratio has a (smaller) opposite effect.

Table A1: Lower fertility and higher government debt

Variable	Moment	Models					
		(i)	(ii)	(iii)	(iv)	(v)	(vi)
Debt		Baseline	Baseline	Higher	Higher	Higher	Higher
Fertility		Baseline	Lower	Lower	Lower	Lower	Lower
Def. Benefit		Baseline	Baseline	Baseline	Zero	Zero	Baseline
Social Sec.		Baseline	Baseline	Baseline	Baseline	Lower	Baseline
Ret. Age		65	65	65	65	65	67
r^B	Mean	1.26%	1.18%	2.26%	3.22%	2.95%	2.70%
r^B	St. Dev.	1.21%	1.23%	1.06%	1.11%	1.00%	1.03%
r^K	Mean	4.77%	4.58%	5.04%	5.73%	5.47%	5.43%
r^M	Mean	7.53%	7.26%	7.24%	7.71%	7.46%	7.58%
r^M	St. Dev.	19.41%	19.44%	19.44%	18.35%	17.79%	19.10%
$r^M - r^B$	Mean	6.28%	6.09%	4.98%	4.48%	4.51%	4.88%
$\frac{Mean(r^M - r^B)}{Std.Dev.(r^M - r^B)}$		0.32	0.31	0.26	0.24	0.25	0.26
Output growth	St. Dev.	3.56%	3.56%	3.56%	3.58%	3.52%	3.55%
Cons. growth (all)	St. Dev.	3.03%	3.03%	2.90%	2.51%	2.36%	2.80%
Cons. growth (A)	St. Dev.	2.62%	2.63%	2.49%	1.89%	1.74%	2.33%
Cons. growth (B)	St. Dev.	3.58%	3.58%	3.43%	3.25%	3.08%	3.38%
k	Mean	4.75	4.68	4.45	4.23	4.42	4.59
$k_{private}$	Mean	4.02	3.96	3.77	4.23	4.42	4.02
b/y	Mean	0.39	0.40	0.61	0.62	0.61	0.59
c/y	Mean	0.65	0.66	0.67	0.65	0.64	0.66
k/y	Mean	2.28	2.28	2.22	2.14	2.20	2.21
Participation (all)	Mean	55.6%	53.8%	52.6%	55.9%	58.1%	50.2%
Participation (A)	Mean	17.9%	14.3%	14.0%	22.9%	28.2%	10.6%
Participation (B)	Mean	93.4%	93.3%	91.2%	89.0%	88.0%	89.8%
τ^L_{ss}	Mean	4.84%	5.25%	5.40%	5.54%	4.88%	4.53%
τ^K_{ss}	Mean	1.35%	1.47%	1.54%	1.64%	1.41%	1.30%
τ^L_{db}	Mean	2.25%	2.55%	2.00%	0.0%	0.0%	1.42%
τ^K_{db}	Mean	0.19%	0.23%	0.15%	0.0%	0.0%	0.10%

Table A1 reports asset pricing and macro moments for the baseline economy (column (i)), and five versions of the economy with lower fertility. In column (ii) the replacement ratios for DB pension and social security are constant. In columns (iii) to (vi) we increase the debt to GDP ratio. Column (iv) removes the DB system, in column (v) we also reduce the social security replacement ratio and column (vi) considers an increased retirement age. The asset pricing data is from CRSP, and r^M denotes levered r^K in the model. $k_{private}$ denotes capital stock held by households. The real risk-free rate moments are from Croce et al. (2012). Stock market participation is from the Survey of Consumer Finances. Real consumption, capital and GDP are from the NIPA Tables.

Online Appendix H: Introducing a bequest motive

In the baseline version of the model we did not include a bequest motive. In this appendix we relax that assumption, by introducing the following changes to the model:

1) Household preferences for type-B agents are now given by:

$$V_{a,t} = \left\{ (1-\beta) C_{a,t}^{\frac{1-\gamma}{\theta}} + \beta \left(E_t \left[p_a V_{a+1,t+1}^{1-\gamma} + (1-p_a) (1-\beta)^{\theta} b^{\frac{\theta}{\psi}} (1-\tau_{beq}) * X_{a+1,t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}. \quad (51)$$

where the parameter b determines the intensity of the bequest motive.[37] We only consider a bequest motive for the type-B agents simply because type-A households save very little during their lives and thus adding a bequest motive would lead to very awkward wealth accumulation profiles for this group. Precisely because they accumulate limited wealth, adding a bequest motive for those agents would not meaningfully impact the results.

2) The tax rate on bequests (τ_{beq}) is decreased from 100% to an approximate historical average of 50%.

3) The resulting bequests are pooled and distributed equally between all type-B age-20 households. Distributing bequest wealth at the initial age is a common simplifying assumption in models with aggregate uncertainty, as it avoids having to model agents' expectations about future bequests.

We recalibrate the model under this specification targeting the same moments as before. The strength of the bequest motive $b^{\theta/\psi}$ is set to 2 (b=1.5). We counteract the corresponding increase in savings by decreasing the discount rate of the type-B agents to 0.958.

Table A2 repeats the analysis from Table 6 for the alternative model with bequests.[38] Not only do we obtain the same qualitative conclusions as before, but the quantitative comparisons are also very similar for all cases.

[37] It can be interpreted as as the ratio of optimal cash-on-hand to optimal consumption in the final period of life if the length between two periods approaches zero (Kraft et al. (2022))

[38] The strength of the bequest motive could be lower in new equilibria, as the number of children per family has fallen. We keep this parameter unchanged since there is guidance in the literature on how to properly calibrate such an effect. Arguably, incorporating such an adjustment would make the results closer to those in the baseline analysis since it would drive the bequest motive towards zero in the new equilibria.

Table A2: Bequest Motive

Variable	Moment	Models					
		(i)	(ii)	(iii)	(iv)	(v)	(vi)
Debt		Baseline	Higher	Higher	Higher	Higher	Higher
Mortality		Baseline	Lower	Lower	Lower	Lower	Lower
Fertility		Baseline	Lower	Lower	Lower	Lower	Lower
Def. Benefit		Baseline	Baseline	Baseline	Zero	Zero	Baseline
Social Sec.		Baseline	Baseline	Lower	Baseline	Lower	Baseline
Ret. Age		65	65	65	65	65	70
r^B	Mean	1.52%	1.47%	-0.67%	2.71%	1.77%	2.34%
r^B	St. Dev.	1.14%	1.07%	1.22%	1.14%	1.00%	1.03%
r^K	Mean	4.85%	4.37%	3.45%	4.93%	3.87%	4.94%
r^M	Mean	7.47%	6.66%	6.71%	6.68%	5.53%	6.99%
r^M	St. Dev.	19.39%	20.35%	20.90%	18.34%	17.76%	19.20%
$r^M - r^B$	Mean	5.95%	5.19%	7.38%	3.97%	3.76%	4.64%
$\frac{Mean(r^M - r^B)}{Std.Dev.(r^M)}$		0.31	0.26	0.35	0.22	0.21	0.24
Output growth	St. Dev.	3.55%	3.57%	3.59%	3.59%	3.54%	3.56%
Cons. growth (all)	St. Dev.	2.96%	3.13%	3.43%	2.46%	2.41%	2.82%
Cons. growth (A)	St. Dev.	2.52%	2.77%	3.17%	1.90%	1.73%	2.35%
Cons. growth (B)	St. Dev.	3.50%	3.61%	3.80%	3.11%	3.15%	3.35%
k	Mean	4.72	4.24	4.80	4.06	4.92	4.82
k_{hh}	Mean	4.01	3.32	3.65	4.06	4.92	4.18
b/y	Mean	0.39	0.65	0.62	0.66	0.61	0.58
c/y	Mean	0.67	0.71	0.67	0.69	0.66	0.69
k/y	Mean	2.27	2.25	2.43	2.19	2.47	2.28
Participation (all)	Mean	56.4%	55.8%	64.9%	58.8%	62.6%	52.7%
Participation (A)	Mean	14.9%	15.0%	32.5%	21.6%	29.8%	8.6%
Participation (B)	Mean	97.8%	96.6%	97.3%	95.9%	95.5%	96.8%
τ^L_{ss}	Mean	4.84%	6.76%	5.19%	7.24%	4.73%	4.57%
τ^K_{ss}	Mean	1.35%	1.91%	1.36%	2.10%	1.22%	1.27%
τ^L_{db}	Mean	2.14%	3.29%	4.67%	0.00%	0.00%	1.51%
τ^K_{db}	Mean	0.18%	0.24%	0.48%	0.00%	0.00%	0.10%

Table A2 reports asset pricing and macro moments for the baseline economy (column (i)), and different versions of the economy with lower fertility, higher government debt and lower mortality (columns (ii) to (iv)). In Column (ii) social security and defined benefit replacement ratios are unchanged. In column (iii) we decrease the social security replacement ratio, while in column (iv) the DB pension has been shut down. In column (v) we consider both simultaneously. Column (vi) considers an increase in retirement age. The asset pricing data is from CRSP, and r^M denotes levered r^K in the model. k_{hh} denotes the capital stock held by households. The moments for the real risk-free rate are from Croce et al. (2012). Stock market participation is from the Survey of Consumer Finances. Real consumption, capital and GDP are from the NIPA Tables.

Online Appendix I: Consumption Equivalence-Scale Adjustment

This appendix describes how we incorporate changes in household composition into preferences. In the baseline model, households have Epstein-Zin-Weil preferences over consumption. With the equivalence-scale adjustment, the recursion for a household of type i and age a is

$$V^i_{a,t}(x) = \left[(1-\beta_i) \left(\frac{C^i_{a,t}(x)}{\eta^m_a} \right)^{1-1/\psi_i} + \beta_i \left(\mathbb{E}_t \left[p_a \left(V^i_{a+1,t+1}(x') \right)^{1-\gamma_i} \right] \right)^{\frac{1-1/\psi_i}{1-\gamma_i}} \right]^{\frac{1}{1-1/\psi_i}}, \tag{52}$$

where β_i is the discount factor, γ_i is relative risk aversion, ψ_i is the elasticity of intertemporal substitution, and p_a is the conditional survival probability from age a to age $a+1$. The only change relative to the baseline preference specification is that household consumption is replaced by adult-equivalent consumption, $C^i_{a,t}/\eta^m_a$.

We construct the age profile of equivalence scales from household composition by age of the household head. Following Lusardi et al. (2017) and Choukjmane and De Silva (2026), let j_a denote the average number of adults and k_a the average number of children under age 18 in households headed by age-a individuals. The raw equivalence scale is

$$\hat{\eta}_a = \frac{(j_a + 0.7k_a)^{0.75}}{(2+0.7)^{0.75}}. \tag{53}$$

We use the profile reported in the replication files of Choukjmane and De Silva (2026), which is based on the public replication files of Lusardi et al. (2017). The available profile covers ages 26 to 100. Since households enter our model at age 20, we extend the profile to ages 20–25 by linear extrapolation using the first two available age points:

$$\hat{\eta}_a = \hat{\eta}_{26} + (a-26)(\hat{\eta}_{27} - \hat{\eta}_{26}), \qquad a = 20, \ldots, 25. \tag{54}$$

We normalize the profile so that its minimum equals one:

$$\eta^0_a = \frac{\hat{\eta}_a}{\min_{a'} \hat{\eta}_{a'}}. \tag{55}$$

This normalization preserves the life-cycle shape of the equivalence-scale profile while expressing all values relative to the lowest household-composition need over the life cycle. The baseline economy uses η^0_a.

For the low-fertility counterfactuals, we adjust the equivalence scale to reflect the smaller number of dependents implied by the fertility path. Let F^0_a denote the baseline fertility component associated with age a, and let F^m_a denote the corresponding component in counterfactual economy m. We adjust only the part of the equivalence scale above one:

$$\eta^m_a = 1 + (\eta^0_a - 1)\frac{F^m_a}{F^0_a}. \tag{56}$$

Thus, lower fertility reduces the equivalence scale at ages when households are expected to have fewer dependents, while leaving the minimum value normalized to one. The baseline column uses η^0_a, and the low-fertility columns use η^m_a.

Table A3: Consumption equivalence-scale adjusted

Variable	Moment	Models					
		(i)	(ii)	(iii)	(iv)	(v)	(vi)
Debt		Baseline	Higher	Higher	Higher	Higher	Higher
Mortality		Baseline	Lower	Lower	Lower	Lower	Lower
Fertility		Baseline	Lower	Lower	Lower	Lower	Lower
Def. Benefit		Baseline	Baseline	Baseline	Zero	Zero	Baseline
Social Sec.		Baseline	Baseline	Lower	Baseline	Lower	Baseline
Ret. Age		65	65	65	65	65	70
R^B	Mean	1.20%	1.04%	-2.25%	3.16%	2.30%	2.60%
R^B	St. Dev.	1.26%	1.02%	1.48%	1.13%	0.98%	1.02%
R^K	Mean	4.62%	3.81%	2.84%	5.47%	4.61%	5.32%
R^M	Mean	7.31%	5.98%	6.84%	7.30%	6.43%	7.46%
R^M	St. Dev.	19.43%	20.53%	21.13%	18.34%	17.76%	19.15%
$R^M - R^B$	Mean	6.11%	4.93%	9.09%	4.14%	4.14%	4.86%
$\frac{Mean(R^M-R^B)}{Std.Dev.(R^M-R^B)}$		0.32	0.24	0.42	0.23	0.24	0.26
Output growth	St. Dev.	3.57%	3.58%	3.60%	3.58%	3.53%	3.56%
Cons. growth (all)	St. Dev.	3.09%	3.26%	3.32%	2.47%	2.41%	2.83%
Cons. growth (A)	St. Dev.	2.74%	2.96%	3.18%	1.95%	1.71%	2.42%
Cons. growth (B)	St. Dev.	3.59%	3.71%	3.66%	3.11%	3.19%	3.35%
k	Mean	4.82	4.48	4.98	3.83	4.52	4.64
k_{hh}	Mean	4.07	3.55	3.82	3.83	4.52	4.03
b/y	Mean	0.39	0.64	0.61	0.67	0.63	0.59
c/y	Mean	0.65	0.69	0.64	0.68	0.65	0.67
k/y	Mean	2.30	2.33	2.49	2.11	2.34	2.22
Participation (all)	Mean	50.2%	48.9%	63.5%	54.3%	59.6%	48.2%
Participation (A)	Mean	6.0%	4.8%	31.2%	18.5%	29.3%	4.9%
Participation (B)	Mean	94.4%	93.0%	95.8%	90.1%	89.8%	91.5%
τ^L_{ss}	Mean	4.84%	6.76%	5.19%	7.24%	4.73%	4.57%
τ^K_{ss}	Mean	1.33%	1.84%	1.32%	2.18%	1.28%	1.31%
τ^L_{db}	Mean	2.32%	3.72%	5.55%	0.00%	0.00%	1.38%
τ^K_{db}	Mean	0.20%	0.30%	0.77%	0.00%	0.00%	0.09%

Table 6 reports asset pricing and macro moments for the baseline economy (column (i)), and different versions of the economy with lower fertility, higher government debt and lower mortality (columns (ii) to (iv)). In Column (ii) social security and defined benefit replacement ratios are unchanged. In column (iii) we decrease the social security replacement ratio, while in column (iv) the DB pension has been shut down. In column (v) we consider both simultaneously. Column (vi) considers an increase in retirement age. The asset pricing data is from CRSP, and R^M denotes levered R^K in the model. k_{hh} denotes the capital stock held by households. The moments for the real risk-free rate are from Croce et al. (2012). Stock market participation is from the Survey of Consumer Finances. Real consumption, capital and GDP are from the NIPA Tables.

Table A4: Recalibration

Variable	Moment	Models		
		(i)	(ii)	(iii)
CEA		No	Yes	Yes
Debt		Baseline	Baseline	Baseline
Mortality		Baseline	Baseline	Baseline
Fertility		Baseline	Baseline	Baseline
Def. Benefit		Baseline	Baseline	Baseline
Social Sec.		Baseline	Baseline	Baseline
Ret. Age		Baseline	Baseline	Baseline
r^B	Mean	1.26%	1.90%	1.20%
r^B	St. Dev.	1.21%	1.02%	1.26%
r^K	Mean	4.77%	5.15%	4.62%
r^M	Mean	7.53%	7.72%	7.31%
r^M	St. Dev.	19.41%	19.33%	19.43%
$r^M - r^B$	Mean	6.28%	5.82%	6.11%
$\frac{Mean(r^M - r^B)}{Std.Dev.(r^M - r^B)}$		0.32	0.30	0.32
Output growth	St. Dev.	3.56%	3.56%	3.57%
Cons. growth (all)	St. Dev.	3.03%	2.99%	3.09%
Cons. growth (A)	St. Dev.	2.62%	2.57%	2.74%
Cons. growth (B)	St. Dev.	3.58%	3.54%	3.59%
k	Mean	4.75	4.58	4.82
k_{hh}	Mean	4.02	3.91	4.07
b/y	Mean	0.39	0.40	0.39
c/y	Mean	0.65	0.66	0.65
k/y	Mean	2.28	2.23	2.30
Participation (all)	Mean	55.6%	49.5%	50.2%
Participation (A)	Mean	17.9%	5.2%	6.0%
Participation (B)	Mean	93.4%	93.7%	94.4%
τ^L_{ss}	Mean	4.84%	4.84%	4.84%
τ^K_{ss}	Mean	1.35%	1.38%	1.33%
τ^L_{db}	Mean	2.25%	1.96%	2.32%
τ^K_{db}	Mean	0.19%	0.16%	0.20%