Intergenerational Discounting and Inequality

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November 28, 2025

Motivation

- Policies today affect present and future generations:
  - Typically, some gain, other lose from the policy.
- Examples:
  - Mitigation of climate change;
  - Public debt reduction;
  - Research and development for technological change;
  - Pension reform.

Motivation

- What policies are better? The answers depend on two aspects of **intergenerational justice**:
  - The attitude toward earlier vs. later generations—*discounting*;
  - The attitude toward better-off vs. worse-off generations—*inequality*.

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  - The attitude toward earlier vs. later generations—*discounting*;
  - The attitude toward better-off vs. worse-off generations—*inequality*.
- *But:* The set of attitudes we typically adopt for economic policy evaluation and analysis is quite narrow.

Focus of the paper

- We develop a new, large family of theories of intergenerational justice.
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- We show that **discounting and inequality** attitudes are generally independent of each other and **can be disentangled**.
- This **orthogonality property** holds across a wide variety of cases, and provides significantly more general (tractable) criteria.

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- The **advantage of our separation** is that we can present a wide variety of cases that go beyond exponential discounting and additive separability.
- Examples:
  - Non-constant social impatience;
  - Positional attitudes to inequality between generations.

Our contribution

1. We develop **"modular" theories of intergenerational justice**:
   - 1.1 *Module 1* exclusively deals with **discounting**;
   - 1.2 *Module 2* exclusively deals with **inequality**;
   - 1.3 Any theory is fully described by these two modules;
   - 1.4 Many new theories emerge;
   - 1.5 The most common theories fit our approach.
     - Exponentially-discounted utilitarianism as special case.

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2. We **axiomatically characterize** the class of modular theories of intergenerational justice.

Our contribution

3. We provide **a transparent framework to understand and choose from** a large class of social welfare functions.
4. We relate the social welfare function to measures of **(in)equality and their trade-off with efficiency** and the **generalized Ramsey rule**.
5. We offer **new definitions** of social impatience and inequality aversion.

Literature

- The relationship between discounting and inequality remains understudied (Stern, 2007; Nordhaus, 2007; Dasgupta, 2008).
- The literature has focused on **discounting attitudes**:
  - Pareto efficiency implies some sort of discounting (Diamond, 1965).
- Our perspective on **inequality attitudes** is borrowed from the leaky bucket experiment popularized by Okun (1975).

Technical remarks

Agenda

- Disentanglement;
- ~~Characterization;~~
- Implications;
- ~~Discounting and inequality;~~
- Conclusion.

Disentanglement

A consumption stream

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- A theory of intergenerational justice is represented by a **social welfare function**

$$W : C \to \mathbb{R}.$$

We assume that the social welfare function is continuous.

Change of perspective

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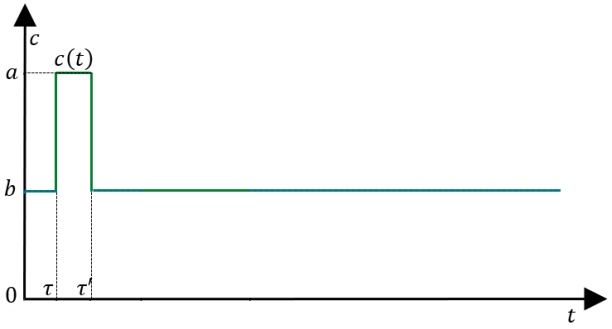

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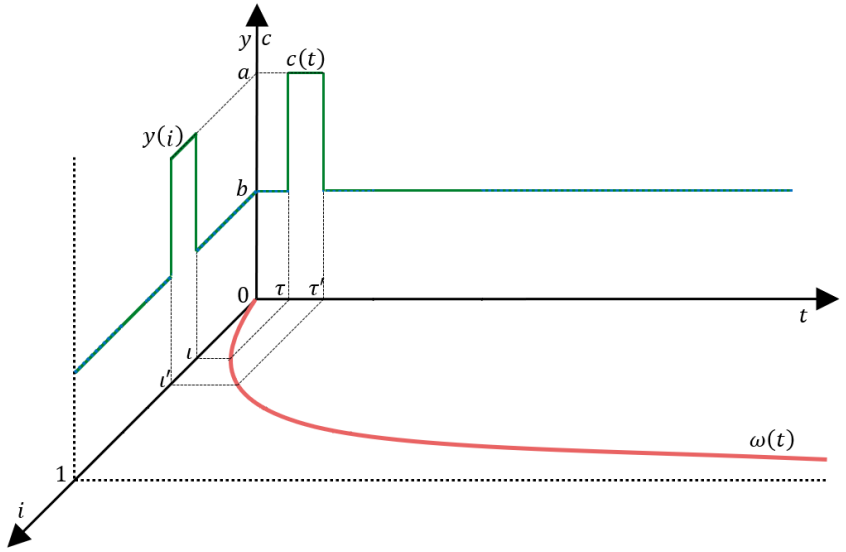

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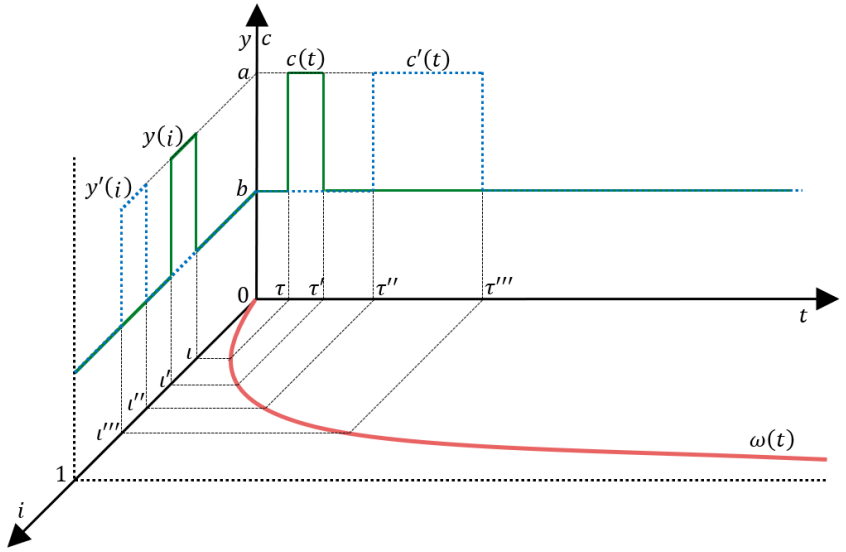

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- The central step of our analysis is to *transform* consumption streams in "calendar time" T to consumption streams in "equivalent time" $I \equiv [0, 1)$.
  - The duration of each generation is *expanded/ compressed* up to the point where all consumption has the **same value**.
- *Anticipating the results:* since each instant of consumption in equivalent time has the same value, we can apply any law invariant inequality averse aggregation of consumption.

The time-discounting function

- We say that consumption streams admit a representation in **equivalent time**, if there exists a strictly increasing function $\omega : T \to I$, such that permuting the transformed consumption is a matter of social indifference.

$$W(c) = W(c') \text{ whenever } c = y(\omega), c' = y'(\omega), y = \Pi y'.$$

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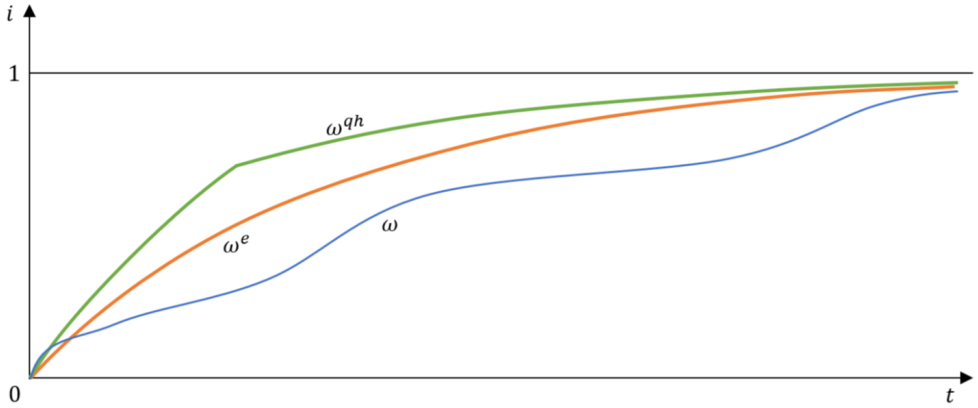
- We call ω the **time-discounting function**.

The time-discounting function



$$\omega^e(t) = 1 - e^{-\rho t} \qquad \omega^{qh}(t) = \begin{cases} \beta\kappa(1 - e^{-\rho t}), & \text{for } t \in [0, \tau] \\ 1 - \kappa e^{-\rho t}, & \text{for } t > \tau, \end{cases} \text{ with } \kappa = 1/(\beta + (1-\beta)e^{-\rho t})$$

The aggregator

- How to aggregate consumption streams in equivalent time?
- Given the social welfare function W and the time-discounting function ω, the **aggregator** $V : Y \to \mathbb{R}$ immediately follows by setting

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- Note that the aggregator is *law invariant* in equivalent time by definition.
- Thus, V could be any standard (atemporal) social welfare function.

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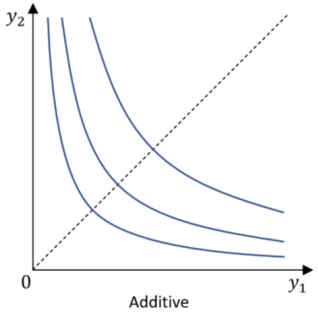
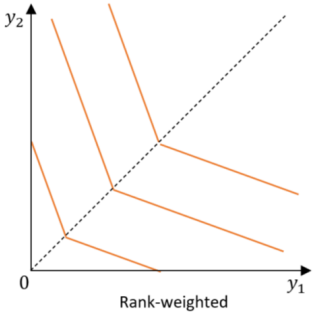


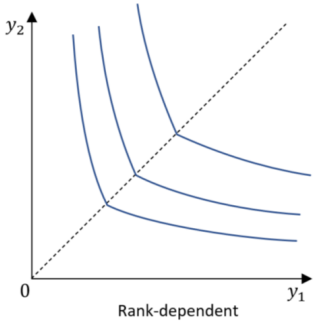
Additive

$$V^a = \int_0^1 \frac{(y_i)^{1-\eta}}{1-\eta} di$$



Rank-weighted

$$V^{rw} = (\gamma + 1) \int_0^1 \left(1 - r(i)\right)^{\gamma} y_i di$$



Rank-dependent

$$V^{rd} = (\gamma + 1) \int_0^1 \left(1 - r(i)\right)^{\gamma} \frac{(y_i)^{1-\eta}}{1-\eta} di$$

The modular social welfare function

- We say that W is **modular** if consumption streams can be represented in equivalent time and, thus, there exist an ω and V induced by W.
- Importantly, the most common welfare criteria adopted in the literature are modular.

Example formulas

Implications

Some modular theories of intergenerational justice

W	V	Additive V^a	Rank-weighted V^{rw}	Rank-dependent V^{rd}
ω		$\int_0^1 f(y_i)di$	$\int_0^1 g(r(i))y_i di$	$\int_0^1 g(r(i))f(y_i)di$
Exponential ω^e	$1-e^{-\rho t}$	EDU	NEW	NEW
Quasi-hyperbolic ω^{qh}	$\begin{cases}\beta k(1-e^{-\rho t}) & t \le \tau \\ 1-ke^{-\rho t} & t > \tau\end{cases}$		NEW	NEW
General ω			NEW	NEW

Efficiency–equity representation

- **Proposition 1.** Assume W is modular, Paretian and scale invariant. Then, the following decomposition emerges:

$$W = \overline{y} \cdot E(y).$$

- Here, $\overline{y}$ is the mean, $y(\omega) = c$, and E is a measure of equity (or 1 minus inequality), building on Atkinson (1970).

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- Here, $\overline{y}$ is the mean, $y(\omega) = c$, and E is a measure of equity (or 1 minus inequality), building on Atkinson (1970).
- Thus, social welfare can be decomposed in two main aspects:
  - *the time attitude* (ω),
  - *the efficiency-equity trade-off* ($\overline{y} \cdot E$).

Generalized Ramsey rule

- **Proposition 2.** Assume W is modular and Paretian, and ω, V are "differentiable". Then, the social discount rate follows:

$$SDR_t = \frac{1}{t}\ln(\omega'(0)/\omega'(t)) + \frac{1}{t}\ln(\mu(0;y)/\mu(\omega(t);y)).$$

- The first term captures the "pure time preference", embodied by the time-discounting function.
- The second term captures inequality adjustment ($\mu(i;y)$ is the marginal of V at i given y).

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- As a special case, the standard Ramsey rule obtains: $SDR_t = \rho + \eta g_t^c$.
- For parametric rank-dependent families: $SDR_t^{erd} = \rho + \eta g_t^c + \gamma g_t^r$.

Conclusion

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- We propose modular theories of intergenerational justice.
- Distinctive features:
  - It builds on and highlights a variety of attitudes towards discounting and inequality;
  - It is very general and flexible, and enriches the toolbox of economic analysis. At the same time, the parametric specifications of these theories remain suitable for investigation of optimal policies.
- Estimation, application, and generalization of the modular theories also open many avenues for future research.