Asset Pricing, not Equity Pricing

Chunjie Wang*

This Version: October 2, 2024
Latest Version Link

Abstract

This paper shows that building characteristics-managed factors with firms' asset returns greatly reduces the number of factors necessary to explain the cross section. A 5-factor model based on asset returns explains 62.4% of the variation in 100 factors, whereas an 88-factor model using equity returns explains only 38.6%. In the out-of-sample, the asset-based implied mean-variance-efficient (MVE) portfolio achieves a Sharpe ratio of 1.2, compared to 0.75 for its equity-based counterpart. The parsimonious asset-based model explains equity returns better than the equity-based model, as it reduces the number of equity anomalies to 15, compared to 23 for the latter. The non-linear transformation of returns caused by leverage increases the loadings of firms with high leverage on the equity-based factors, exposes these factors to firm-level systematic risks that would not arise in asset-based factors, and contributes to the factor zoo.

*chunjie.wang@hhs.se

Introduction

Starting with Fama and French (1993), researchers have proposed hundreds of candidate factors based on firm characteristics to explain cross-sectional stock returns, forming the factor zoo (Cochrane, 2011). The factors are typically characteristics-managed portfolios, in which the weights of assets depend on conditioning characteristics such as size and book-to-market ratio. This body of literature aims to understand the fundamental sources of risk to which firms are exposed. However, empirical tests predominantly construct factors using equity returns, which are non-linear transformations of firms' asset returns. Higher leverage increases the quantity of risk that equities are exposed to and overweights highly-levered firms in the construction of the equity-based factors. This unintentionally exposes factors to systematic risks that are not meant to be captured by the corresponding characteristics and results in redundant factors. In this paper, I test whether a sparser and more efficient representation of the stochastic discount factor (SDF) is feasible using factors built with asset returns.

I perform a comparative analysis of 100 characteristic-managed factors built from equity returns versus the same factors built from asset returns. I employ a linear model with coefficients regularization that allows the joint evaluation of a large number of factors. This approach implements an economically founded optimization process that filters out factors contributing less to the covariance matrix. By comparing the optimized model structures and their out-of-sample performance between equity-based and asset-based factors, I find that the asset-based factor model is sparser and more efficient in explaining not only asset returns but also equity returns.

I start by analytically showing that when the SDF can be represented by a single factor built from asset returns, the corresponding equity-based model fails to explain the cross section. Instead, an alpha arises and necessitates additional terms in the SDF to address the non-existent anomaly.

This raises the question to which extent is the proliferation of factors in the literature attributed to this effect. To tackle the high-dimensional challenge, I adopt a Bayesian approach following Kozak, Nagel, and Santosh (2020) (hereafter "KNS") that allows me to estimate the SDF's coefficients on potentially hundreds of characteristics-managed factors, letting the data determine which and how many factors to include before estimating their coefficients. Should my argument holds, the asset-based SDF should require fewer factors and exhibits a higher Sharpe ratio than the equity-based SDF.

To implement the test, I build 100 firm-level predictive characteristics suggested by Green, Hand, and Zhang (2017) for all non-financial firms listed in the U.S. from 1951 to 2022. I unlever equity returns using Merton (1974)'s model. Specifically, I estimate the market value of firms' assets on each day following the iterative procedure by Vassalou and Xing (2004). Then, I construct asset-based (equity-based) daily factors managed by the 100 characteristics. Each factor is a zero-cost long-short portfolio of asset (equity) returns weighted by the corresponding characteristic. I use Merton model as the baseline because it is the simplest within the class of models that allow for a non-linear relation between equity returns and leverage. This provides expositional convenience and facilitates intuition regarding our empirical results.

As robustness tests, I adopt two alternative unlevering methods using the weighted average cost of capital (WACC): In one approach, I assume all debts are risk-free and approximate asset returns as the average of stock returns and the 3-month U.S. treasury rates, weighted by market capitalization and book value of total liability. In another, I approximate asset returns as the average of stock returns and corporate bond returns, weighted by market capitalization and market value of corporate bonds. Firm-level bond returns are aggregated from individual bonds' month-over-month price change plus accrued interest and coupon payments. The market value of a bond is estimated with amount outstanding, scaled by dollar volume to par-value volume ratio. Throughout the paper, unless otherwise specified, the asset-based factors refer to the Merton method. I also report the results for the other two unlevering methods, which are qualitatively similar. Despite challenges using bond data — such as data quality, market segmentation, and institutional details that might hinder the reconstruction of asset returns using stock and bond returns — unlevering returns with bonds provides even stronger evidence that many anomalies disappear once correctly accounting for the leverage effect.

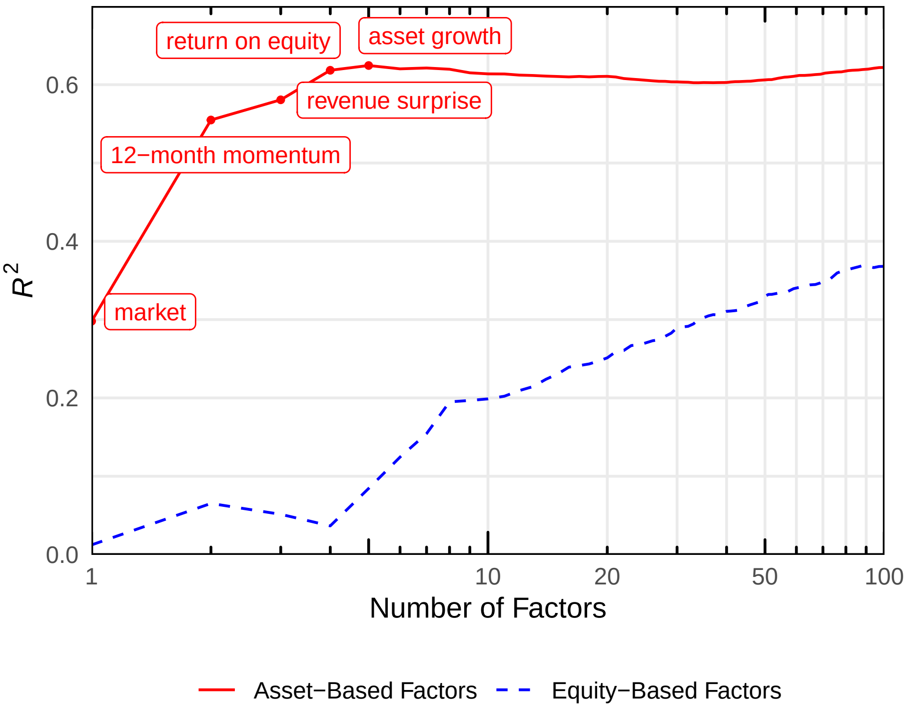
With the sets of candidate factors established, I employ a Bayesian prior motivated by KNS to shrink the cross section in search for a data-driven optimal SDF. Specifically, the prior distribution links the first and second moments of the candidate factors, leading the posterior to impose stronger penalties on the coefficient estimates for factors linked to low-eigenvalue principal components (PCs).

To identify the optimal factor model, I employ a cross-validation procedure: I repeatedly partition the data by withholding a subset of samples and estimating the factor model on the remaining data. In each iteration, I vary the number of factors included and adjust the penalty levels applied to the coefficients. These two parameters that are fixed before the estimation of coefficients are called hyperparameters. Each combination of the two hyperparameters corresponds to one model. The procedure to estimate coefficients follows the LARS-EN algorithm by (Zou and Hastie, 2005) that progressively removes factors that contribute least to explaining the covariance matrix until the target number of factors is achieved. I evaluate the performance of each model by measuring the R^2 on the withheld samples. This process allows me to assess how well each model generalizes to unseen data. The model that achieves the highest R^2 in these validation sets is considered to provide the optimal SDF representation for this specific set of factors. Comparing the number of factors and the out-of-sample (OOS) performance in their respective optimal SDFs of shows which factors, asset-based or equity-based, are more efficient proxies for the underlying risks.

I find the optimal equity-based SDF consists of 88 factors but only explains 36.9% of the cross-sectional variation. This result coroborates the work of KNS, who show an optimal 49-factor model explains 23.5% of the cross-sectional variation among 50 factors. It suggests that a large number of characteristics-managed factors are required in the SDF to adequately capture the pricing information, and that sparsity is generally elusive in the equity return space. In constrast, my analysis with asset-based SDF yields a distinct outcome: the optimal asset-based SDF consists of only 5 factors that can explain 62.4% of the variation. Only market, 12-month momentum, return on equity, asset growth, and revenue surprise are required in the asset-based SDF.

Figure 1

This figure compares asset-based and equity-based SDFs by their cross-validated R^2 to explain the 100 characteristics-managed factors when the number of factors increases. Penalties are applied according to KNS. Factor selection and coefficient estimation follows the LARS-EN algorithm. The red texts indicate when corresponding factors first enter the asset-based SDF.



I show that the asset-based SDF is closer to the efficient frontier than the equity-based SDF, even though it entails a trading strategy that utilizes significantly fewer observable firm characteristics. In the OOS test from 2005 to 2022, the asset-based implied MVE portfolio achieves a Sharpe ratio of 1.20, compared to 0.75 for the equity-based portfolio, similar to the 0.71 in KNS. Moreover, the asset-based MVE portfolio outperforms the equity-based version by 7% in annualized alpha over the same period, irrespective of the benchmark. On the other hand, as the benchmark shifts from the equity market portfolio to the Fama-French 4 Factors and to the asset market portfolio, the magnitude and statistical significance of the alphas progressively decrease.

To better isolate anomalies from the market risk premium, I orthogonalize all factors against the market factor and re-estimate the SDFs using beta-neutral factors. In the optimized asset-based model, the number of factors is further shrunk to three: 12-month momentum, return on equity, and revenue surprise. However, the model's R^2 drops from 62.4% to 20.0% and allowing more factors only further decreases the number. In contrast, the optimal equity-based SDF with beta-neutral factors retains 99 factors, explaining 39.5% of the variation, similar to the 36.9% before orthogonalization. These findings suggest the market plays a more critical role in the unlevered space, as removing it significantly reduces the model's pricing power. Many factors are relevant only because of their market components. These results point to a more parsimonious CAPM-like model in the asset return space.

I find the asset-based model price equity returns better than equity-based model. In the out-of-sample, I regress each of the 100 equity-based factors on the following three benchmarks: equity market portfolio, equity-based MVE portfolio, and asset-based MVE portfolio. The numbers of statistically significant alphas are 69, 23, 15 respectively. The asset-based model, despite having only 5 characteristics-managed factors, performs slightly better than the equity-based 88-factor model.

Overall, asset-based factor models provide sparser and closer representations of the marginal rate of substitution than the traditional equity-based models. Equities can be viewed as call options on the firm's underlying assets, with the face value of its debts as the strike price. Its expected return is non-linearly magnified by debt. Ignoring this transformation risks exposing characteristics-managed factors to unintended systematic risks and introducing redundant factors.

Literature Review

Doshi et al. (2019) attribute certain stock pricing anomalies to the non-linear transformation of asset returns. However, their strategies of portfolio sortings and Fama and MacBeth (1973) (hereafter "FMB") regressions limit the joint test of a wide collection of factors due to overfitting, and therefore are silent on the number of priced risks in the SDF. In contrast, the technique I adopt from KNS allows me to jointly test 100 factors and compare the performance of asset-based SDF against equity-based SDF in the out-of-sample. I show that the leverage effect contributes to dozens of factors in the factor zoo.

Recent literature also starts to implicitly address the leverage effect by acknowledging that securities representing claims on the same underlying assets of firms should be priced under the same framework. Bali, Beckmeyer, and Goyal (2023) and Chen et al. (2024) reveal pervasive common risk factor structures that jointly explain the risk-return tradeoff across stocks, bonds, and options. However, the techniques employed in their papers, Instrumented-PCA (Kelly, Pruitt, and Su, 2019) and Regressed-PCA (Chen, Roussanov, and Wang, 2023), embody the opposite treatment of signals compared to unlevering. These noval PCA methods do not aim to preserve observable characteristics. Instead, they aggregate risk proxies from more cross-security information. Higher-ordered PCs are by construction better than characteristics-based factors in summarizing variations and these papers essentially expand the ingredients from which PCs are sourced to proxy firm-level risks. In contrast, my paper explains the cross section with fewer characteristics, not more.

This paper also revisits the stream of literature on the factor zoo of stock pricing. For example, Feng, Giglio, and Xiu (2020) account for model selection mistakes due to omitted variables and raise the bar to evaluate marginal contribution of new factors. Kozak, Nagel, and Santosh (2020) obtain a sparse SDF by aggregating information of 49 predictors into a small number of PCs. In contrast, this paper makes an economically motivated attempt to reduce the dimensionality by targeting the non-linearity introduced by the option-theoretic feature of equity.

This paper also contributes to the expanding body of literature within machine learning applications to empirical asset pricing. Many techniques requires as many signals as possible and achieve great success in predicting returns (Gu, Kelly, and Xiu, 2020; Bali et al., 2023; Bianchi, Büchner, and Tamoni, 2021). My paper aims to explain rather than predict returns. Specifically, I retain characteristics-managed factors before my estimation and distinguish risk prices (systematic risks) and risk quantities (leverage-induced amplification) with machine learning techniques. I shrink the cross section to a small number of interpretable factors.

Last but not least, this paper connects to the theoretical literature that rationalize a small number of factors to sufficiently describe the SDF (Lin and Zhang, 2013; Carlson, Fisher, and Giammarino, 2004; Zhang, 2005). These papers speculate what are the fundamental sources of risks and are predominantly silent on capital structure, suggesting that empirical tests should be conducted at the firm level using asset returns rather than equity returns (Hou, Xue, and Zhang, 2015). I propose a sparse asset-based SDF with 5 characteristics-managed factors, rather than latent ones. This facilitates clearer economic interpretation of the surviving factors and offers empirical evidence to future theoretical models.

The remainder of the paper is structured as follows: Section I discusses why constructing characteristics-managed factors using equity returns is problematic. Section II introduces the Bayesian approach for factor selection and SDF estimation. Section III describes the data and the unlevering process. Section IV reports and compares the optimal models for equity- and asset-based SDFs, as well as the out-of-sample performance of their implied MVE portfolios. Section V shows the robustness test with two alternative unlevering methods. Section VI concludes.

I. Factor-Mimicking Portfolios in Two Return Spaces

I start by outlining the basic framework for characteristics-based asset pricing. Next, I explain the leverage-induced non-linear transformation of equity returns and analytically prove that in an equity-based single factor model is not sufficient to explain an asset-based single factor economy. Finally in the section, I conduct a simulation in an one-factor economy and reveal a non-existent anomaly (alpha) that an equity-based SDF fails to explain.

A. Characteristics-Managed Portfolios as Factors

Let r_t denote the stack of N excess returns. The conditional pricing equation is:

$$0 = \mathrm{E}_{t-1}\left[M_t \cdot r_t\right] . \tag{1}$$

Along the lines of Hansen and Jagannathan (1991), one can find an SDF in the linear span of excess returns as[1]

$$M_t = 1 - a'_{t-1}\left(r_t - \mathrm{E}_{t-1}[r_t]\right) \tag{2}$$

by solving the $N \times 1$ vector a_{t-1} that satisfies Equation (1). To obtain models with empirical content, characteristics-based asset pricing models parametrize the SDF loadings as

$$a_{t-1} = Z_{t-1} b \tag{3}$$

[1] Appendix D details the derivation.

where Z_{t-1} is a $N \times H$ matrix that collects measureable characteristics such as size, book-to-market ratio, profitability, etc. The idea is to reduce the SDF's dimensionality from N to H, which is the number of characteristics that predict cross-sectional returns. This leads to an unconditional pricing equation

$$M_t = 1 - b'(F_t - \mathrm{E}_{t-1}[F_t])\,, \tag{4}$$

where

$$F_t = Z'_{t-1} r_t\,. \tag{5}$$

Each entry in the $H \times 1$ factor matrix F_t is a linear combination of excess returns weighted by one characteristic, forming a characteristic-managed portfolio. In empirical tests, each factor is built as a zero-investment long-short portfolio, such as SMB and HML, whose composition changes every period in response to the characteristic. It aims to capture risks beyond the systematic risks explained by other factors. While researchers have focused extensively on which characteristics to include in Z_{t-1}, the equally important decision of which r_t to use when constructing the SDF is often overlooked.

Consider building an equity-based factor. Debt amplifies the quantity of risks borne by firms. If the characteristic correlates with leverage, firms in one leg of the factor on average are exposed to higher systematic risks that are already priced by other factors. This creates a mismatch between the long and short legs, exposing the factor to systematic risks unrelated to the corresponding characteristic.

B. Leverage Effect on Returns and Equity-Based Factors

This subsection decomposes equity returns into asset returns and the leverage effects and shows that if the economy is priced by a single characteristic-managed factor built from asset returns, it cannot be priced by the its equity counterpart.

To simplify the analysis of leverage effect, I assume the risk-free rate in the economy is 0 and consider the classic model from Black and Scholes (1973) and Merton (1974) (hereafter BSM). The distribution of firm assets value at the end of any finite time interval is log normal. The variance of the rate of return on the firm's assets is constant.

A firm's equity can be viewed as a call option on the firm's underlying assets, with the face value of its debts as the strike price. The relationships between the risk premia of these securities are highly non-linear. It is easier to see the decomposition of the expected excess return in continuous time:[2]:

$$\mathrm{E}\left[\tilde{r}_e\right] = \mathrm{E}\left[\tilde{r}_a\right] + \left[\mathcal{N}(d_1)\frac{V}{E} - 1\right]\mathrm{E}\left[\tilde{r}_a\right] \,, \tag{6}$$

where E is the market value of equity, V is the market value of the firm, $\tilde{r}_a$ is the instantaneous excess return of asset, $\tilde{r}_e$ is the instantaneous excess return of equity, and $\mathcal{N}(d_1)$ is the probability of default (delta). The first term on the right-hand side of Equation (6) compensates for firm-level risks, and the second term compensates for the leverage risk borne by shareholders, which is an amplification of firm-level risks. The effect is non-linear in the debt.

Figure 2 Panel (a) illustrates the relationship between expected asset, equity, and debt returns at debt maturity. Similar figure is presented in textbooks such as Berk and DeMarzo (2007) on capital structure.

[2] Appendix A provides detailed derivation.

Figure 2: Leverage and Cost of Capital

Panel (a) of this figure illustrates the relationships between leverage and cost of asset, equity, and debt as a firm levers up. Leverage is defined as the the ratio of face value of debt (B) over the market value of firm. The excess asset return of the example firm is assumed to have a mean of 6%, a volatility of 10%, and follows normal distribution. Panel (b) of this figure illustrates the probability density function for equity returns for two identical firms except for their leverage. Suppose the economy has only two firms, one with 0% and the other with 50% leverage, an asset-based factor $w_1 r_{1,a} - w_2 r_{2,a}$ has no exposure to the systematic risk while an equity-based factor $w_1 r_{1,a} - w_2 r_{2,e}$ has unintended exposure when factors are zero-investment portfolios $w_1 = w_2$.

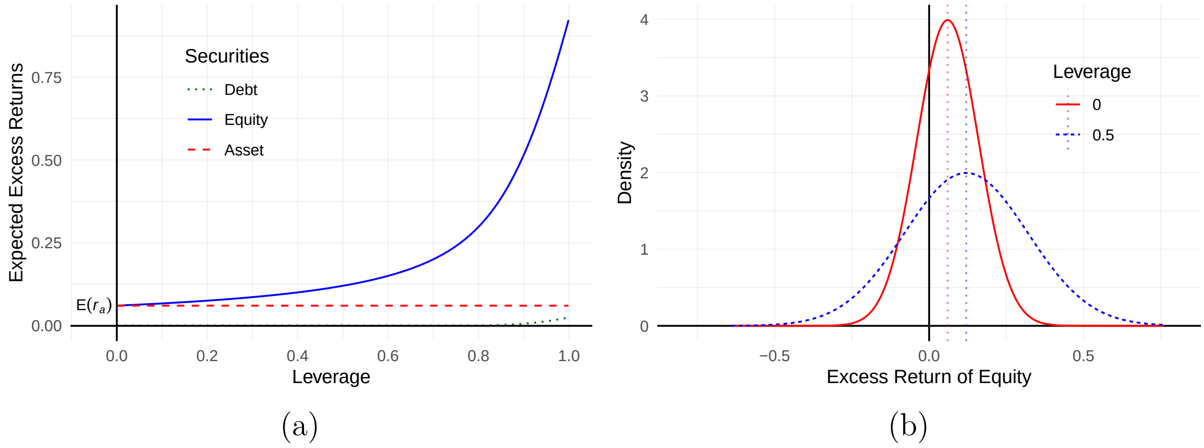


Figure 2 Panel (b) reports the distribution of equity returns under two different financial policies. Their difference already provides an intuition why the choice of return space has non-trivial impact on the characteristic-managed portfolios in Equation (5). When a characteristic is correlated to the leverage, the leverage effect is stronger for firms in one leg of the portfolio. As a result, firm weights are determined not just by the characteristic but also by leverage, effectively overweighting risks borne by higher-levered firms.

Officially, consider a single factor economy with N firms. The factor is $F = w' r_a$ where w is the $N \times 1$ weight vector and r_a is the $N \times 1$ asset return vector. The beta representation of Equation (4) is [3]

$$\bar{r_a} = \frac{\Sigma w}{w'\Sigma w} w'\bar{r_a}\,, \tag{7}$$

where $\bar{r_a}$ and Σ are mean and covariance matrix of r_a.

Denote equity returns as r_e, and $r_e = Dr_a$ where D is a diagonal matrix. Each element in D refers to the leverage-induced return transformation for the corresponding firm in r_a. The covariance matrix of r_e is $D\Sigma D$. If the equity-based single factor SDF that is managed by the same characteristic (same weights w) is also feasible, then the following equation must hold:

$$D\bar{r_a} = \frac{D\Sigma Dw}{w'D\Sigma Dw} w'D\bar{r_a}\,. \tag{8}$$

However, the only solution to Equation (8) is when D scales the $\bar{r_a}$ and Σ proportionally. D is a scalar multiple of the identity matrix I:

$$D = cI\,. \tag{9}$$

From Equation (6), we know that D does not satisfy the condition unless there is no cross-sectional variation in leverage and volatility. In other words, we cannot use the same characteristic-managed portfolio, but built with equity returns, to recover the SDF. Note that since my paper focuses only on characteristics-managed factors that are interpretable, I do not rescale w, which would lead to a latent factor.

I conduct a Monte-Carlo simulation to show the consequence of building an equity-based factor in an asset-based single factor economy. The first and second moments of the simulated factor are 10% and 20%. The moments of leverage matches its empirical

[3] j-th row of $\bar{r}_a$ is: $\mathrm{E}[r_{a,j}] = \frac{\mathrm{Cov}[F, r_{a,j}]}{\mathrm{Var}[F]}\mathrm{E}[F] = \beta_{j,F}\mathrm{E}[F]$.

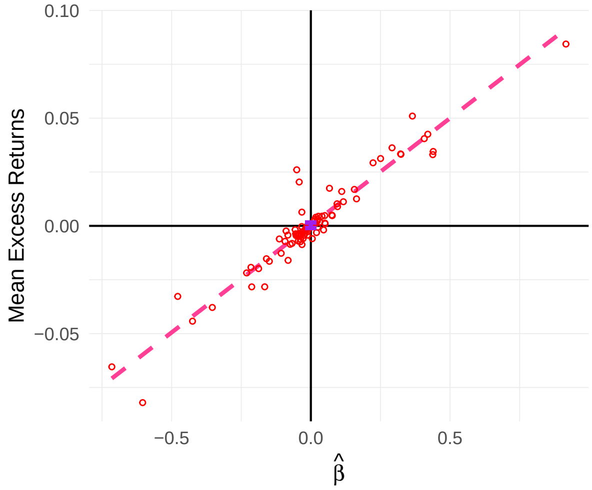
moments in the data[4].

Specifically, I first backout the first and second moments of asset returns for 100 firms such an single factor prices the cross section. Next, I impose leverage structure on these firms. Then, I simulate the asset returns for the cross section for 1000 periods ($r_{a,t}$), and calculate the equity returns ($r_{e,t}$), asset-based factor return ($F_{a,t} = w'r_{a,t}$), and equity-based factor return ($F_{e,t} = w'r_{e,t}$). Finally, I run FMB regressions in the asset and equity space respectively. In the first step, time-series regressions are performed to estimate factor loadings (betas) for each asset. In the second step, cross-sectional regressions use these estimated betas to estimate the risk premia (factor prices) across different time periods, averaging the results to obtain standard errors. Figure 3 reports the second stage for the FMB methods. The slope is the estimated price of the risk and the intercept is the alpha. Panel (a) indicates that the asset-based factor correctly prices the economy with statistically insignificant alpha. On the other hand, Panel (b) suggests an annualized alpha of 2.8%, which actually reflects the non-linear leverage effect.

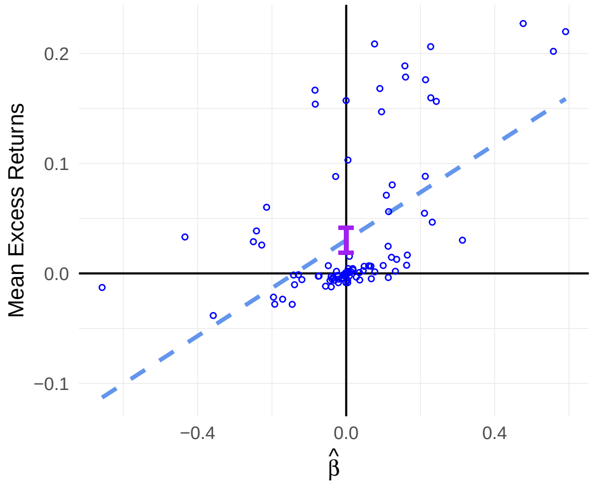
[4] Appendix B details the setup.

Figure 3: A Non-Existent Anomaly

This figure shows the FMB second stage regression results in an asset-based single factor Monte-Carlo simulated economy. Panel (a) reports the cross-sectional regression result of $\bar{r}_{a,i} = \alpha_a + \lambda_a \hat{\beta}_{a,i}$ where $\hat{\beta}_{a,i}$ is the coefficient estimate for the time series regression $r_{a,i,t} = \eta_{a,t} + \beta_{a,i} F_{a,t}$. Subscripts a, i, t represent asset returns, firm identifier ($N = 100$), and time ($T = 1000$) respectively. $F_a = w_1 r_{a,1} + \cdots + w_N r_{a,N}$ is the asset-based factor. Panel (b) reports the same result but for equity returns and equity-based factor. The relations between $r_{e,i}$ and $r_{a,i}$ are derived under Black-Schole-Merton model. While Panel (a) recovers the true SDF, Panel (b) generates an non-existent anomaly that captures the leverage effect.



(a) Asset-Based Factor



(b) Equity-Based Factor

II. A Bayesian Approach for Factor Discipline

If the leverage effect discussed in Section I contributes to the factor zoo, the SDF in the asset return space should be more characteristics-sparse than in the equity space. This requires a regularization technique that can handle the joint evaluation of many factors in a high-dimensional setting, with built-in filters to allow sparsity. Traditional portforlio sortings or FMB regressions struggle to accomplish these tasks. Therefore, I employ the economically motivated Bayesian approach by KNS to regularize the fitting procedure. This section outlines the approach.

Inserting the unconditional pricing equation (Equation (4)) into $\mathrm{E}[M_t F_t] = 0$ solves the coefficient b of the factor model:

$$b = \Sigma^{-1}\mathrm{E}\left[F_t\right] = \left(\Sigma\Sigma\right)^{-1}\Sigma\mathrm{E}\left[F_t\right] \tag{10}$$

where $\Sigma \equiv \mathrm{E}\left[\left(F_t - \mathrm{E}\left[F_t\right]\right)\left(F_t - \mathrm{E}\left[F_t\right]\right)'\right]$. b is the coefficients in a cross-sectional regression of the factors' population mean on its variance-covariance matrix. Empirically, regressions of the sample equivalents[5] are used to estimate the coefficients

$$\hat{b} = \bar{\Sigma}^{-1}\bar{\mu} = \left(\bar{\Sigma}\bar{\Sigma}\right)^{-1}\bar{\Sigma}\bar{\mu}\,. \tag{11}$$

A. Economically Motivated Bayesian Model by KNS

The risk of overfitting becomes substantial when a large number of candidate factors are considered. With the expansion of a factor model comes a higher propensity of picking up noises and performing poorly in the out-of-sample. Regularziation methods are needed for model selection and mitigation of overfit. KNS argue that the main source of overfit comes from the sample mean estimator $\hat{\mu}$, not from covariance. I proceed under the assumption that $\bar{\Sigma} = \Sigma$ and tackle with the imprecision of $\bar{\mu}$ by introducing a prior[6]:

$$\mu \sim \mathcal{N}\left(0, \frac{\kappa^2}{\tau}\Sigma^2\right) \tag{12}$$

where $\tau = \mathrm{tr}\left[\Sigma\right]$ is the trace of the variance-covariance matrix and κ governs the strength of the prior. This prior implies an economically plausible notion that there exists a connection between the first and second moments of factor returns. Specifically, Sharpe ratios of factors associated with high-eigenvalue PCs should be higher than those associ-

[5] $\bar{\mu} = \frac{1}{T}\sum_{t=1}^{T} F_t$, and $\bar{\Sigma} = \frac{1}{T}\sum_{t=1}^{T}\left(F_t - \bar{\mu}\right)\left(F_t - \bar{\mu}\right)'$.

[6] This family of priors is widely used in earlier literature. See Pástor (2000), Pástor and Stambaugh (2000), and Liechty, Harvey, and Liechty (2008).

ated with low-eigenvalue PCs[7]. It is statistically in line with many asset classes including stock returns that a few high-eigenvalue PCs account for most return variance while the contribution of the rest is neglegible.

With normal prior and likelihood, the Bayesian posterior mean and variance of b with a sample size of T are[8]:

$$\hat{b} = (\Sigma + \gamma I)^{-1} \bar{\mu} \tag{13}$$

$$\mathrm{Var}(b) = \frac{1}{T} (\Sigma + \gamma I)^{-1} \tag{14}$$

where $\gamma = \frac{\tau}{\kappa^2 T}$. Compared with $\hat{b}$ in Equation (11), Equation (13) shrinks cofficients toward 0, similar to ridge regressions (Hastie et al., 2009). The effect is disproportionately stronger for factors associated with low-eigenvalue PCs. In simpler words, the KNS estimators are more intolerant toward coefficients of factors that contribute less to the cross-sectional variations. κ (or equivalently, γ) regularizes the fitting process in an economically plausible way. In fact, Equation (13) is the closed form solution to minimizing the HJ-distance (Hansen and Jagannathan, 1991) with L^2 penalty:

$$\hat{b} = \underset{b}{\operatorname{argmin}} \, (\hat{\mu} - \Sigma b)' \, \Sigma^{-1} \, (\hat{\mu} - \Sigma b) + \gamma b' b \, . \tag{15}$$

To implement the estimation, the value of κ is needed. Parameters like κ are called hyperparameters in machine learning language. They control the behavior of a fitting process and are not learned from data directly. Instead, they are tuned between sessions for better OOS performance.

I adopt a standard K-fold cross-validation (CV) method for hyperparameter tuning.

[7]To see this, think of factors as orthogonized PCs and $\Sigma^{-1/2}\mu \sim \mathcal{N}\left[0, (\kappa^2/\tau)\Sigma\right]$.

[8]Inserting the Bayesian posteror $\hat{\mu} \sim \mathcal{N}\left[(\gamma I + \Sigma)^{-1} \bar{\mu}, \frac{1}{T}\left(\Sigma^{-1} + \gamma \Sigma^{-2}\right)^{-1}\right]$ into Equaition (11).

(1) First, I divide the historical data into training and testing period, and further contiguously divide training data into K equal subsamples; (2) Then, for each possible κ, I compute $\hat{b}$ applying Equation (13) for $K-1$ of these subsamples and evaludate the OOS performance on the single withheld subsample via $R^2_{oos} = 1 - [\bar{\mu}_o - \bar{\Sigma}_o \hat{b}][\bar{\mu}_o - \bar{\Sigma}_o \hat{b}]' / [\bar{\mu}_o' \bar{\mu}_o]$ where subscript o indicates sample moments from the withheld subsample; (3) Next, I repeating this procedure K times, each time treating a different subsample as the OOS data. I average the R^2_{oos} across these K estimates; and (4) Finally, I choose the optimal hyperparameter κ that generates the highest average of R^2_{oos} and evaluate the model performance on the testing period.

Throughout the CV process, I chose $K = 3$ following KNS as a compromise between the estimation error in $\hat{b}$ and $\bar{\Sigma}_o$. Based on the prior (Equation (12)), κ has a natural economic interpretation. It is the square root of the expected maximum squared Sharpe ratio:

$$\kappa = \left(\mathrm{E} \left[\mu \Sigma^{-1} \mu \right] \right)^{\frac{1}{2}} . \tag{16}$$

Optimal κ's will be generated during the estimations of asset- and equity-based factors. A higher optimal κ not only suggests a better-behaved data as there is less necessity for regularization (lower λ), but also signals a closer proximity to the efficient frontier.

So far, the Bayesian approach shrinks coefficients to almost but not exact zero, keeping all factors in the model. To reinforce my hypothesis that redundant factors might arise from constructing SDF with equity-based returns, another penalty to filter out certain factors is desirable. I follow KNS and add an additional L^1 penalty on top of L^2:

$$\hat{b} = \underset{b}{\operatorname{argmin}} \left(\hat{\mu} - \Sigma b \right)' \Sigma^{-1} \left(\hat{\mu} - \Sigma b \right) + \gamma_2 b' b + \gamma_1 \sum_{i=1}^{H} |b_i| \, . \tag{17}$$

Due to the geometry of L^1 norm, Equation (17) accomplishes automatic factor selection without imposing that the SDF is necessarily sparse. In other words, the number of factors with non-zero coefficients is another hyperparameter that is optimized through the CV process and serves as an indicator of the comparison between asset- and equity-based SDFs. I solve the optimization problem in Equation (17) using LARS-EN algorithm (Zou and Hastie, 2005).

III. Data and Asset Returns

I obtain daily stock returns from CRSP for all firms listed in the NYSE, AMEX, and NASDAQ. I supplement the data with the three-month Treasury-bill rate from FRED as proxy for risk-free rate from which I calculate individual excess returns.

While there are at least hundreds of stock-level predictive signals in published research[9], I build upon the list of Green, Hand, and Zhang (2017) after weighing feasibility and quality. I construct a large set of 100 firm-level characteristics based on the cross-section of stock returns literature[10]. Appendix E lists the source to these characteristics. Data on firm equities, financial statements, and macroeconomic variables is retrieved from CRSP, Compustat, and Amit Goyal[11] to build the characteristics. To obtain predictor matrix on a daily frequency, I forward fill quarterly or annual accounting-based characteristics.

I exclude in the data financial firms with SIC code between 6000 and 6999 and small-firms whose market caps are below 0.01% of the aggregate market[12]. My sample begins in January 1951 and ends in December 2022 (71 years) and includes 7422 firms that on average account for 74.4% of the total market value.

[9]Harvey, Liu, and Zhu (2016) studied 316 firm characteristics and common factors.
[10]The Machine Learning Toolbox by Adrien d'Avernas, Martin Waibel, and Chunjie Wang.
[11]Amit Goyal's website.
[12]The illiquidity from small stocks might contaminate the analyses. Financial firms that can sustain very high leverage might also drive the results. I include financial firms in robustness tests, in which the sample expands to 8004 unique stocks and accounts for 90.3% of the total market value on average. The results persist.

Now that I have obtained equity returns and firm-level characteristics, I apply the Merton (1974) model to unlever the returns. This model is the simplest baseline to account for the leverage effect. If this straightforward approach already produces a sparser SDF, it would support my argument that the factor zoo is partly driven by the non-linearity of return transformation. In Section V, I use two additional methods as robustness checks: first, by assuming corporate debts are risk-free, and second, by reconstructing asset returns using corporate bond data. Both yield results consistent with the Merton case.

Asset returns are the changes in market value of firms. Under the assumption that the total value of a firm follows GBM, the Merton model argues that the equity of the firm is a call option on the underlying value of the firm with a strike price equal to the face value of the firm's debt that can be priced by

$$E = V\mathcal{N}(d_1) - e^{-r_f T} B\mathcal{N}(d_2) \tag{18}$$

$$d_1 = \frac{\ln(V/B) + (r_f + 0.5\sigma_v^2)T}{\sigma_v \sqrt{T}} \tag{19}$$

$$d_2 = d_1 - \sigma_v \sqrt{T} \tag{20}$$

where E is the market value of equity, V is the market value of the firm, σ_v and σ_e are the volatilities of the assets and equities, B is the face value of debt, T is debt's time-to-maturity, and r_f is the instantaneous risk-free rate. The model also implies that σ_e and σ_v are related by[13]:

$$\sigma_e = \left(\frac{V}{E}\right) \mathcal{N}(d_1)\sigma_v \,. \tag{21}$$

The Merton model translates the volatility and market value of equity into those of firm's asset with Equation (18) and (21). All variables except σ_v and V are either known or can

[13] Under the GBM assumption, the equity value satisfies the time-series process: $dV = \mu_v V dt + \sigma_v V dz$ where dz is a standard Wiener process. It follows from Ito's Lemma and Equation (18) that $\sigma_e = \left(\frac{V}{E}\right)\frac{\partial E}{\partial V}\sigma_v$. In the BSM model, $\frac{\partial E}{\partial V} = \mathcal{N}(d_1)$.

be estimated: (i) E is the product of the firm's shares outstanding and its current stock price; (ii) σ_e is measured by the annualized realized volatility of daily stock returns in each month; (iii) B is the sum of the firm's current liabilities and one half of its long-term liabilities; (iv) r_f is measured by the annualized return on three-month Treasury-bill rate; and (v) $T = 1$.[14]

Instead of solving this two-equation system directly, I implement the iterative procedure in Vassalou and Xing (2004) and Crosbie and Bohn (2003)[15] to avoid a statistical challenge posed by acute movements of market leverage: (1) I guess an initial value of $\tilde{\sigma}_v = \sigma_e[E/(E+B)]$ and insert it into Equation (18) to infer the market value of each firm $\tilde{V}$ every day for the previous month; (2) I calculate the implied log return on assets each day and and use the returns series to generate new estimates $\tilde{\sigma}_v$; and (3) Iterate the steps until $\tilde{\sigma}_v$ converges so the absolute difference in adjacent $\tilde{\sigma}_v$'s is less than 10^{-3}.

Now that I obtain a panel of asset returns $(r_{a,t})$ from converged $\tilde{V}$, the last step before implementing the estimation from Equation (17) is to build factors. I rank-normalize the predictor matrix Z_{t-1} such that each factor is a zero-investment long-short portfolio. For each predictor at each time, I obtain the rank-transformed value as the ratio of a firm's rank in the predictor over the number of firms. Next, I normalize the value by first demeaning the rank-transformed predictor cross-sectionally then dividing the value by the sum of absolute deviations from the mean of all firms. Along with the characteristics-based factors, an additional market factor is added to capture the level of risk premia. I

[14]The estimations of T and B are also adopted by Chang, d'Avernas, and Eisfeldt (2021) and Gilchrist and Zakrajšek (2012). As a robustness check, I also apply another set of estimations following Bharath and Shumway (2008) in which B is the total liabilities and σ_e is measured by annualized realized volatility of daily stock returns in each year. The results remain consistent.

[15]Bharath and Shumway (2008); Gilchrist and Zakrajšek (2012); Chang, d'Avernas, and Eisfeldt (2021) also adopt this procedure.

can then construct asset-based factors $F_{a,t}$ following

$$F_{a,t} = Z'_{t-1} \cdot r_{a,t} \,. \tag{22}$$

Testing $F_{a,t}$ under the Bayesian approach discussed in Section II allows me to compare the number of factors in the optimized model and its out-of-sample performance with its equity counterpart

$$F_{e,t} = Z'_{t-1} \cdot r_{e,t} \,. \tag{23}$$

The Merton model entails the assumption of full market integration. Stock holders have complete access to the debt market of firms and there exist SDFs that price stock, bond, and loan market simultaneously. In reality, however, loan lenders are almost exclusively banks and the majority of corporate bonds are held by institutions such as insurance companies and pension funds. Debt markets are also much more geographically segmented compared to the stock market. Even though households have access to the debt market through bond ETFs and certain mutual funds, they usually trade a portfolio of diversified bonds that do not allow flexible weight adjustments to reconstruct asset returns for specific firms. Consequently, $F_{a,t}$ is not directly tradable as it is theoretical. Despite these concerns, This method provides a solid benchmark and computational convenience. It does not require debt data, which tends to be incomprehensive and sometimes erroneous. It provides asset returns for almost all public firms on a daily basis from decades before the advent of a remotely reliable corporate bonds dataset. Therefore, I use Merton model as the baseline.

IV. Empirical Results

A. Model Selections

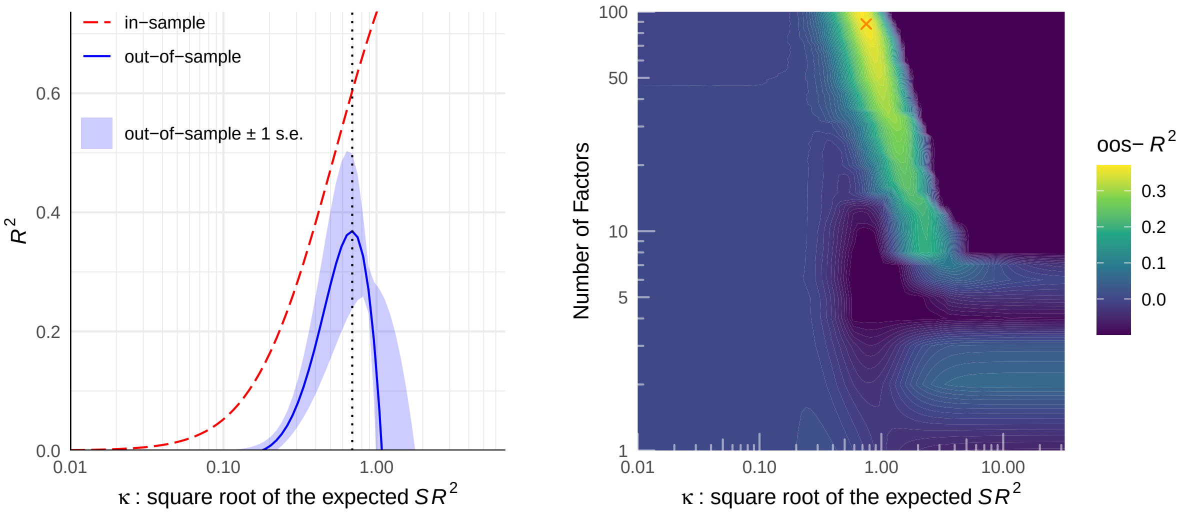
Figure 4 presents the OOS R^2 from the CV process for equity-based factors under both L^2-only and L^1-L^2 specifications with a range of hyperparameters.

When all factors are considered, the data calls for a sizable L^2-shrinkage to explain 36.8% of the OOS variation (Panel (a)). The in-sample (IS) R^2 decreases as I address the concerns of overfitting by imposing higher strength of the penalty in the Bayesian approach (higher λ and lower κ). As κ approaches 0, I use little IS information during coefficients estimation thereby IS R^2 converges to 0. On the contrary, relying too much (large κ) or too little (small κ) IS data leads bad OOS explanation. The model is optimized when κ is set around 0.69.

Similar OOS R^2 is only possible with the inclusion of most factors after allowing for sparsity (Panel (b)). The optimal number of factors is 88, indicating little redundancy across factors. A small subset of these portfolios cannot span the SDF regardless of κ. Forcing a sparse model would risk losing pricing information, shown as the significant drop in OOS R^2 moving down the plot. Despite my longer sample and wider collection of anomalies, the result is consistent with KNS that showed all 49 factors they built survived the shrinkage.

Figure 4: Equity-Based Factors F_e: R^2 under Singular- and Dual-Penalty

This figure presents the R^2 that explain the cross-sectional variation of 100 equity-based daily factors from 1951 to 2022. The OOS R^2 is derived from a 3-fold cross validation process under different combinations hyperparameters. Panel (a) only employs L^2 penalty of which the strength is measured by prior root expected SR^2 (κ). Panel (b) also employs L^1 penalty of which the strength is measured by the number of factors. Hyperparameters corresponding to highest OOS R^2 are marked in the figure. Axes of hyperparameters are ploted on logarithmic scale.

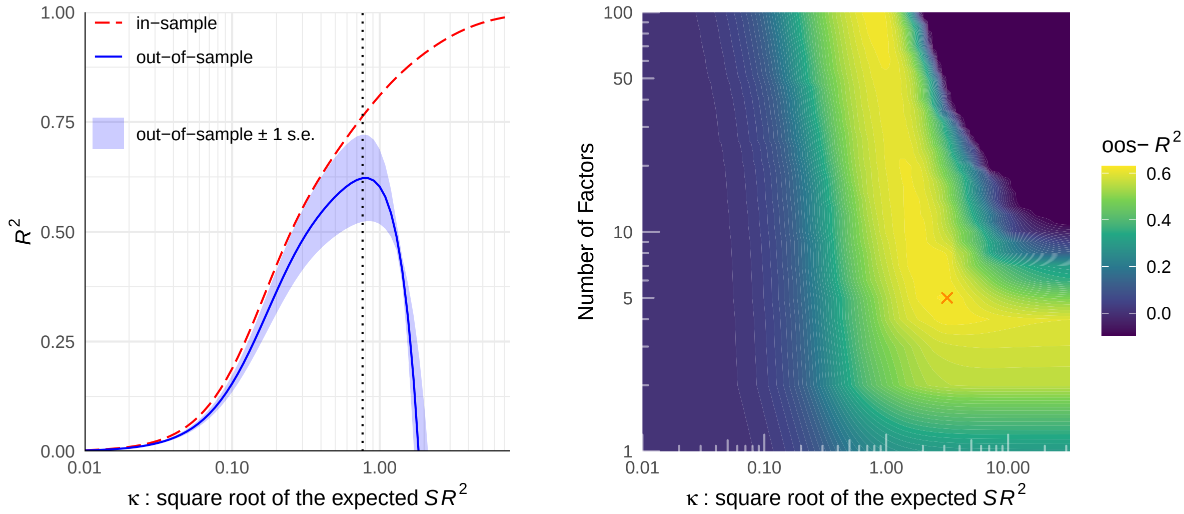


(a) L^2 penalty (b) L^1-L^2 penalties

Figure 5 presents the OOS R^2 under the same specifications and hyperparameters for asset-based factors. The situation is quite different: OOS R^2 is overall higher, indicating that asset-based factors in general carry more pricing information. While the optimal κ under singular penalty is comparable to its equity-based counterparty (0.77 versus 0.69), it is much higher under dual-penalty (3.16 versus 0.75), indicating that less supervision from L^2 penalty is needed when training F_a: the concern of overfitting is milder and there is less noise in this return space. Most importantly, It only requires 5 factors to peak the OOS R^2 at 62.4%: market, 12-month momentum, return on equity, asset growth, and revenue surprise. Compared with the optimized 88-factor equity-based SDF with 36.9% OOS R^2, asset-based factors call for a much sparser SDF and explains a much higher proportion of OOS variations. High R^2 in the contour plot covers a much wider area compared to Figure 4, implying some robustness in the vinicity around the optimal specification: it is inconsequential whether to include or exclude a few anomalies, or whether to have a slightly different root expected SR^2. In other words, additional factors provide little marginal benefit. They might introduce unnecessary noises and overfit the IS data, thus requiring higher level of regularization evidenced by the growing optimal L^2 penalty as more factors are included (the yellow strip runs from the southeast to the northwest in Figure 5 Panel (b)). Summing up, asset-based factors explain higher percentage of the cross-sectional variation with fewer factors, suggesting some anomalies in the literature are due to the omission of non-linear transformation of returns.

Figure 5: Asset-Based Factors F_a: R^2 under Singular- and Dual-Penalty

This figure presents the R^2 that explain the cross-sectional variation of 100 asset-based daily factors from 1951 to 2022. The OOS R^2 is derived from a 3-fold cross validation process under different combinations hyperparameters. Panel (a) only employs L^2 penalty of which the strength is measured by prior root expected SR^2 (κ). Panel (b) also employs L^1 penalty of which the strength is measured by the number of factors. Hyperparameters corresponding to highest OOS R^2 are marked in the figure. Axes of hyperparameters are ploted on logarithmic scale.



(a) L^2 penalty

(b) L^1-L^2 penalties

I then compare the asset- and equity-based factors in the optimized models. Equation (14) gives a closed form solution to the posterior standard error for the coefficient estimates under the singular penalty of κ. I report top 10 most significant factors in Table I for both sets of factors. Market and momentums are priced high in both return spaces. The t-statistics are low for most factors, but what is important is the joint significance of these factors and the explanatory power of the SDF constructed from them.

Table I: Coefficient Estimates under L^2 Penalty

Under a singular regularization of root expected SR^2 (κ), this table lists top 10 of 100 coefficient estimates and t-statistics corresponding to the CV-implied optimal prior, sorted by the absolute values of the t-statistics.

equity-based factors (F_e)			asset-based factors (F_a)		
factors	b	t-stat	factors	b	t-stat
market	4.181	4.458	12-month momentum	4.737	2.696
12-month momentum	3.551	2.654	revenue surprise	4.283	2.260
1-month momentum	-3.211	-2.451	return on equity	3.408	1.797
change in 6-month momentum	-2.915	-2.129	market	2.013	1.774
change in shares outstanding	-2.254	-1.521	6-month momentum	2.499	1.416
earnings to price	1.907	1.319	size (industry-adjusted)	-2.395	-1.260
R&D to sales	1.646	1.190	change in employees (industry-adjusted)	2.209	1.156
return on equity	1.626	1.142	volatility of liquidity (share turnover)	2.193	1.151
maximum daily return	-1.643	-1.123	change in 6-month momentum	-1.978	-1.107
number of earnings increase	1.594	1.065	number of earnings increase	2.088	1.082

A rank of factor importance when allowing for sparsity is more informative. Without a closed-form solution to the standard error of estimates under dual-penalty similar to Equation (14), I rank the factor importance by their earliest entry into the the SDF when allowing higher dimensions (moving from bottom to top in the contour plots in Figure 4 and 5). Comparing components in levered and unlevered sparse SDFs of the same length, factors only appearing in the equity-based SDF are either capturing economically-founded risks that are specific to the stock market, or the non-linear return transformation from the leverage. I rank each characteristic by its first entry into the SDF when relaxing the number of admitted factors, setting κ corresponding to the number at optimum. Table II demonstrates top 10 factors for each return spaces. For instance, most momentum-based factors appear in a sparse stock pricing model but all are trivial after unlevering except 12-month momentum. On the other hand, there is evidence that labor, investment, and profitability measures are of much higher ranks among asset-based factors.

Table II: Factor Importance Ranked under Dual Penalty

This table reports top 10 factors ranked by the their earliest entry into the SDF when allowing for higher number of factors under dual-penalty. Factors are selected to generate highest OOS R^2 from 3-fold cross validation process with 100 candidate factors. The other hyperparameter root expected SR^2 (κ) is set at respective optimum.

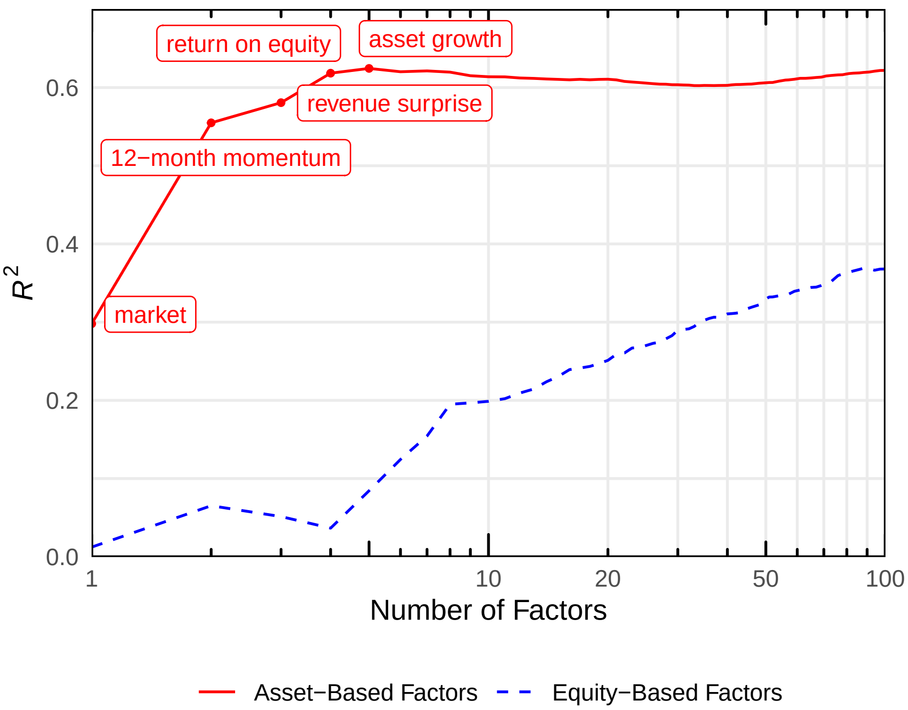
ranking	equity-based factors (F_e)	asset-based factors, Merton (F_a)
1	market	market
2	12-month momentum	12-month momentum
3	1-month momentum	revenue surprise
4	6-month momentum	return on equity
5	change in shares outstanding	asset growth
6	sales to price	employee growth rate
7	industry momentum	change in employees (industry-adjusted)
8	change in 6-month momentum	earnings volatility
9	earnings to price	maximum daily return
10	maximum daily return	size (industry-adjusted)

Even though it is out of the scope of this paper to inspect the economic interpretation behind how specific factors might have unintendedly capture the leverage effect, rather, I aim to measure the joint severity of the issue, this exercise still sheds a light on what anomalies need further investigation on why they diverge in their ranks in asset versus stock pricing.

To facilitate the comparison between factor sets, Figure 6 extracts a slice from the contour plots from Panel (b) of Figure 4 and 5 along the optimal κ for a given number of factors. As the figure shows, OOS R^2 only starts rising substantially for equity-based factors toward the right of the plot when more than 4 factors are admitted. This is consistent with KNS that showed it is never too much to add an additional factor proposed from the literature into the equity-based SDF. There is little redundancy in the return space. In constrast, a 5-factor model accounts for most variation in the asset return space and very sparse models perform remarkably well. The marginal effect of adding an additional factor into the SDF is trivial after the 5-factor model and its OOS R^2 is much higher than that of the equity-based SDF. An optimized asset-based 5-factor SDF explains more variation than an equity-based 88-factor SDF optimized under the same approach.

Figure 6: OOS R^2 to Explain 100 Managed Portfolios

This figure compares asset-based and equity-based SDFs by their cross-validated R^2 to explain the 100 characteristics-managed factors when the number of factors increases. L^2 penalty level κ is optimized for respective number of factors. Factor selection and coefficient estimation follows the LARS-EN algorithm. The red texts indicate when corresponding factors first enter the asset-based SDF.



Asset growth measures investment while revenue surprise and return on equity measures profitability. Therefore, the 5-factor asset-based model that consists of 12-month momentum, return on equity, asset growth, and revenue surprise also provides some evidence on production-based theories such as Hou, Xue, and Zhang (2015) and Lin and Zhang (2013) that argue high investment relative to low expected profitability must imply low costs of capital, and low investment relative to high expected profitability must imply high costs of capital.

Market captures the level of equity or asset risk-premia. To focus on understanding the factors that help explain cross-sectional anomalies, I orthogonize every characteristics-based factors with respect to the market factor to examine the incremental power of other factors[16]. I denote these beta-neutral factors as $\tilde{F}_{e,t}$ and $\tilde{F}_{a,t}$ and employ the exact same Bayesian methods.

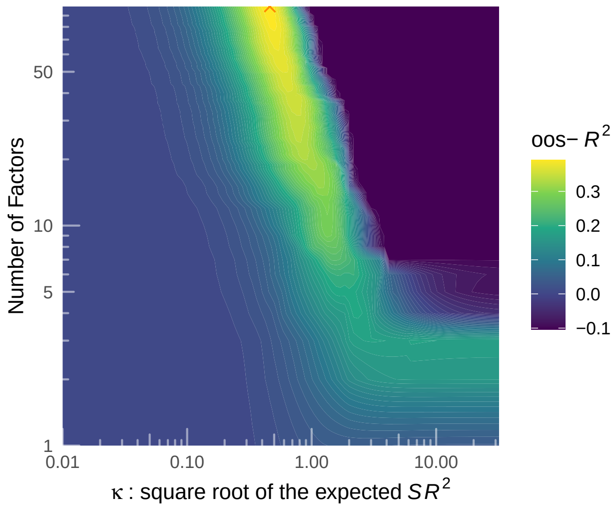
Figure 7 presents the OOS R^2 from the CV process for equity- and asset-based beta-neutral factors under L^1-L^2 specifications with a range of hyperparameters. Similar as before, the data still calls for the inclusion of most equity-based factors, the optimal specification in the asset-based SDF further reduces the dimensionality to three: 12-month momentum, return on equity, and revenue surprise. There are two important differences from F_a. First, the OOS R^2 of $\hat{F}_a$ nose dives across all specifications of hyperparameters after orthogonizing against market. This sharp drop is not observed in the asset-based models. In other words, including market in the factor model significantly enhances the OOS performance in the asset return space, highlighting its contribution to explain the cross-sectional variation. Second, Panel (b) suggests that high OOS R^2 area clusters in the low dimension. In contrast, high OOS R^2 area covers both low and high dimensions when market is included: even though additional factors from F_a add little incremental benefit on top of the optimized 5-factor model, adding more factors does not impede the OOS prediction as long as a higher level of L^2 is imposed. On the contrary, additional factors from $F_{\breve{a}}$ on top of the optimized 3-factor model negative impact in OOS prediction in the beta-neutral case. The two differences are not observed in the equity return space after orthogonizing against the market. These results suggest that there is much less left to explain on top of the market in the asset return space, and many factors are mostly composed of noises after removing the market element from the

[16]For each characteristics-based factor, I run a time-series regression on the market: $F_t \sim \beta \text{MKT}_t + \alpha$; then I calculate the corresponding beta-neutral factor as the remainder: $\tilde{F}_t = F_t - \hat{\beta}\text{MKT}_t$. For equity-based factor set F_e, the market refers to the value-weighted stock market returns; for asset-based sets, it refers to the value-weighted asset returns.

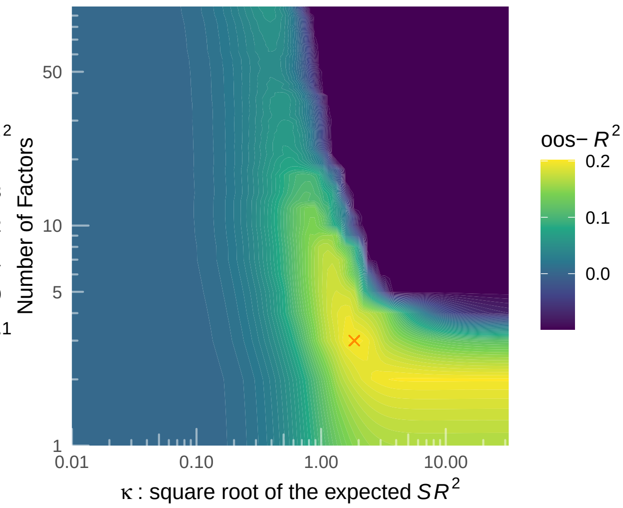
factors because including them exacerbates the over-fitting problem.

Figure 7: OOS R^2 under Dual-Penalty for Beta-Neutral Factors

This figure reports the OOS R^2 under different hyperparameters from 3-fold cross validation process using 100 factors. Panel (a) depicts the result for beta-neutral equity-based factors ($\tilde{F}_{e,t}$) and Panel (b) for beta-neutral asset-based factors ($\tilde{F}_{a,t}$). Hyperparameters corresponding to highest OOS R^2 are marked in the figure. Axes are ploted on logarithmic scale.



(a) equity-based $\tilde{F}_e$

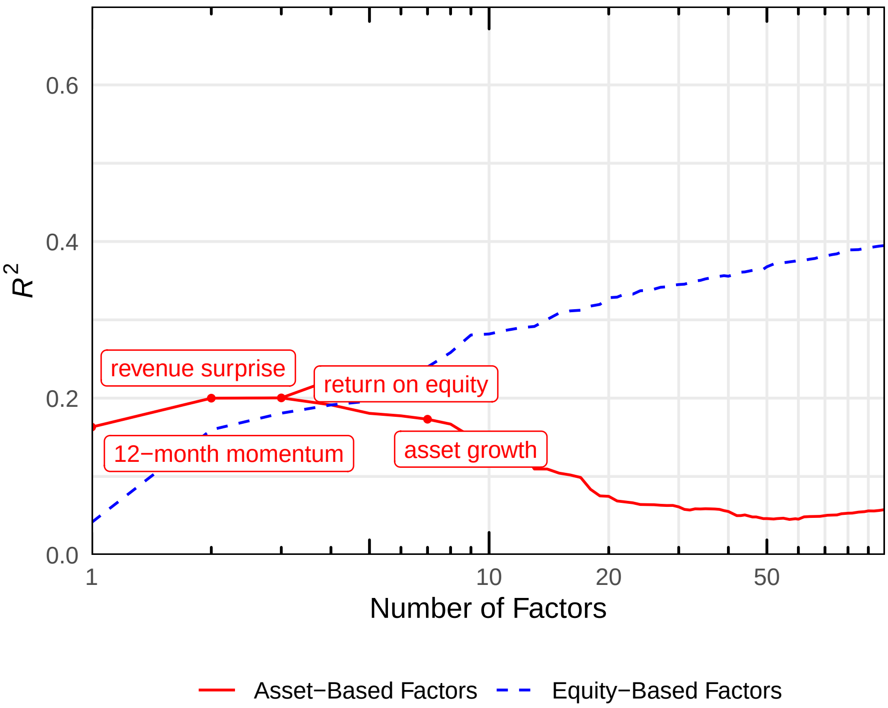


(b) asset-based $\tilde{F}_a$

Similar to Figure 6, Figure 8 takes a cut in the contour plots of Figure 7 along the ridge of maximal OOS R^2 from bottom to top where we optimize L^2 shrinkage (κ) for each level of sparsity. While the trend remains similar for the rest compared to pre-orthogonizing cases, asset-based factors displays an apparent decline. OOS R^2 keeps rising toward the right of the plot by including more equity-based factors. Beta-neutral asset-based factors have lower explanatory power for cross-sectional variations not only compared to beta-neutral levered factors, but also to pre-orthogonizing asset-based factors. These results point to a CAPM-like parsimonious factor model in the asset return space whilst equity-based factor models leave many anomalies unanswered.

Figure 8: OOS R^2 to Explain 100 Managed Portfolios (Beta-Neutral)

This figure compares asset-based and equity-based SDFs by their cross-validated R^2 to explain the 100 characteristics-managed factors orthogonized against the market factor when the number of factors increases. L^2 penalty level κ is optimized for respective number of factors. Factor selection and coefficient estimation follows the LARS-EN algorithm. The red texts indicate when corresponding factors first enter the asset-based SDF.



B. OOS Performance of SDF-implied MVE Portfolios

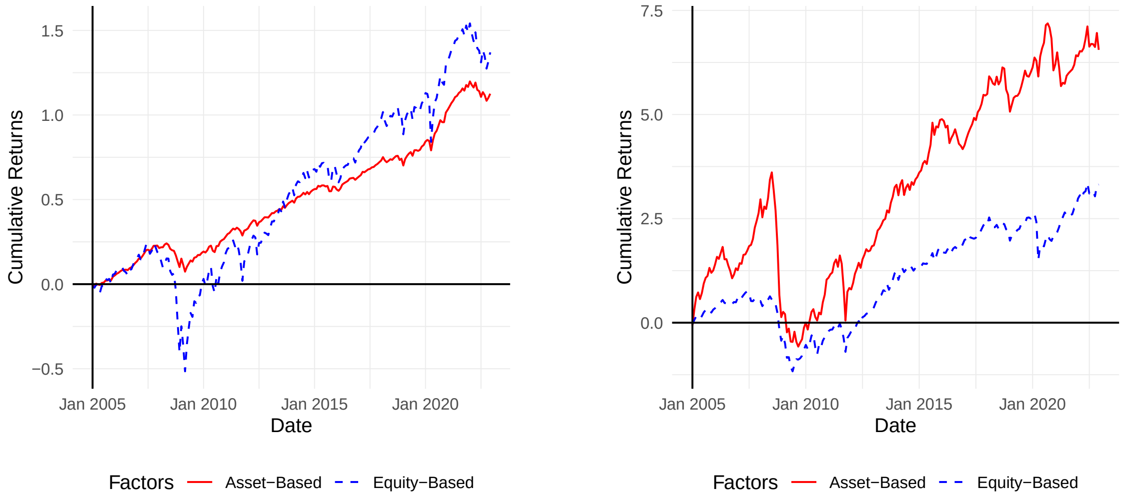
The discussion so far is restricted to hyperparameter tuning. Once κ and number of factors are set to the optimums, I can proceed to build MVE portfolios and compare their OOS performance. I re-estimate $\hat{b}$ under the optimal hyperparameter excluding a time window as testing period. Following KNS, I set the testing period for daily factors from January 2005 to December 2022[17]. The OOS MVE portfolo for each factor set is given by:

$$MVE_t = \hat{b}' \cdot F_t \,. \tag{24}$$

Table III and Figure 9 report the cumulative log returns and the Sharpe ratios for markets and SDF-implied MVE portfolios for factors.

Figure 9: Cumulative Return of Markets and SDFs-implied MVE portfolios

This figure demonstrates the OOS cumulative logarithmic returns of Market (Panel (a)) and SDF-implied MVE (Panel (b)) portfolios from 2005 to 2022. Model hyperparameters (κ and number of factors) are optimized using data from 1951 to 2004.



(a) market portfolios (b) MVE portfolios

[17] In robustness tests, I also set different start dates for the testing period: January 2000 and January 2010. The results persist.

Table III: Cumulative Returns and Sharpe Ratios of MVEs and MKTs in Testing Periods

This table reports the cumulative logarithmic returns and Sharpe ratios of 4 portfolios: market portfolios and implied MVE portfolios in both asset and equity return spaces. MVE portfolios are optimized with sample from February 1951 to December 2004. Empirical standard errors are derived from bootstrapping with 1000 resamples and reported in the parentheses.

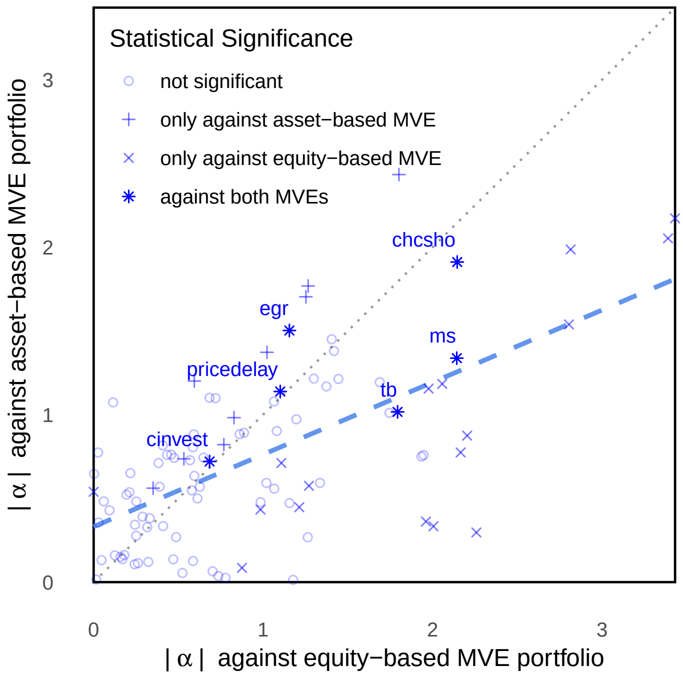
	market portfolios		implied MVE portfolios	
	equity	asset	equity	asset
log cumulative return	1.31	1.10	3.30	6.60
Sharpe ratio	0.46	0.98	0.75	1.20
bootstrap s.e.	(0.24)	(0.24)	(0.23)	(0.25)

Between the market returns, stock market has the highest mean and volatility. This is no surprise because market portfolios long riskier securities and short safer ones. Equities as call options to firms assets, are exposed to higher systematic risks. Under classic asset pricing theory, MVE frontier is an equivalent representation of the SDF thereby carrying the highest Sharpe ratio, thus I focus on comparing the Sharpe ratio between portfolios. In an bootstrapping exercise, I randomly resample returns of the same length with replacement from all spaces and calculate their respective Sharpe ratio. Repeating the step for 1000 times yields empirical standard errors, which I report in the parentheses in Table III. Equity market is less efficient than asset markets, with lower Sharpe ratio. On the other hand, after we consider efficient factor-mimicking portfolios, the cumulative returns all drastically increase. On average equity-based SDF consisting of 88 factors still underperforms compared to 5 asset-based factors that were optimized under the same Bayesian methods (0.75 versus 1.20).

Now I formally inspect alphas for various combinations of testing assets and benchmarks. For each individual equity-based factor, I first regress its excess return on the equity-based implied MVE portfolio using OOS data and collect the resulting alpha. Next, I perform a similar regression of the same excess return but on the asset-based implied MVE portfolio to obtain its alpha. Finally, I compare these alphas. If the number of statistical significant alphas is smaller when tested against one benchmark, it implies that this benchmark serves better as the MVE portfolio. Figure 10 reports the result. Only 15 anomalies are not resolved when tested against asset-based MVE portfolio that is managed by only 5 characteristics, compared to 23 anomalies when tested against equity-based MVE portfolio that is managed by 88 characteristics. There are 6 alphas subsumed by neither benchmarks. They are corporate investment, price delay, earnings growth rate, tax income to book income, financial performance score, and growth in common shareholder equity. Overall, the asset-based MVE implied portfolio not only prices asset returns, it also serves a better benchmark and resolve many anomalies when tested against a wide collection of equity portfolios.

Figure 10: Alphas of 100 Equity Portfolios against Asset- and Equity-Based SDFs

This figure reports the absolute value of annualized alphas in percentage of 100 equity-based characteristics-managed portfolios, tested against the equity-based MVE portfolio (x-axis) and against the asset-based MVE portfolio (y-axis). When the benchmark is asset portfolio, there are only 15 alphas still statistically significant, with an average of 0.74%; when the benchmark is equity portfolio, there are 23 alphas statistically significant, with an average of 0.94%.



(a)

In the previous subsection, I briefly touched upon how unlevering equity returns might bring us closer to CAPM. Therefore, I test MVE protfolios against various markets: the stock market return, Fama-French 4 factors returns, and asset market returns. If my hypotheses are correct that many anomalies arise because they are not tested against the correct market, then assets' α's should drop in both economic scale and statistcal significance along these benchmarkes that employ progressively better measures of the market. I choose the MVE portfolios as testing assets to further investigate their performance.

The results are indeed consistent with my story.

Table IV reports the annualized abnormal returns α's (in %) from time-series regressions of MVE portfolios on benchmark portfolios. Overall, the market portfolios vary from least to most plausible (from top to bottom), as suggested by the drop of magnitude and significance of α for all testing assets; In addition, the testing assets enjoy gradually higher abnormal returns (from left to right), as suggested by the increase of magnitude and significance of α for all benchmark portfolios. To be more specific, I unsurprisingly uncover the anomalies studied extensively in stock pricing literature by regressing a 88-factor model on market (1st row, 1st column), comparable to the result in KNS. However, once you correctly construct a new market return in the asset return space, the anomaly vanishes (3rd row, 1st column), thereby CAPM explains away many anomalies proposed in the literature. There still exists unexplained returns: after all, this improved market return cannot explain all the variations of the asset-based SDF implied MVE portfolios (Panel (a): last row, last column). The good news is, I shrink the 87 cross-sectional anomalies into 4; the bad news is: the alpha is material. Nevertheless, we are much closer to a parsimonious CAPM specification after unlevering.

Table IV: Annualized α of MVE Portfolios against Various Benchmarks

This table measures the distance to CAPM by checking the annualized α (in %) from regressing SDF-implied MVE portfolios on various market portfolios and Fama-French 4 factors (market, size, value, and profitability). The SDFs are optimized on dual-penalty. MVE portfolio and benchmark returns are normalized to have the same standard deviation as the aggregate stock market for better comparison. Standard errors are reported in parentheses.

benchmarks \test assets	equity-based MVE	asset-based MVE
equity market	5.50^{*}	12.76^{***}
	(3.03)	(3.95)
Fama-French 4 Factors	3.53^{*}	9.65^{***}
	(2.03)	(2.70)
asset market	-1.92	7.88^{***}
	(3.20)	(4.10)

*p<0.1; **p<0.05; ***p<0.01

V. Robustness

As robustness tests, I use two alternative methods to unlever equity returns. To further assess the effectiveness of the dimensionality reduction from the unlevering process, I also perform a principal component rotation on equity-based factors, which is designed to produce a more sparse SDF. The PCs of these equity-based factors serve as my third set of factors. In this section, I introduce the introduce the construction of the three sets of factors and then evaluate them under the same Bayesian approach. The results are qualitative the same as the Merton method: fewer asset-based factors are able to better explain the cross section.

A. Unlever with risk-free

This unlevering method omits default risks. This measurement of asset-based factors is computed by

$$F_{\breve{a},t} = Z'_{t-1} \cdot [\mathbf{1}_N - L_{t-1}]' \cdot r_{e,t} \tag{25}$$

where L_t is an $N \times 1$ book leverage matrix defined as the ratio of book liabilities to the sum of book liabilities and market caps. This naïve approach assumes $r_{d,t} = \mathbf{0}_N$ at all times. Due to the cross-sectional heterogeneity of leverage, the difference between the coefficient estimates of $F_{\breve{a},t}$ and $F_{e,t}$ is nontrivial. The risk-free investments in the long leg differ from short leg thereby altering the positions for stocks in the cross section: they are not proportional to Z_{t-1} anymore. Doshi et al. (2019) adopted the method in their primary analyses and found it sufficient to subsume value and volatility premium.

$F_{\breve{a},t}$ stands for the opposite end of market segmentation compared to $F_{a,t}$. $F_{\breve{a},t}$ assumes equity holders have no access to debt markets at all. They can, however, adjust the weights of stocks in their portfolios to roughly approximate asset returns. Unlike $F_{a,t}$, $F_{\breve{a},t}$ is directly tradable in the stock market.

B. Unlever with corporate bonds

In practice, investors especially institutional ones can build unlevered portfolios to certain extent by participating in multiple markets. With bond transactions data from FINRA's TRACE and bond issue and issuer characteristics from Mergent FISD, I can build monthly corporate bond returns and reconstruct empirical asset returns as a mixture of stocks and bonds returns after merging with CRSP.

Unlike equity research that shares a common clean dataset, building empirical corporate bond returns involves numerous active judgements that make the cleaning procedure complex. Transactions in TRACE are self-reported by bond dealers and various types of errors are common. I delete transactions marked as errors using standard filters following the guideline of the WRDS Bond Database (2017). Their procedures are discussed in detail in Dick-Nielsen (2009), Dick-Nielsen (2014), and Asquith et al. (2013). Specifically, after the initial filtering, I merge TRACE by CUSIP with Mergent FISD that contains information on bond issues and calculate the return for bond k in month t as month-over-month percentage price change plus the accrued coupon interest between coupon payment dates:

$$r_{k,t} = \frac{Pr_{k,t} + AI_{k,t} + C_{k,t}}{Pr_{k,t-1} + AI_{k,t-1}} - 1 \tag{26}$$

where $Pr_{k,t}$ is the volume-weighted average price on the last day at which the bond was traded in month t, $AI_{k,t}$ is the accrued interest, and $C_{k,t}$ is the coupon payment, if applicable. The literature treats differently remaining potential errors, especially the outliers. Dick-Nielsen et al. (2023) point out that the treatment of extreme returns are crucial in the pricing power of bond factors. They manually check 5000 largest returns in absolute value and identify 292 errors. Mistakenly deleting or winsorizing the remaining 4708 large returns is likely one of the reasons why pricing powers of most factors fail to replicate. On the other hand, the average (median) absolute returns of the 292 errors is 32000390% (56%) and thus keeping all the outliers is also likely to have a substantial impact on any analysis using the TRACE dataset. WRDS winsorizes returns at 1% level to reduce the impact of extreme numbers while I skip the step. Instead, I identify and delete the 292 erroneous bond-month observations by Dick-Nielsen et al. (2023)[18] and restrict the bond sample by the following criteria: (1) float-rate bonds; (2) not under Rule 144a; and (3) Bond Type is equal to US Corporate Convertible (CCOV), US Corporate Debentures (CDEB), US Corporate Medium Term Note (CMTN), US Corporate Medium Term Note Zero (CMTZ), or US Corporate Paper (CP). I aggregate

[18] In practice, I mark the bond-month observations that would be winsorized in the WRDS bond return database. I remove any marked returns that do not exist in the dataset of Dick-Nielsen et al. (2023).

the firm-level of corporate bond return as

$$r_{b_{i,t}} = \sum_{k \in \mathcal{R}_{i,t}} w_{k,t-1} r_{k,t} \tag{27}$$

where $\mathcal{R}_{i,t}$ is the set of firm i's bonds with non-missing returns during the month t, and $w_{k,t-1} = mv_{k,t-1} / \sum_{j \in \mathcal{R}_{i,t-1}} mv_{j,t-1}$ is the fraction of the market value of bond k among all bonds of firm i in the previous month $t-1$. Next, I can calculate the empirical asset returns on the firm-level as the value-weighted average of stock and bond returns, assuming that the bond returns are representative of the entire debts of the firm. I denote the asset returns as $r_{\ddot{a},t}$. Corresponding factors is given by

$$F_{\ddot{a},t} = Z'_{t-1} \cdot r_{\ddot{a},t} \,. \tag{28}$$

My treatment of bond returns differs from Dick-Nielsen et al. (2023). They only include bonds that are traded within the last five trading days per month. If a firm has a bond that is not traded within the window, they impute this missing return from the observed average bond return using a duration adjustment. In constrast, my dataset has neither the window restriction or imputation. As the corporate bond market does not display high-level of trading activity, I intend to keep as many true transactions as possible so that the corresponding factors serve as a tradable robustness check for the Merton-unlevered factor. This different treatment also introduces a slight gap during the data filtering. There might be outliers in my dataset that have not been manually inspected if they are traded before the window. Nevertheless, I deem tradability the priority for the exercise and this approach represents the most suited option available for this paper.

$F_{\ddot{a},t}$ is not only tradable as a mixture of bonds and stocks, but also properly accounts for the default risks. Despite me keeping more observations, the dataset is notably limited in both cross section and time series. It begins in July 2002 and is only monthly. Daily frequency would further decimate the average number of observations in the sample per period from an already small 663. This limitation directly shakens the estimation of the covatiance matrix of factors $\Sigma_{\ddot{a}}$, whose stability is one of the central pillars of the Bayesian approach. The unlevering approach also relies heavily on the bold assumption that returns backed out from TRACE reasonably represent the compensation for the credit risk when loans are likely to have different superiority than bonds and are often convenant attached. Nevertheless, $F_{\ddot{a},t}$ accommoadates realistic heterogeneity in how the same characteristics instrument the sensitivity of different securities. In this case, markets are partially segmented.

C. PCs of levered factors

KNS find that applying a PC rotation on equity-based factors results in a sparser SDF under the Bayesian approach. While PCs by construction reduce the dimensionality of the cross section, the transformations also diminishes the interpretability of economically motivated factors and does not explain why certain predictors have powers whilst others do not. I facilitate the comparision by considering PCs of $F_{e,t}$ as another set of factors:

$$P_{e,t} = Q_e' F_{e,t} \tag{29}$$

where Q_e is the matrix of eigenvectors of Σ_e.

Summing up, my robustness test includes one sets of daily characteristics-based factors ($F_{\breve{a},t}$), one set of monthly characteristics-based factor ($F_{\ddot{a},t}$), and one set of daily PC ($P_{e,t}$), built from the same set of 100 predictors. Table V compare them against $F_{a,t}$ and $F_{e,t}$ used in previous sections.

Table V: Comparison of Asset- and Equity-Based Factors

This table compares the advantages and drawbacks of five sets of factors used in Bayesian approach discussed in Section II.

	F_e	F_a	$F_{\breve{a}}$	$F_{\ddot{a}}$	P_e
return types	equity	asset (Merton)	asset (risk-free)	asset (bonds)	PCs of equity
sample period	1951-2022	1970-2022	1970-2022	2002-2022	1951-2022
frequency	daily	daily	daily	monthly	daily
average firms per period	930	917	921	663	-
default risks	-	✓	✗	✓	-
tradability	✓	✗	✓	✓	✓
economic intepretation	✓	✓	✓	✓	✗
market segmentation	-	fully integrated	fully segmented	partially segmented	-

D. Model Selections

The optimal SDF implied by the other two sets of asset-based factors ($F_{\breve{a}}$, $F_{\ddot{a}}$) and the PCs of equity-based factors (P_e) are sparser than F_e but less so than F_a. The figures depicting their OOS R^2 against different hyperparameters can be found in Appendix F and key hyperparameters are collected in Panel (a) of Table VI. Even though model improvement can be achieved through optimizing Z (*e.g.* principal component or partial least squares), there seems to be higher marginal gain by simply unlevering the return, even just under the assumption that all debts are risk-free, or reconstructing the empirical asset returns with limited access to the corporate bond returns.

Table VI: Optimal Hyperparameters under Singular- and Dual-Penalty

This table collects CV-implied optimal hyperparameters for 10 sets of factors built from the same 100 characteristics. They are one equity-based factor (F_e), three asset-based factors (Merton: F_a, stock plus risk-free rate: $F_{\breve{a}}$, and stock plus bond: $F_{\ddot{a}}$), one PCs of equity-based factor (P_e), and their respective beta-neutral factors after orthogonizing against the market. For L^2-only regularization, the hyperparameter is the root expected SR^2 (κ); For L^1-L^2 regularization, the hyperparameters include κ and the number of factors.

Panel (a): factors including market

		F_e	F_a	$F_{\breve{a}}$	$F_{\ddot{a}}$	P_e
L^2 Penalty	κ	0.69	0.77	0.45	1.84	0.69
L^1-L^2 Penalty	κ	0.75	3.16	0.64	3.88	0.75
	number of factors	88	5	42	21	27

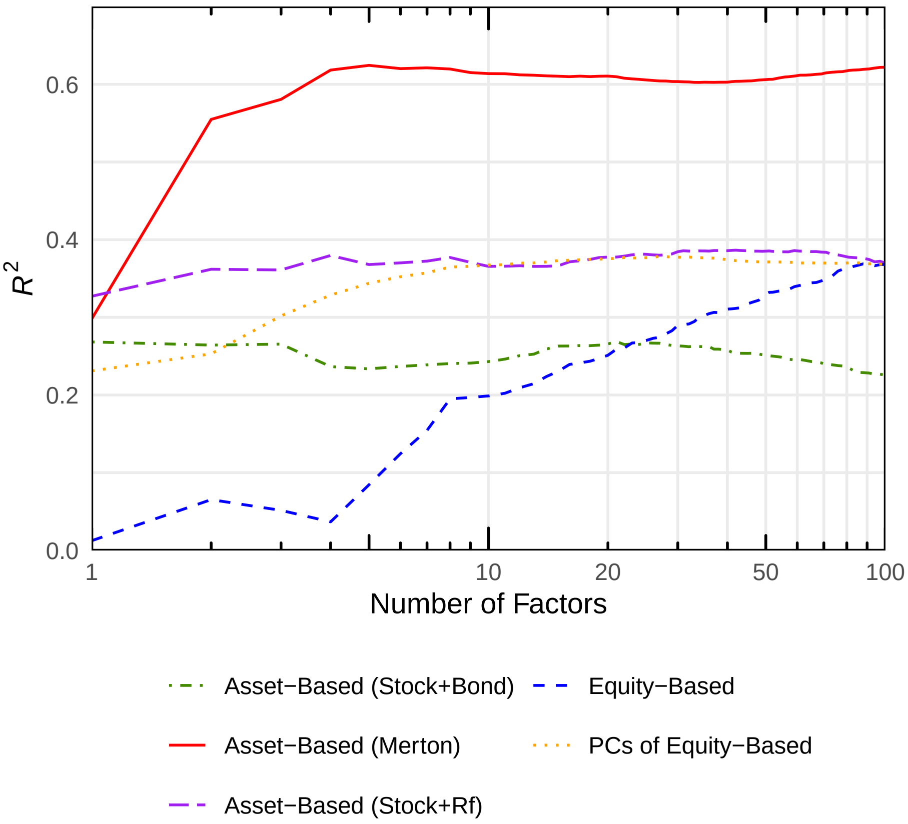
Panel (b): beta-neutral factors

		$\tilde{F}_e$	$\tilde{F}_a$	$\tilde{F}_{\breve{a}}$	$\tilde{F}_{\ddot{a}}$	$\tilde{P}_e$
L^2 Penalty	κ	0.46	0.36	0.33	0.01	0.46
L^1-L^2 Penalty	κ	0.46	1.84	1.44	32.00	0.46
	numebr of factors	99	3	15	1	59

Building upon Figure 6, Figure 11 reports the OOS R^2 of alternative sets of factors to explain the cross section by allowing for more factors estimated from Equation 17. Both alternative asset-based factors and PCs of equity-based factors are in between equity-based factors and Merton-unlevered asset-based factors in terms of sparsity: fewer factors are sufficient to capture the cross-sectional variation but the ceiling of OOS R^2 is similar to the levered space. This result is striking: it provides evidence that even if the stock market and the debt market are entirely segmented and equity holders have zero access to the bond market, they can still benefit from reweighting the economically-motivated portfolios with a leverage matrix. Correctly accounting for firm-level, instead of stock-level risks, allows a small number of factor-mimicking portfolios that can compete against complicated synthetic portfolios built from conventional dimension reduction techniques. In reality, debt and stock markets are neither entirely segmented nor integrated. Investors have limited access to the bond and loan market. Reconstructing firm assets from a mixture of corporate bonds and stocks allows investors to halve the number of characteristics-managed portfolios. This is evidenced by the training result of bond-unlevered factors. it drastically reduces the required number from 88 to 21. Note that the OOS R^2 of bond-unlevered asset-based factors is not directly comparable to the rest sets of results due to their frequency discrepancy. Nevertheless, it sends out the message that investors can construct an efficient portfolio from a mix of bonds and stocks managed by a fewer economically founded predictors.

Figure 11: OOS R^2 to Explain 100 Managed Portfolios

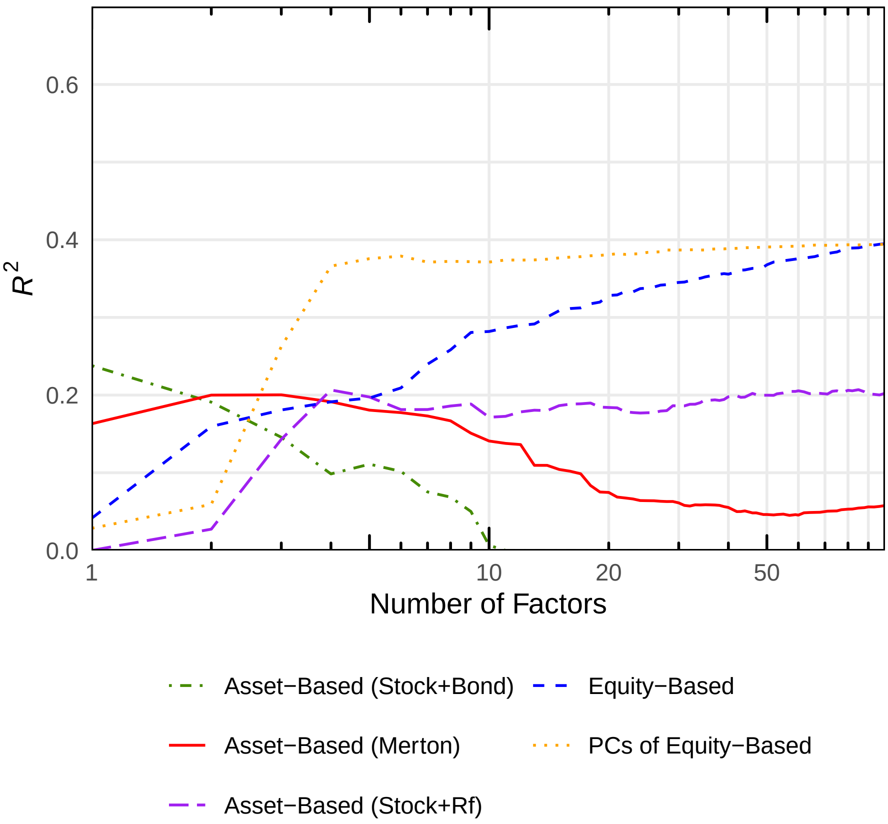
This figure compares asset-based and equity-based SDFs by their cross-validated R^2 to explain the 100 characteristics-managed factors when the number of factors increases. L^2 penalty level κ is optimized for respective number of factors. Factor selection and coefficient estimation follows the LARS-EN algorithm.



For the alternative asset-based factors, I also report the results after orthogonizing every characteristics-based factors with respect to the market factors in Panel (b) of Table VI and Figure 12. I denote these beta-neutral factors as $\tilde{F}_{\breve{a},t}$, $\tilde{F}_{\breve{a},t}$, and $\tilde{P}_{e,t}$. For $\tilde{F}_{\breve{a}}$ and $\tilde{P}_e$, OOS R^2 already peak when fewer than a dozen factors are considered. Interestingly, red (Merton-unlevered asset-based factors) and green (bond-unlevered asset-based factors) almost move in the opposite directions to the blue line (equity-based factors): as one keeps learning from more equity-based factors in the data, more asset-based data only adds noises to the training. Notably, the OOS R^2 of $\tilde{F}_{\ddot{a}}$ dropping below 0 (a reminder that it is not directly comparable to the rest due to the frequency discrepancy) likely arises from the interplay of several causes. Unlevering indeed helps shrink the cross-section, and that the data contains the most noise due to the sample limitation. Figure F.4 in Appendix F reports the OOS R^2 of the beta-neutral contour map. The results are qualitatively the same compared to the Merton-unlevered asset-based factors.

Figure 12: OOS R^2 to Explain 100 Managed Portfolios (Beta-Neutral)

This figure compares asset-based and equity-based SDFs by their cross-validated R^2 to explain the 100 characteristics-managed factors orthogonized against the market factor when the number of factors increases. L^2 penalty level κ is optimized for respective number of factors. Factor selection and coefficient estimation follows the LARS-EN algorithm. The red texts indicate when corresponding factors first enter the asset-based SDF.



E. OOS Performance of SDF-implied MVE Portfolios

I test the implied-MVE portfolios for the alternative sets of asset-based factors. The testing period is daily from January 2005 to December 2022, except for bond-unlevered asset-based factors, which is monthly from Jan 2015 to December 2022. Table VII reports the Sharpe ratios for alternative asset-based market and implied MVE portfolios.

Compared to the main exercise, these two alternative asset-based factors are directly tradable. Reconstructed market assets are more efficient than either bonds or stocks. With 21 factors, investors are able to increase their Sharpe ratio (0.75 versus 1.28) compared to investing in 88 stock-only factors: investing in firm assets managed by two dozons of firm-level characteristics are significantly more efficient than either bonds or stocks managed by a zoo of factors. Unlike Merton-unlevered factors, my building of bond-unlevered factors prioritizes tradability, as discussed in Section III. Therefore, these results provide strong evidence that asset pricing outperforms stock pricing.

Table VII: Sharpe Ratios of MVEs and MKTs in Testing Periods

This table reports Sharpe ratios of market portfolios and implied MVE portfolios in alternative asset return spaces. MVE portfolios are optimized with sample from Feb 1951 to December 2004 for daily risk-free-unlevered asset-based factors and from Feb 2002 to December 2014 for monthly bond-unlevered asset-based factors. Empirical standard errors are derived from bootstrapping with 1000 resamples and reported in the parentheses.

Panel (a): 2015 - 2022, daily

	market portfolios		implied MVE portfolios	
	equity	asset (stock + risk-free)	equity	asset (stock + risk-free)
Sharpe ratio	0.46	0.68	0.75	0.75
bootstrap s.e.	(0.24)	(0.24)	(0.23)	(0.24)

Panel (b): 2015 - 2022, monthly

	market portfolios		implied MVE portfolios	
	equity	asset (stock + bond)	equity	asset (stock + bond)
Sharpe ratio	0.55	0.82	0.75	1.28
bootstrap s.e.	(0.37)	(0.38)	(0.39)	(0.44)

I also test alternative asset-based MVE protfolios against various markets in the OOS data. The result is reported in Table VIII. The pattern from my main analysis persists:

the alphas decrease from top to bottom and increase from left to right. They suggest a more efficient market factor in the asset space.

Table VIII: Annualized α of MVE Portfolios against Various Benchmarks

This table measures the distance to CAPM by checking the annualized α (in %) from regressing SDF-implied MVE portfolios on various market portfolios and Fama-French 4 factors (market, size, value, and profitability). The SDFs are optimized on dual-penalty. MVE portfolio and benchmark returns are normalized to have the same standard deviation as the aggregate stock market for better comparison. Standard errors are reported in parentheses.

Panel (a): 2005 - 2022, daily

benchmarks \test assets	equity-based MVE	asset-based MVE (stock + risk-free)	asset-based MVE (Merton)
equity market	5.50^{*}	6.43^{**}	12.76^{***}
	(3.03)	(2.53)	(3.95)
Fama-French 4 Factors	3.53^{*}	4.99^{**}	9.65^{***}
	(2.03)	(2.04)	(2.70)
asset (stock + risk-free) market	2.31	3.11	11.16^{***}
	(3.13)	(2.76)	(4.14)
asset (Merton) market	-1.92	-1.48	7.88^{***}
	(3.20)	(2.86)	(4.10)

Panel (b): 2015 - 2022, monthly

benchmarks \test assets	equity-based MVE	asset-based MVE (stock + bond)
equity market	7.20^{*}	12.71^{***}
	(3.69)	(3.84)
Fama-French 4 Factors	5.14^{*}	10.74^{***}
	(2.55)	(3.65)
asset (stock + bond) market	-1.86	9.71^{***}
	(2.07)	(3.32)

$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

VI. Conclusion

Most economically founded stock pricing factors reveal risk profile on a firm level. Building factor-mimicking portfolios in the equity return space does not reflect the fundamental macroeconomic risks that are proxies for the utility growth. Equities are call options to the firms' assets. I analytically show that the non-linear return structures can distort the risk exposure of characteristics-managed portfolios and gives rise to redundant factors.

In this paper, I shrink the cross-sectional variations of equity returns and various asset returns with the same set of 100 firm return characteristics proposed by the literature. I employ an economically-motivated Bayesian prior to regularize the high dimensions and compare the optimal models across return spaces. Cross validated 5-factorasset-based SDF outperforms its 88-factor equity counterpart in sparsity, R^2, OOS Sharpe ratio, as well as market alphas. A mixture of bonds and stocks managed by 21 characteristics allows investors to drastically increase the highest Sharpe ratio managed by a zoo of factors should they only consider stocks in their portfolio. A CAPM-like parsimonious factor model exists in the asset return space, evidenced by a much lower explanatory power of asset-based factors after orthogonizing against the market, and a drastic decrease in the number of significant alphas.

These results attribute a substantial number of stock pricing anomalies to the leverage effect. It is much more economically coherent to price with asset-based factors.

References

Asquith, P., A. S. Au, et al. (2013). "The market for borrowing corporate bonds". In: *Journal of Financial Economics* 107(1), pp. 155–182.

Bali, T. G., H. Beckmeyer, and A. Goyal (2023). "A Joint Factor Model for Bonds, Stocks, and Options". In: *Stocks, and Options (October 1, 2023).*

Bali, T. G., H. Beckmeyer, et al. (2023). "Option return predictability with machine learning and big data". In: *The Review of Financial Studies* 36(9), pp. 3548–3602.

Berk, J. B. and P. M. DeMarzo (2007). *Corporate finance.* Pearson Education.

Bharath, S. T. and T. Shumway (2008). "Forecasting default with the Merton distance to default model". In: *The Review of Financial Studies* 21(3), pp. 1339–1369.

Bianchi, D., M. Büchner, and A. Tamoni (2021). "Bond risk premiums with machine learning". In: *The Review of Financial Studies* 34(2), pp. 1046–1089.

Black, F. and M. Scholes (1973). "The pricing of options and corporate liabilities". In: *Journal of political economy* 81(3), pp. 637–654.

Carlson, M., A. Fisher, and R. Giammarino (2004). "Corporate investment and asset price dynamics: Implications for the cross-section of returns". In: *The Journal of Finance* 59(6), pp. 2577–2603.

Chang, H., A. d'Avernas, and A. L. Eisfeldt (2021). "Bonds vs. equities: Information for investment". In: *Equities: Information for Investment (April 15, 2021).*

Chen, Q., N. Roussanov, and X. Wang (2023). *Semiparametric conditional factor models: Estimation and inference.* Tech. rep. National Bureau of Economic Research.

Chen, Z., N. L. Roussanov, et al. (2024). "Common Risk Factors in the Returns on Stocks, Bonds (and Options), Redux". In: *Bonds (and Options), Redux (January 21, 2024).*

Cochrane, J. H. (2011). "Presidential address: Discount rates". In: *The Journal of finance* 66(4), pp. 1047–1108.

Coval, J. D. and T. Shumway (2001). "Expected option returns". In: *The journal of Finance* 56(3), pp. 983–1009.

Crosbie, P. and J. Bohn (2003). "Modeling default risk: Modeling methodology". In: *KMV corporation*.

Dick-Nielsen, J. (2009). "Liquidity biases in TRACE". In: *The Journal of Fixed Income* 19(2), p. 43.

Dick-Nielsen, J. (2014). "How to clean enhanced TRACE data". In: *Available at SSRN 2337908*.

Dick-Nielsen, J., P. Feldhütter, et al. (2023). "Corporate bond factors: Replication failures and a new framework". In: *Available at SSRN*.

Doshi, H., K. Jacobs, et al. (2019). "Leverage and the cross-section of equity returns". In: *The Journal of Finance* 74(3), pp. 1431–1471.

Fama, E. F. and K. R. French (1993). "Common risk factors in the returns on stocks and bonds". In: *Journal of financial economics* 33(1), pp. 3–56.

Fama, E. F. and J. D. MacBeth (1973). "Risk, return, and equilibrium: Empirical tests". In: *Journal of political economy* 81(3), pp. 607–636.

Feng, G., S. Giglio, and D. Xiu (2020). "Taming the factor zoo: A test of new factors". In: *The Journal of Finance* 75(3), pp. 1327–1370.

Galai, D. and R. W. Masulis (1976). "The option pricing model and the risk factor of stock". In: *Journal of Financial economics* 3(1-2), pp. 53–81.

Gilchrist, S. and E. Zakrajšek (2012). "Credit spreads and business cycle fluctuations". In: *American economic review* 102(4), pp. 1692–1720.

Green, J., J. R. Hand, and X. F. Zhang (2017). "The characteristics that provide independent information about average US monthly stock returns". In: *The Review of Financial Studies* 30(12), pp. 4389–4436.

Gu, S., B. Kelly, and D. Xiu (2020). “Empirical asset pricing via machine learning”. In: *The Review of Financial Studies* 33(5), pp. 2223–2273.

Hansen, L. P. and R. Jagannathan (1991). “Implications of security market data for models of dynamic economies”. In: *Journal of political economy* 99(2), pp. 225–262.

Harvey, C. R., Y. Liu, and H. Zhu (2016). “... and the cross-section of expected returns”. In: *The Review of Financial Studies* 29(1), pp. 5–68.

Hastie, T., R. Tibshirani, et al. (2009). *The elements of statistical learning: data mining, inference, and prediction.* Vol. 2. Springer.

Hou, K., C. Xue, and L. Zhang (2015). “Digesting anomalies: An investment approach”. In: *The Review of Financial Studies* 28(3), pp. 650–705.

Kelly, B. T., S. Pruitt, and Y. Su (2019). “Characteristics are covariances: A unified model of risk and return”. In: *Journal of Financial Economics* 134(3), pp. 501–524.

Kozak, S., S. Nagel, and S. Santosh (2020). “Shrinking the cross-section”. In: *Journal of Financial Economics* 135(2), pp. 271–292.

Liechty, J., C. R. Harvey, and M. W. Liechty (2008). “Bayes vs. resampling: A rematch”. In: *Journal Of Investment Management* 6(1).

Lin, X. and L. Zhang (2013). “The investment manifesto”. In: *Journal of Monetary Economics* 60(3), pp. 351–366.

Merton, R. C. (1974). “On the pricing of corporate debt: The risk structure of interest rates”. In: *The Journal of finance* 29(2), pp. 449–470.

Pástor, L. (2000). “Portfolio selection and asset pricing models”. In: *The Journal of Finance* 55(1), pp. 179–223.

Pástor, L. and R. F. Stambaugh (2000). “Comparing asset pricing models: an investment perspective”. In: *Journal of Financial Economics* 56(3), pp. 335–381.

the WRDS Bond Database (2017). *WRDS Corporate Bond Database Manual*. WRDS Research. URL: https://wrds-www.wharton.upenn.edu/documents/248/WRDS_Corporate_Bond_Database_Manual.pdf.

Vassalou, M. and Y. Xing (2004). "Default risk in equity returns". In: *The journal of finance* 59(2), pp. 831–868.

Zhang, L. (2005). "The value premium". In: *The Journal of Finance* 60(1), pp. 67–103.

Zou, H. and T. Hastie (2005). "Regularization and variable selection via the elastic net". In: *Journal of the Royal Statistical Society Series B: Statistical Methodology* 67(2), pp. 301–320.

.

Appendix A. Equity Returns and Asset Returns

Parts of the analysis in this section is based on Black and Scholes (1973) and Galai and Masulis (1976).

From stochastic calculus the dollar return on an option, and thus the dollar return on the equity can be described as

$$dE = \frac{\partial E}{\partial V} dV + \frac{1}{2} \frac{\partial^2 E}{\partial V^2} \sigma_v^2 V^2 dt + \frac{\partial E}{\partial t} dt \,, \tag{A.1}$$

where E is the market value of equity, V is the market value of the firm, σ_v is the volatility asset return volatility. Dividing dE by E and substituting for instantaneous returns $\tilde{r}_e$, we obtain from Equation (A.1)

$$\tilde{r}_e = \frac{\partial E}{\partial D} \frac{V}{E} \tilde{r}_a \,. \tag{A.2}$$

BSM assumes the distribution of firm assets value at the end of any finite time interval is log normal and that the variance of the rate of return on the firm's assets is constant. The value for equity is

$$E = V\mathcal{N}(d_1) - e^{-r_f T} B\mathcal{N}(d_2) \tag{A.3}$$

$$d_1 = \frac{\ln(V/B) + (r_f + 0.5\sigma_v^2)T}{\sigma_v \sqrt{T}} \tag{A.4}$$

$$d_2 = d_1 - \sigma_v \sqrt{T} \,. \tag{A.5}$$

where B is the face value of debt, T is debt's time-to-maturity, and r_f is the instantaneous risk-free rate.

Under BSM, delta equals $\mathcal{N}(d_1)$:

$$\frac{\partial E}{\partial D} = \mathcal{N}(d_1) \,. \tag{A.6}$$

Thus, we obtain

$$\tilde{r}_e = \mathcal{N}(d_1)\frac{V}{E}\tilde{r}_a\,. \tag{A.7}$$

Appendix B. A Non-Existent Anomaly in the Equity Space

This appendix demonstrate through a Monte-Carlo simulation that an alpha arises when one test an equity-based factor in FMB regressions while the true economy is priced by an asset-based single factor.

A. Generate moments for a single factor economy

I begin by setting the mean and volatility of the asset-based factor $F = w'r_a$ to 10% and 20% respectively. For $N = 100$ firms, I generate a firm characteristic that is independently and identically distributed (i.i.d.). This characteristic is then demeaned to form the weight vector w. I generate leverage with a mean of 30% and standard deviation of 15% following a truncated normal distribution. The leverage and w has a correlation of 0.2. Next, I generate a random symmetric positive semi-definite matrix, normalized to serve as the correlation matrix for the asset returns. Using this correlation matrix and the weight vector w, I derive the covariance matrix Σ for the asset returns, ensuring that the variance of F is 20%. Finally, I compute the mean returns for the firms $\bar{r}_a$ using Equation (7).

B. Monte-Carlo simulation

With $\bar{r}_a$ and Σ, I simulate asset returns for 10000 periods following multi-variate normal distribution in a Monte-Carlo simulation. For each observation, I calculate equity returns following BSM. Lastly, I obtain w-weighted asset and equity portfolio returns.

C. FMB and alphas

Appendix C. Expected Equity Returns and Debts

This appendix details the derivation of the relation between expected excess stock return and the face value of debt of a firm following Coval and Shumway (2001).

Assuming the existence of an SDF that prices all assets with:

$$1 = \mathrm{E}\left[M \cdot R\right] \tag{C.1}$$

where R is the gross return of any asset, and M is the strictly positive SDF. Denote the firm's face value of debt as B and firm's market value of assets on maturity date as v that is a random varible followsing probability density distribution $f(v)$. The expected excess equity return is

$$\mathrm{E}[r_e(B)] = \frac{\mathrm{E}[\max(v-B,0)]}{\mathrm{E}[M \cdot \max(v-B,0)]} - 1 \tag{C.2}$$

$$= \frac{\int_{v=B} (v-B) f(v) \partial v}{\int_{m=0} \int_{v=B} m(v-B) f(v,m) \partial v \partial m} - 1 \tag{C.3}$$

$$= \frac{\int_{v=B} (v-B) \left[1 - \mathrm{E}[M|v]\right] f(v) \partial v}{\int_{v=B} (v-B) \mathrm{E}[M|v] f(v) \partial v} \tag{C.4}$$

where $f(v,m)$ is the joint distribution of the asset value and the SDF. Applying Leibniz inegral rule, the derivative of expected net returns with respect to the debt can be expressed as

$$
\begin{aligned}
&\frac{\partial \mathrm{E}[r_e(B)]}{\partial B} \\
&= \frac{\int_{v=B}(v-B)f(v)\partial v \cdot \int_{v=B}\mathrm{E}[M|v]f(v)\partial v - \int_{v=B}(v-B)\mathrm{E}[M|v]f(v)\partial v \cdot \int_{v=B} f(v)\partial v}{\int_{v=B}(v-B)\mathrm{E}[M|v]f(v)\partial(s)} \qquad \text{(C.5)} \\
&= \frac{\int_{v=B}\frac{v-B}{1-F(B)}f(v)\partial v \cdot \int_{v=B}\frac{\mathrm{E}[M|v]}{1-F(B)}f(v)\partial v - \int_{v=B}\frac{(v-B)\mathrm{E}[M|v]}{1-F(B)}f(v)\partial v}{\left[\int_{v=B}(v-B)\,\mathrm{E}\left[M|v\right]\frac{f(v)}{1-F\left(B\right)}\partial v\right]^2} \qquad \text{(C.6)}
\end{aligned}
$$

where $F(v)$ is the corresponding cumulative density for $f(v)$. The numerator and denominator of Equation (C.6) are composed of several conditional expectations that can be rewritten as

$$
\frac{\mathrm{E}[M|v>B]\cdot \mathrm{E}[v-B|v>B] - \mathrm{E}[\mathrm{E}(M|v)(v-B)|v>B]}{\left(\mathrm{E}[\mathrm{E}(M|v)(v-B)|v>B]\right)^2} \qquad \text{(C.7)}
$$

that can be further simplified as

$$
\frac{-\mathrm{Cov}\left[\mathrm{E}\left(M|v\right), v-B|v>B\right]}{\left(\mathrm{E}[\mathrm{E}(M|v)(v-B)|v>B]\right)^2}\,. \qquad \text{(C.8)}
$$

When the SDF moves against the underlying firm's market value of assets conditional on the firm being solvent, the derivative is positive.

Appendix D. Characteristics-Based Factor Model

I postulate a stochastic discount factor (SDF) that is projected to the payoff space $\underline{X}$ to be a linear function of the shocks to the payoffs (Hansen and Jagannathan, 1991):

$$x^* = \mathrm{E}[x^*] + (x - \mathrm{E}[x])'a\,. \tag{D.1}$$

For any arbitrary asset whose payoff is x, its price p satisfies

$$p = \mathrm{E}[xx^*] \tag{D.2}$$
$$= \mathrm{E}[x^*]\mathrm{E}[x] + \mathrm{E}[x(x - \mathrm{E}[x]]')a \tag{D.3}$$
$$= \mathrm{E}[x^*]\mathrm{E}[x] + \mathrm{E}\left[(x - \mathrm{E}[x])(x - \mathrm{E}[x])'\right]a \tag{D.4}$$
$$= \mathrm{E}[x^*]\mathrm{E}[x] + \Omega a\,. \tag{D.5}$$

We can solve for a and insert it back to Equation (D.1):

$$a = \Omega^{-1}\left(p - \mathrm{E}[x^*]E[x]\right) \tag{D.6}$$
$$x^* = \mathrm{E}[x^*] + \left(p - \mathrm{E}[x^*]\mathrm{E}[x]\right)'\Omega^{-1}\left(x - \mathrm{E}[x]\right)\,. \tag{D.7}$$

Considering excess returns in the payoff space and pick a random zero-beta rate $R_f = 1$ gives

$$p = 0 \tag{D.8}$$
$$x = r \tag{D.9}$$
$$E(x^*) = \frac{1}{R_f} = 1\,. \tag{D.10}$$

Denote the SDF as M, then Equation (D.7) becomes

$$M = 1 - \mathrm{E}[r]\Omega^{-1}\left(r - \mathrm{E}[r]\right) . \tag{D.11}$$

In a multiperiod world

$$M_t = 1 - a'_{t-1}\left(r_t - \mathrm{E}_{t-1}[r_t]\right) \tag{D.12}$$

where a_{t-1} is the product of two expectations($\mathrm{E}_{t-1}[r]$ and $\mathrm{E}_{t-1}^{-1}\left[(r - \mathrm{E}[r])(r - \mathrm{E}[r])'\right]$), known at $t-1$ before r_t are realized at t. Characteristics-based asset pricing models assume the loadings on the return shocks are linear combinations of return predictors (*e.g.* firm characteristics and macroeconomic variables) and parametrize a_{t-1} as

$$a_{t-1} = Z_{t-1}b \tag{D.13}$$

where Z is a $N \times H$ predictors matrix. N is the number of assets in the economy and H is the number of predictors. To clarify the decomposition, I assume only two predictors $i_{n,t}$ and $j_{n,t}$ in the economy where the subscripts represent the cross section and time series respectively:

$$a_{t-1} = \begin{bmatrix} b_i i_{1,t-1} + b_j j_{1,t-1} \\ b_i i_{2,t-1} + b_j j_{2,t-1} \\ \cdots \\ b_i i_{N,t-1} + b_j j_{N,t-1} \end{bmatrix} = \begin{bmatrix} i_{1,t-1} & j_{1,t-1} \\ i_{2,t-1} & j_{2,t-1} \\ \cdots & \\ i_{N,t-1} & j_{N,t-1} \end{bmatrix} \cdot \begin{bmatrix} b_i \\ b_j \end{bmatrix} = Z_{t-1}b\,. \tag{D.14}$$

Inserting Equation (D.13) back to Equation (D.12), gives

$$
\begin{aligned}
M_t &= 1 - b' Z'_{t-1} \left(r_t - \mathrm{E}_{t-1}[r_t] \right) && \text{(D.15)} \\
&= 1 - \begin{bmatrix} b_i & b_j \end{bmatrix} \cdot \begin{bmatrix} i_{1,t-1} & i_{2,t-1} & \cdots & i_{N,t-1} \\ j_{1,t-1} & j_{2,t-1} & \cdots & j_{N,t-1} \end{bmatrix} \begin{bmatrix} r_{1,t} - \mathrm{E}_{t-1}[r_{1,t}] \\ r_{2,t} - \mathrm{E}_{t-1}[r_{2,t}] \\ \cdots \\ r_{N,t} - \mathrm{E}_{t-1}[r_{N,t}] \end{bmatrix} && \text{(D.16)} \\
&= 1 - b' \begin{bmatrix} i_{1,t-1}(r_{1,t} - \mathrm{E}_{t-1}[r_{1,t}]) + \cdots + i_{N,t-1}(r_{N,t} - \mathrm{E}_{t-1}[r_{N,t}]) \\ j_{1,t-1}(r_{1,t} - \mathrm{E}_{t-1}[r_{1,t}]) + \cdots + j_{N,t-1}(r_{N,t} - \mathrm{E}_{t-1}[r_{N,t}]) \end{bmatrix} && \text{(D.17)} \\
&= 1 - b' \left(\begin{bmatrix} i_{1,t-1} r_{1,t} + \cdots + i_{N,t-1} r_{N,t} \\ j_{1,t-1} r_{1,t} + \cdots + j_{N,t-1} r_{N,t} \end{bmatrix} - \begin{bmatrix} i_{1,t-1} \mathrm{E}_{t-1}[r_{1,t}] + \cdots + i_{N,t-1} \mathrm{E}_{t-1}[r_{N,t}] \\ j_{1,t-1} \mathrm{E}_{t-1}[r_{1,t}] + \cdots + j_{N,t-1} \mathrm{E}_{t-1}[r_{N,t}] \end{bmatrix} \right) && \text{(D.18)} \\
&= 1 - b'(F_t - \mathrm{E}_{t-1}[F_t]) \,. && \text{(D.19)}
\end{aligned}
$$

Each element in F_t is a linear combination of excess returns weighted by one predictor, thus also tradable such that

$$\mathrm{E}[M_t \cdot F_t] = 0 \,. \tag{D.20}$$

Solving the system of Equation (D.19) and (D.20) results in the coefficient b of the factor model:

$$b = \Sigma^{-1} \mathrm{E}\left[F_t\right] = \left(\Sigma\Sigma\right)^{-1} \Sigma \mathrm{E}\left[F_t\right] \tag{D.21}$$

where $\Sigma \equiv \mathrm{E}\left[\left(F_t - \mathrm{E}\left[F_t\right]\right) \left(F_t - \mathrm{E}\left[F_t\right]\right)' \right]$. Empirically, b is the coefficients in a cross-sectional regression of the factors' population mean on its variance-covariance matrix.

In a special case, demeaning Z_{t-1} cross-sectionally converts factors F_t into zero-investment long-short portfolios since $i_{1,t-1} + i_{2,t-1} + \cdots + i_{N,t-1} = 0$.

Appendix E. Firm Characteristics

This table lists the firm characteristics I include in the predictor matrix Z_t. Most are compiled by Green, Hand, and Zhang (2017).

Table E.1: Details of the Characteristics

No.	Firm characteristic	Author(s)	Year, Journal
1	size	Banz	1981, JFE
2	beta (weekly)	Fama & MacBeth	1973, JPE
3	beta (daily)	Fama & MacBeth	1973, JPE
4	idiosyncratic volatility (daily)	Ali, Hwang & Trombley	2003, JFE
5	beta squared (daily)	Fama & MacBeth	1973, JPE
6	beta squared (weekly)	Fama & MacBeth	1973, JPE
7	change in 6-month momentum	Gettleman & Marks	2006, WP
8	dollar trading volume	Chordia, Subrahmanyam & Anshuman	2001, JFE
9	idiosyncratic volatility (weekly)	Ali, Hwang & Trombley	2003, JFE
10	industry momentum	Moskowitz & Grinblatt	1999, JF
11	1-month momentum	Jegadeesh & Titman	1993, JF
12	6-month momentum	Jegadeesh & Titman	1993, JF
13	12-month momentum	Jegadeesh	1990, JF
14	36-month momentum	Jegadeesh & Titman	1993, JF
15	price delay	Hou & Moskowitz	2005, RFS

Table E.1: Details of the Characteristics (Continued)

No.	Firm characteristic	Author(s)	Year, Journal
16	share turnover	Datar, Naik & Radcliffe	1998, JFM
17	absolute accruals	Bandyopadhyay, Huang & Wirjanto	2010, WP
18	working capital accruals	Sloan	1996, TAR
19	# years since first Compustat coverage	Jiang, Lee & Zhang	2005, RAS
20	asset growth	Cooper, Gulen & Schill	2008, JF
21	cash flow to debt	Ou & Penman	1989, JAE
22	cash productivity	Chandrashekar & Rao	2009, WP
23	cash flow to price	Desai, Rajgopal & Venkatachalam	2004, TAR
24	cash flow to price (industry-adjusted)	Asness, Porter & Stevens	2000, WP
25	change in asset turnover (industry-adjusted)	Soliman	2008, TAR
26	change in shares outstanding	Pontiff & Woodgate	2008, JF
27	change in employees (industry-adjusted)	Asness, Porter & Stevens	1994, WP
28	change in inventory	Thomas & Zhang	2002, RAS
29	change in profit margin (industry-adjusted)	Soliman	2008, TAR
30	convertible debt indicator	Soliman	2008, TAR
31	current ratio	Ou & Penman	1989, JAE
32	depreciation PP&E	Holthausen & Larcker	1992, JAE
33	dividend initiation	Michaely, Thaler & Womack	1995, JF
34	dividend omission	Michaely, Thaler & Womack	1995, JF

Table E.1: Details of the Characteristics (Continued)

No.	Firm characteristic	Author(s)	Year, Journal
35	dividend to price	Litzenberger & Ramaswamy	1982, JF
36	growth in common shareholder equity	Richardson, Sloan, Soliman & Tuna	2005, JAE
37	earnings to price	Basu	1977, JF
38	gross profitability	Novy-Marx	2013, JFE
39	growth in capital expenditures	Anderson & Garcia-Feijoo	2006, JF
40	growth in long-term net operating assets	Fairfield, Whisenant & Yohn	2003, TAR
41	industry sales concentration	Hou & Robinson	2006, JF
42	employee growth rate	Bazdresch, Belo & Lin	2014, JPE
43	capital expenditures and inventory	Chen & Zhang	2010, JF
44	leverage	Bhandari	1988, JF
45	growth in long-term debt	Richardson, Sloan, Soliman & Tuna	2005, JAE
46	size (industry-adjusted)	Asness, Porter & Stevens	2000, WP
47	operating profitability	Fama & French	2005, JFE
48	organizational capital	Eisfeldt & Papanikolaou	2013, JF
49	% change in capital expenditures (industry-adjusted)	Abarbanell & Bushee	1998, TAR
50	% change in current ratio	Ou & Penman	1989, JAE
51	% in depreciation	Holthausen & Larcker	1992, JAE
52	% change in gross margin - % change in sales	Abarbanell & Bushee	1998, TAR
53	% change in quick ratio	Ou & Penman	1989, JAE

Table E.1: Details of the Characteristics (Continued)

No.	Firm characteristic	Author(s)	Year, Journal
54	% change in sales - % change in inventory	Abarbanell & Bushee	1998, TAR
55	% change in sales - % change in A/R	Abarbanell & Bushee	1998, TAR
56	% change in sales - % change in SG&A	Abarbanell & Bushee	1998, TAR
57	% change in sales-to-inventory	Ou & Penman	1989, JAE
58	percent accruals	Hafzalla, Lundholm & Van Winkle	2011, TAR
59	financial statements score	Piotroski	2000, JAR
60	quick ratio	Ou & Penman	1989, JAE
61	R&D increase	Eberhart, Maxwell & Siddique	2004, JF
62	R&D to market capitalization	Guo, Lev & Shi	2006, JBFA
63	R&D to sales	Guo, Lev & Shi	2006, JBFA
64	real estate holdings	Tuzel	2010, RFS
65	return on invested capital	Brown & Rowe	2007, WP
66	sales to cash	Ou & Penman	1989, JAE
67	sales to inventory	Ou & Penman	1989, JAE
68	sales to receivables	Ou & Penman	1989, JAE
69	secured debt	Valta	2016, JFQA
70	secured debt indicator	Valta	2016, JFQA
71	sales growth	Lakonishok, Shleifer & Vishny	1994, JF
72	sin stocks	Hong & Kacperczyk	2009, JFE

Table E.1: Details of the Characteristics (Continued)

No.	Firm characteristic	Author(s)	Year, Journal
73	sales to price	Barbee, Mukherji, & Raines	1996, FAJ
74	debt capacity/firm tangibility	Almeida & Campello	2007, RFS
75	tax income to book income	Lev & Nissim	2004, TAR
76	abnormal earnings announcement volume	Lerman, Livnat & Mendenhall	2007, WP
77	cash holdings	Palazzo	2012, JFE
78	change in tax expense	Thomas & Zhang	2011, JAR
79	corporate investment	Titman, Wei & Xie	2004, JFQA
80	earnings announcement return	Kishore, Brandt, Santa-Clara & Venkatachalam	2008, WP
81	number of earnings increase	Barth, Elliott & Finn	1999, JAR
82	return on assets	Balakrishnan, Bartov & Faurel	2010, JAE
83	earnings volatility	Francis, LaFond, Olsson & Schipper	2004, TAR
84	return on equity	Hou, Xue & Zhang	2015, RFS
85	revenue surprise	Kama	2009, JBFA
86	accrual volatility	Bandyopadhyay, Huang & Wirjanto	2010, WP
87	cash flow volatility	Huang	2009, JFE
88	financial statement score	Mohanram	2005, RAS
89	bid-ask spread	Amihud & Mendelson	1989, JF
90	illiquidity	Amihud	2002, JFM
91	maximum daily return	Bali, Cakici & Whitelaw	2011, JFE

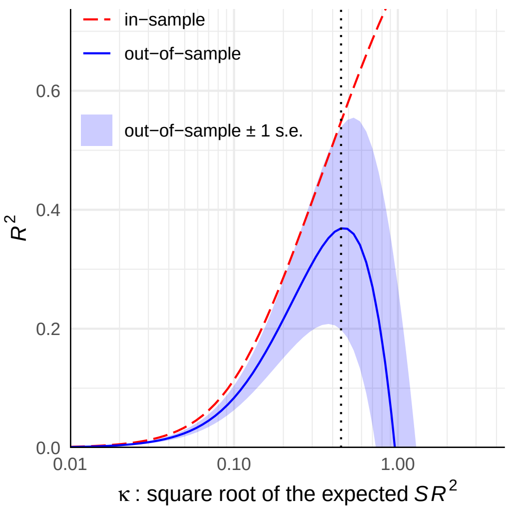
Table E.1: Details of the Characteristics (Continued)

No.	Firm characteristic	Author(s)	Year, Journal
92	return volatility	Ang, Hodrick, Xing & Zhang	2006, JF
93	volatility of liquidity (dollar trading volume)	Chordia, Subrahmanyam & Anshuman	2001, JFE
94	volatility of liquidity (share turnover)	Chordia, Subrahmanyam & Anshuman	2001, JFE
95	zero trading days	Liu	2006, JFE
96	book-to-market	Rosenberg, Reid & Lanstein	1985, JPM
97	book-to-market (industry-adjusted)	Asness, Porter & Stevens	2000, WP
98	skewness	Condard, Robert & Ghysels	2012, JF
99	kurtosis	Condard, Robert & Ghysels	2012, JF

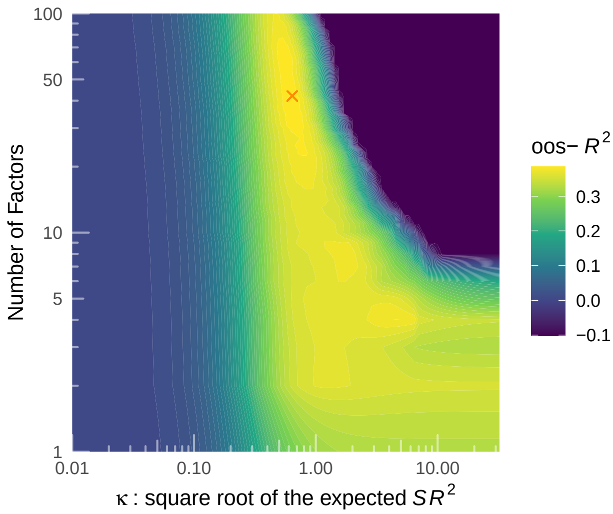
Appendix F. Figures

Figure F.1: Riskless-Unlevered Factors $F_{\check{a}}$: R^2 under Singular- and Dual-Penalty

This figure presents the R^2 that explain the cross-sectional variation of 100 asset-based daily factors from 1970 to 2022. The OOS R^2 is derived from a 3-fold cross validation process under different combinations hyperparameters. Panel (a) only employs L^2 penalty of which the strength is measured by prior root expected SR^2 (κ). Panel (b) also employs L^1 penalty of which the strength is measured by the number of factors. Hyperparameters corresponding to highest OOS R^2 are marked in the figure. Axes of hyperparameters are ploted on logarithmic scale.



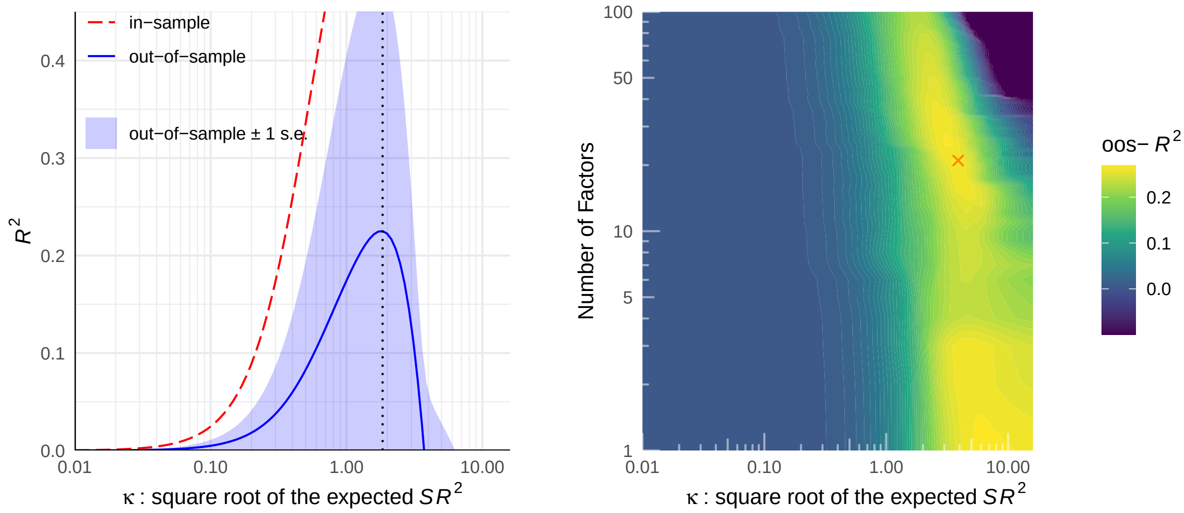
(a) L^2 penalty



(b) L^1-L^2 penalties

Figure F.2: Bond-Unlevered Factors $F_{\ddot{a}}$: R^2 under Singular- and Dual-Penalty

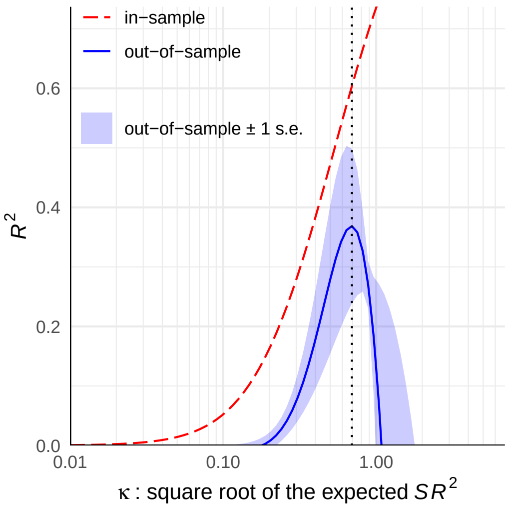
This figure presents the R^2 that explain the cross-sectional variation of 100 asset-based monthly factors from 2002 to 2022. The OOS R^2 is derived from a 3-fold cross validation process under different combinations hyperparameters. Panel (a) only employs L^2 penalty of which the strength is measured by prior root expected SR^2 (κ). Panel (b) also employs L^1 penalty of which the strength is measured by the number of factors. Hyperparameters corresponding to highest OOS R^2 are marked in the figure. Axes of hyperparameters are ploted on logarithmic scale.



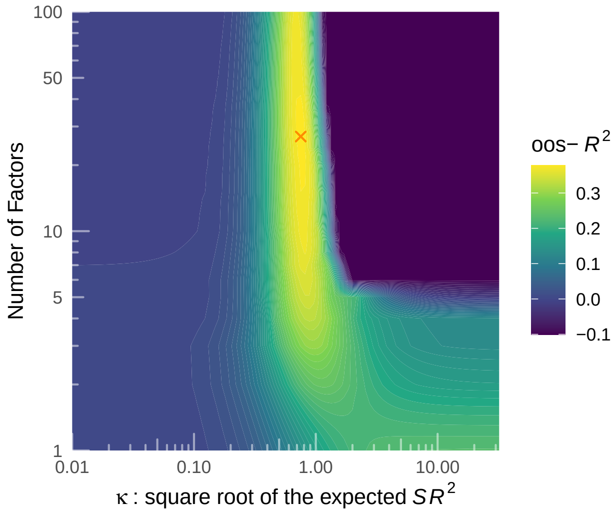
(a) L^2 penalty (b) L^1-L^2 penalties

Figure F.3: Principal Components P_e: R^2 under Singular- and Dual-Penalty

This figure presents the R^2 that explain the cross-sectional variation of 100 PCs of equity-based daily factors from 1951 to 2022. The OOS R^2 is derived from a 3-fold cross validation process under different combinations hyperparameters. Panel (a) only employs L^2 penalty of which the strength is measured by prior root expected SR^2 (κ). Panel (b) also employs L^1 penalty of which the strength is measured by the number of factors. Hyperparameters corresponding to highest OOS R^2 are marked in the figure. Axes of hyperparameters are ploted on logarithmic scale.



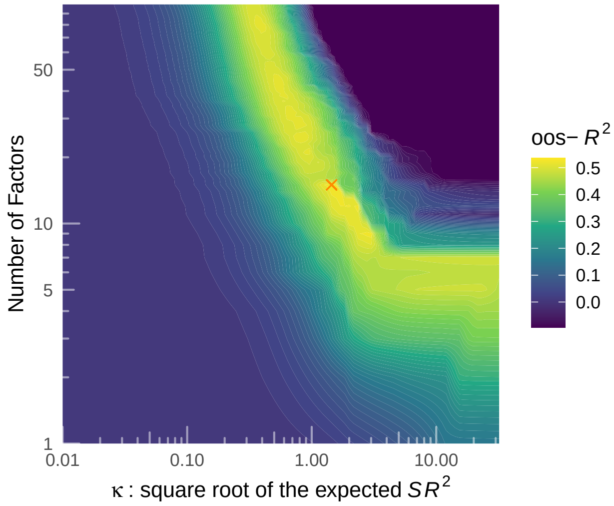
(a) L^2 penalty



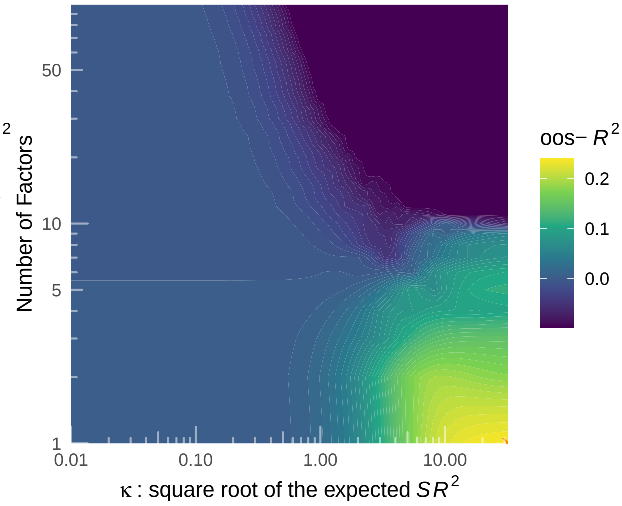
(b) L^1-L^2 penalties

Figure F.4: OOS R^2 under Singular- and Dual-Penalty for Beta-Neutral Factors

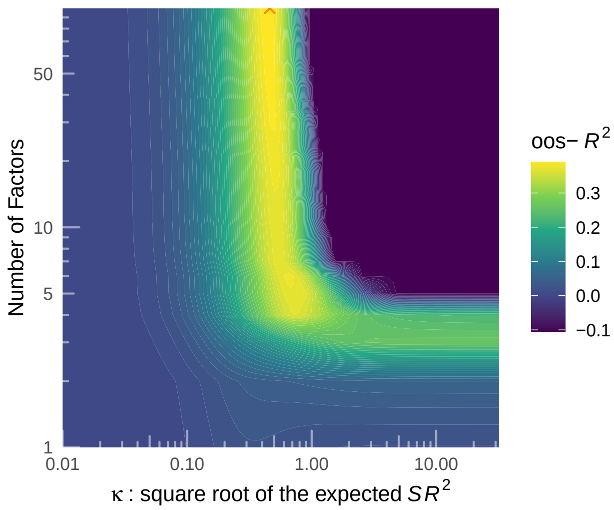
This figure reports the OOS R^2 under different hyperparameters from 3-fold cross validation process using 100 factors. Panel (a) depicts the result for beta-neutral riskless-unlevered asset-based factors ($\tilde{F}_{\breve{a}}$), Panel (b) for beta-neutral bond-unlevered asset-based factors ($\tilde{F}_{\ddot{a}}$), and Panel (c) for PCs of beta-neutral equity-based factors ($\tilde{P}_e$). Hyperparameters corresponding to highest OOS R^2 are marked in the figure. Axes are ploted on logarithmic scale.



(a) riskless-unlevered $\tilde{F}_{\breve{a}}$



(b) bond-unevered $\tilde{F}_{\ddot{a}}$



(c) PCs of levered $\tilde{P}_e$

Appendix G. Coefficients Estimates and Factor Importance

This section reports coefficients estimates and factor importance for robustness sets of factors.

Market and momentums are robust regardless of return spaces, and so is ROE for bond-unlevered asset-based factors. Bond-unlevered asset-based factors have much higher point estimates of coefficients, partly due to the poorly estimated covariance matrix $\tilde{\Sigma}$ with coarser data over a much shorter sample period. Given the discussion in Section V, these numbers are at best a supplement to the daily factors in Panel (a) and (b).

Table G.1: Coefficient Estimates under L^2 Penalty

Under a singular regularization of root expected SR^2 (κ), this table lists top 10 of 100 coefficient estimates and t-statistics corresponding to the CV-implied optimal prior, sorted by the absolute values of the t-statistics. Panel (a) reports equity-based and Merton-unlevered asset-based factors. Panel (b) reports riskless-unlevered asset-based and PCs of equity-based factors. Panel (c) reports bond-unlevered asset-based factors.

Panel (a)

equity-based factors (F_e)			asset-based factors, Merton (F_a)		
factors	b	t-stat	factors	b	t-stat
market	4.181	4.458	12-month momentum	4.737	2.696
12-month momentum	3.551	2.654	revenue surprise	4.283	2.260
1-month momentum	-3.211	-2.451	return on equity	3.408	1.797
change in 6-month momentum	-2.915	-2.129	market	2.013	1.774
change in shares outstanding	-2.254	-1.521	6-month momentum	2.499	1.416
earnings to price	1.907	1.319	size (industry-adjusted)	-2.395	-1.260
R&D to sales	1.646	1.190	change in employees (industry-adjusted)	2.209	1.156
return on equity	1.626	1.142	volatility of liquidity (share turnover)	2.193	1.151
maximum daily return	-1.643	-1.123	change in 6-month momentum	-1.978	-1.107
number of earnings increase	1.594	1.065	number of earnings increase	2.088	1.082

Panel (b)

asset-based factors, stock + risk-free ($F_{\ddot{a}}$)			PCs of equity-based factors (P_e)		
factors	b	t-stat	PCs	b	t-stat
market	1.871	2.321	PC3	4.973	5.213
12-month momentum	1.498	1.388	PC10	5.710	4.537
1-month momentum	-1.091	-1.004	PC2	-1.530	-2.121
R&D to market capitalization	1.025	0.920	PC5	-2.224	-2.080
change in 6-month momentum	-0.969	-0.882	PC12	2.361	1.814
return on assets	0.929	0.829	PC6	-1.740	-1.551
return on equity	0.886	0.787	PC25	1.963	1.374
change in shares outstanding	-0.866	-0.766	PC28	-1.829	-1.271
return on invested capital	0.767	0.682	PC9	-1.550	-1.242
financial statement score	0.767	0.681	PC14	-1.618	-1.224

Panel (c)

asset-based factors, stock + bond ($F_{\ddot{a}}$)		
factors	b	t-stat
market	18.499	3.834
1-month momentum	10.128	1.838
change in employees (industry-adjusted)	5.901	1.004
6-month momentum	5.321	0.974
skewness	5.591	0.950
change in 6-month momentum	5.292	0.947
sin stocks	4.402	0.827
dollar trading volume	-4.668	-0.803
volatility of liquidity (dollar trading volume)	4.678	0.801
12-month momentum	4.135	0.762

Table G.2: Factor Importance Ranked under Dual Penalty

This table reports top 10 factors ranked by the their earliest entry into the SDF when allowing for higher number of factors under dual-penalty. Factors are selected to generate highest OOS R^2 from 3-fold cross validation process with 100 candidate factors. The other hyperparameter root expected SR^2 (κ) is set at respective optimum.

	equity-based factors (F_e)	asset-based factors, Merton (F_a)
1	market	market
2	12-month momentum	12-month momentum
3	1-month momentum	revenue surprise
4	6-month momentum	return on equity
5	change in shares outstanding	asset growth
6	sales to price	employee growth rate
7	industry momentum	change in employees (industry-adjusted)
8	change in 6-month momentum	earnings volatility
9	earnings to price	maximum daily return
10	maximum daily return	size (industry-adjusted)
	asset-based factors, stock + risk-free ($F_{\breve{a}}$)	asset-based factors, stock + bond ($F_{\ddot{a}}$)
1	market	market
2	12-month momentum	1-month momentum
3	change in shares outstanding	dollar trading volume
4	1-month momentum	volatility of liquidity (dollar trading volume)
5	maximum daily return	skewness
6	earnings to price	change in 6-month momentum
7	asset growth	6-month momentum
8	return volatility	illiquidity
9	change in 6-month momentum	change in employees (industry-adjusted)
10	industry momentum	cash flow to debt