

# Subjective Risk Premia in Bond and FX Markets

Daniel Pesch<sup>1</sup>, Ilaria Piatti<sup>2</sup> and Paul Whelan<sup>3</sup>

<sup>1</sup>Saïd Business School

<sup>2</sup>Queen Mary University of London

<sup>3</sup>Copenhagen Business School

## Abstract

This paper elicits subjective risk premia from an international survey dataset on interest rates and exchange rates. Survey-implied risk premia are *(i)* unconditionally negative for bonds, positive for investment currencies and negative for funding currencies, *(ii)* correlated with (subjective) macro expectations, *(iii)* correlated with quantities of risk, *(iv)* mean-reverting, as opposed to extrapolative; and *(v)* predict future realised returns with a positive sign. Deriving a subjective belief decomposition, we exploit surveys to estimate a multi-country asset pricing model with time-variation in economic uncertainty and three probability measures: the risk-adjusted, the physical and a subjective measure. The estimation quantifies the size of financial market belief distortions and demonstrates that subjective risk premia are well explained by a classical risk-return trade-off.

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Measuring expected returns is a long standing endeavour in financial economics; usually inferred by approximating conditional expectations with projections of realizations onto observable factors. Projection approaches are problematic for a number of well known reasons. For example, projections depend crucially on the researcher's choice of covariates. Moreover, proxying for expected returns with realised returns assumes that information surprises cancel out during the sample period studied, which in the context of trending interest rate markets is unlikely.<sup>1</sup>

This paper advocates an alternative approach. We extract subjective expected excess returns on government bonds and exchange rates from a panel of professional forecasters who are local specialists in their respective domestic markets.

The belief panel is available for the G10 countries at a monthly frequency for the sample period January 1995 - December 2020, and allows us to jointly measure (i) model free real-time risk premia; and (ii) expectation errors across both sovereign bond and exchange rate markets. The joint properties subjective risk premia and expectation errors sheds light on the the validity of existing asset pricing models, and provides guidance for the design of future models that incorporate deviations from full information rationality.

Studying their empirical properties, we show that subjective bond risk premia (*BRP*) are highly correlated across countries. The average cross-country correlation is 62% and they are significantly positive for all country pairs. Second, subjective *BRPs* are negative on average but are volatile and become persistently positive. An unconditionally negative *BRP* is consistent with predictions from many leading equilibrium models.

We explore the cyclical properties of *subjective BRP* through a series of panel regressions on *subjective* expected growth measures, also obtained from surveys, estimating a strong and robust negative relationship; thus, *BRP* are counter-cyclical. This is an important result since leading asset pricing models featuring priced long run risks (Bansal and Yaron, 2004), habit preferences (Campbell and Cochrane, 1999), and rare disasters (Wachter, 2013) predict that risk premia vary cyclically and are increasing in states of high marginal utility (low realised growth or expected growth). Third, subjective *BRPs* are significantly positively linked to the realized volatility of bond returns; thus, survey expectations preserve the basic risk-return relation which predicts a tight link between quantities of risk and compensation for risk. This is a second important take-away since detecting a link between realised returns and measures of volatility is notoriously

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<sup>1</sup>For a detailed discussion of this point see the American Finance Association Presidential Address of Elton (1999).

difficult (see, e.g., Moreira and Muir, 2017 or Eraker, 2018). This result is consistent with Buraschi, Piatti, and Whelan (2021), but while they document a positive link between quantity of risk and subjective bond risk premia for the U.S, we extend this result to all G10 countries.

Results for subjective exchange rate risk premia ( $XRP$ ) are consistent with standard “carry trade” intuition. The average exchange rate risk premium for the standard “investment currencies” within the carry trade, i.e. Switzerland and Japan, is largely negative, while it is largely positive for Norway and Australia. For the remaining countries the average  $XRP$  is closer to zero, but it is still highly time-varying. Exchange rate risk premia also display a large degree of co-movement, with all pairwise correlations being positive and significant, equal to 45% on average. The large cross-sectional correlation between individual countries risk premia, which we document in both fixed income and currency spaces, is consistent with highly globalized markets, in which sources of risk and risk compensations are common across countries. Subjective exchange rate risk premia also display interesting cyclical properties: they are positively linked to expected growth in foreign economies and negatively linked to expected growth in the U.S, consistent with a safe-haven view of the U.S Dollar in crisis periods.

Next, we estimate a set of return predictability regressions in order to characterise the informational content of subjective risk premia as a signal about futures realised returns. As benchmark predictors we consider the interest rate differential and the slope of the yield curve, for the exchange rate and bond realized returns, respectively. Summarising, we show that ex-ante beliefs about returns significantly forecast realised returns, in both economic and statistical terms, and with the correct sign according to rational expectations. At the same time, survey expectations are “unspanned” by interest rate spreads which also significantly predict future excess returns. This implies that while positively correlated with future realised returns survey implied forecast errors are predictable by date  $t$  information, suggesting that beliefs are not fully rational consistent with existing literature (e.g., Frankel and Froot, 1987 and Cieslak, 2017).

Studying the properties of expectation errors we extend the findings of Buraschi, Piatti, and Whelan (2021) showing that for all countries in our sample surveys over-predicted the level of future interest rates, consistent with a downward trend in rates during our sample that was unpredictable ex-ante. For exchange rates, the mean expectational error are close to zero, meaning that there is no systematic bias. Interestingly, forecast errors are highly correlated for across countries. Finally, while errors display only mild persistence sampled at annual horizons, this translates

into predictability by interest rate spreads, consistent with the ‘lack of spanning’ result discussed above.

Finally, we derive a subjective multi-country general equilibrium model in which investors fully optimize their consumption and investment choices conditional on beliefs formed under a subjective measure. We explicitly consider a (exogenous) distortion between the subjective and the physical measure, to allow for forecast errors that are not zero on average. Therefore, we model three probability measures in total: the subjective measure of the agents in the economy, the physical measure of the econometrician and the risk-neutral measure under which prices are computed, assuming no arbitrage and complete markets. Combining subjective risk premia, expectation errors and realised asset prices we estimate the model via simulated method of moments. The estimated model fits subjective risk premia and forecast errors well, with a relatively small distortion between subjective and physical probability measures, which suggests that subjective risk premia follow a classical risk-return trade off regardless of whether survey expectations are rational or not.

**Related Literature:** Our paper is related to a large literature that seeks to measure expected returns. In currency markets, Chernov, Dahlquist, and Lochstoer (2020) argue that expected returns can be summarised by a low dimensional set of signals (interest differential, trend, and mean reversion) while Kremens and Martin (2019) test a theory based expected currency return signal based on the prices of quanto index contracts. In bond markets, Bauer and Hamilton (2018) revisit much of the literature on expected bond return signals and argue that a low dimensional number of principal components span the vast majority of information about bond risk premia. Our paper compliments these works by proposing a summary measure for expected returns across currency and bond markets, extracted directly from expectations, as opposed to inferred from prices.

Piazzesi, Salomao, and Schneider (2015) is probably the first paper to construct subjective (bond) risk premia from surveys while. These authors point out that while statistical measures of bond risk premia are volatile and countercyclical, subjective premia are less volatile and not ‘very’ cyclical. A related recent point is made by Nagel and Xu (2022) who analyse survey-based risk premia across different asset classes. These authors argue subjective risk premia are acyclical. We contribute to the debate on cyclicity, focusing on a cross-section of sovereign bond and currency markets, and demonstrate that subjective risk premia are indeed cyclical, if one considers measures of *subjective* macro growth rates also extracted from surveys.

Deriving a subjective belief decomposition we derive and study an equilibrium model with subjective beliefs. Thus, our paper relates to a growing literature that embeds subjective beliefs and belief distortions in equilibrium models (Brunnermeier and Parker (2005), Chen, Hansen, and Hansen (2020), Maenhout, Vedolin, and Xing (2021) and Bhandari, Borovička, and Ho (2022).) In terms of the estimation, Chernov and Mueller (2012) also exploit information in the term structure of survey-based forecasts of inflation to estimate a term structure model that allows for differences between risk-adjusted, subjective, and physical probability measures.

## I. Data

**Survey Data.** Our survey data is supplied by Consensus Economics (CE). Professional financial market participants submit monthly forecasts of (i) spot exchange rates; and (ii) yields on 10 year government bonds for a variety of countries.<sup>2</sup> We focus on the most heavily-traded G10 currencies vis-a-vis the United States (USD): Australia (AUD), Canada (CAD), Switzerland (CHF), Europe (EUR), United Kingdom (GBP), Japan (JPY), New Zealand (NZD), Norway (NOK) and Sweden (SEK). CE reports projections for two horizons, 3 and 12 months, for both exchange rate and interest rate expectations. We focus on the 12-month forecasts which is the horizon where predictable variation in risk premia is most likely to arise. Forecasts begin in (i) 1990 for the USD, CAD, EUR, GBP, JPY; (ii) in 1995 for AUD, NZD and SEK; and (iii) in 1998 for NOK and CHF. All results in the paper, unless otherwise stated, are based on the period from January 1995 to December 2020, for a total of 300 monthly observations for all countries except NOK and CHF.

CE has maintained a consistent questioning procedure over time and survey respondents face the same questions for each country. Forecasters receive the questionnaire in the first few days of the month, and survey forecasts are collected the second week of every month on Monday and then released by CE three days after on the Thursday of the same week. We sample all yields, spot rates and exchange rates on the date when the survey goes public, i.e. the release date, that is normally around the middle of the month, in order to avoid any look-ahead bias. Moreover, the survey focuses on experts for each region, with respondents generally located in the country for which they are asked to make predictions. Thus, the dataset is comparable across a large

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<sup>2</sup>CE panellists provide par yield forecast which we treat as zero coupon forecasts. Section A.1 in the Online Appendix (OA) discusses the approximation error.

cross-section of countries and is available at monthly frequency for an extended sample period. Section A.1 in the (OA) provides additional details of the CE dataset.

**Realised Data.** We obtain monthly G10 FX spot and 3-month forwards from Thomson Reuters Eikon for the sample period January 1995 to December 2020. For the same panel, we obtain zero coupon bond yields, which are generally available for maturities 3, 6, 9 and 12-months, and 3, 5, 10 and 15 years from Bloomberg. Below we require country-specific yields for three bond maturities: the 12-month rate (risk-free), the 10-year rate, and the 11-year rate. While Bloomberg provides the former two series, it does not provide us with the latter. To remedy this, we fit a cubic spline to all available maturities and sample the desired yield. Realized second moments of bond and FX returns ( $r_t$ ) are measured at daily frequency by computing realized volatility between subsequent survey release dates, which are approximately  $n = 22$  days apart

$$\hat{\sigma}_t^2 = \frac{1}{n-1} \sum_{i=1}^n r_{t-1-n}^2. \quad (1)$$

All volatility estimates are annualised. In addition, in foreign exchange markets, we exploit option-implied risk neutral variances constructed and discussed by Krohn, Mueller, and Whelan (2020).

As part of section V, we also use realised CPI data for the United States, Australia, and Switzerland. These realisations are obtained from FRED, the Australian Bureau of Statistics (ABS), and the Federal Statistical Office of Switzerland, respectively. As the Australian Bureau of Statistics only reports CPI realisations on a quarterly frequency, we compute weighted averages of quarterly observations to obtain monthly estimates.

## II. Framework and Notation

The price of an  $k$ -period bond satisfies the first order condition of a representative investor who forms their beliefs under a subjective measure ( $\mathbb{S}$ ). This measure does not necessarily coincide with the objective measure ( $\mathbb{S} \neq \mathbb{P}$ ); in which case the following decomposition holds

$$P_t^k = E_t^{\mathbb{S}} [M_{t+1} P_{t+1}^{k-1}] \quad (2)$$

$$= \underbrace{\frac{1}{R_t^f} E_t^{\mathbb{P}} [P_{t+1}^{k-1}]}_{\text{objective NPV}} + \underbrace{\text{Cov}_t^{\mathbb{S}} [M_{t+1}, P_{t+1}^{k-1}]}_{\text{subjective RP}} + \underbrace{\frac{1}{R_t^f} [E_t^{\mathbb{S}} [P_{t+1}^{k-1}] - E_t^{\mathbb{P}} [P_{t+1}^{k-1}]]}_{\text{forecast errors}}. \quad (3)$$

where  $M_{t+1}$  is the one period stochastic discount factor. The final line shows that subjective risk premia (studied in section III) and forecast errors (studied in section IV) are flip sides of the same coin. In this section, we introduce notation and formulas used to compute subjective risk premia and later we study forecast errors.<sup>3</sup>

#### A. Risk premia in the fixed income market

Let  $P_t^{(k)}$  be the time  $t$  price of a default risk-free zero-coupon bond of maturity  $k$  years. Spot  $k$ -year yields are then defined as  $i_t^{(k)} = -\frac{\ln P_t^{(k)}}{k}$ . The bond risk premium is defined as the expected excess return on the bond, so we start by computing the realized holding horizon return of a  $k$ -year bond, with a  $j$ -year holding horizon:

$$\ln \frac{P_{t+j}^{(k-j)}}{P_t^{(k)}} = -(k-j)i_{t+j}^{(k-j)} + ki_t^{(k)} \quad (4)$$

The annualised expected excess return on a  $k$ -year bond with a  $j$ -year holding horizon is then:

$$E_t \left[ rx_{t+j}^{(k)} \right] = -\frac{k-j}{j} E_t \left[ i_{t+j}^{(k-j)} \right] + \frac{k}{j} i_t^{(k)} - i_t^{(j)}, \quad (5)$$

where continuously compounded yields are annualised and  $k$  and  $j$  are expressed in years. We denote the bond risk premium for maturity  $k$  and horizon  $j$  by

$$BRP_t^{(j,k)} \equiv E_t \left[ rx_{t+j}^{(k)} \right]. \quad (6)$$

Note that under the expectation hypothesis (EH),  $E_t \left[ i_{t+j}^{(k-j)} \right] = f_t^{(j,k)}$ , where  $f_t^{(j,k)} = \frac{ki_t^{(k)} - ji_t^{(j)}}{k-j}$  is the forward rate for  $k$  periods starting from  $j$  periods from time  $t$ , so that the risk premium for investing in long-term bonds is zero,  $BRP_t^{(j,k)} = 0$ .

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<sup>3</sup>The literature studying return predictability typically interprets expected excess log returns as risk premia. This is not quite correct since risk premia should be measured as expected excess simple returns. Assuming log-normality, arithmetic average returns differs from the geometric average returns by one half the variance (the Jensen's gap). Computing Jensen's terms from BlueChip Financial Forecasts, Buraschi, Piatti, and Whelan (2021) show that the Jensen's gap is tight using survey expectations. In what follows we interpret expected log excess returns as risk premia.

### B. Risk premia in the foreign exchange market

Denote by  $x_{t+j}$  the log of the exchange rate, expressed in US Dollars per unit of foreign currency, and by  $\Delta x_{t+j}$  the change in the log exchange rate from time  $t$  to time  $t + j$ . Therefore, a positive  $\Delta x_{t+j}$  corresponds to a depreciation of the US Dollar relative to the foreign currency. The  $j$ -period interest rate in the foreign country is denoted by  $i_t^{(j)*}$ . The annualised log currency excess return is given by:

$$rx_{t+j}^{FX} = (i_t^{(j)*} - i_t^{(j)}) + \frac{1}{j}\Delta x_{t+j}. \quad (7)$$

The exchange rate risk premium is defined as the conditional expectation of the excess return in Equation (7), i.e.,  $XRP_t^{(j)} \equiv E_t [rx_{t+j}^{FX}]$ :

$$XRP_t^{(j)} = (i_t^{(j)*} - i_t^{(j)}) + \frac{1}{j} (E_t [x_{t+j}] - x_t), \quad (8)$$

where  $E_t [x_{t+j}]$  is the forecast of the exchange rate in  $j$  periods.  $XRP_t^{(j)}$  can be interpreted as the annualised expected excess return of investing for  $j$  periods (using a  $j$ -period instrument, e.g. a 3-month bill if  $j = 0.25$ ) in the foreign currency, financing the investment selling the local currency (e.g. selling a 3-month T-bill). Also note that interest rates are annualised, so we also annualise the exchange rate change by dividing by  $j$ .

According to the uncovered interest rate parity (UIP), high interest rate countries are expected to experience an exchange rate depreciation to equalise expected exchange rate adjusted returns on assets. The idea behind UIP is that when the foreign interest rate is higher than the local interest rate, i.e.,  $i_t^{(j)*} > i_t^{(j)}$ , the foreign currency will depreciate by the difference, i.e.  $x_{t+j}$  decreases by  $i_t^{(j)*} - i_t^{(j)}$ , so that in local currency terms the return on investing in the two countries is exactly the same, i.e.  $XRP$  is zero.

## III. Subjective Risk Premia

We compute subjective bond risk premia and exchange rate risk premia for a twelve-month forecast horizon  $j$  and a bond maturity  $k$  of *eleven years*. Since  $j$  and  $k$  are fixed, in the following we drop them and refer to the premia as  $BRP_t$  (bonds) and  $XRP_t$  (exchange rates).<sup>4</sup>

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<sup>4</sup>CE Surveys provide us with the expected yield on a 10-year bond for a 1-year forecast horizon. In what follows, we compute subjective bond risk premia as the expected change in 11-year log bond prices above the 1-year risk free rate.



### A. Bond Risk Premia

We compute bond risk premia from Equation (6). Figure 1 shows the time series of survey-based  $BRP_t$  and panel (a) of Table I reports summary statistics. Subjective bond risk premia are slightly negative on average, volatile and persistent. For example, the GBP  $BRP$  ranges between -8% around the year 2000 to +10% in the aftermath of the 2008 financial crisis. The JPY  $BRP$  is less volatile but still displays significant persistence ranging between -3% pre financial crisis to +7% post financial crisis.

[ INSERT FIGURE 1 AND TABLE I ]

A negative  $BRP_t$  implies that default-free bonds are perceived as hedges, consistent with predictions from many structural models. Re-writing the definition for  $BRP_t$  above we have the following decomposition

$$BRP_t = 10 \times \left( i_t^{(11)} - E_t \left[ i_{t+1}^{(10)} \right] \right) + \underbrace{\left( i_{t+1}^{(11)} - i_t \right)}_{\text{Term Spread}} . \quad (9)$$

On average, the second term is positive since the (nominal) term spread is typically positive; thus a negative  $BRP_t$  means that the first term in parenthesis is negative. Assume date  $t$  interest rate beliefs can be written as a random walk forecast plus an adjustment  $\phi_t$ . The consensus belief can then be written as  $E_t \left[ i_{t+1}^{(10)} \right] = i_t^{(10)} + \phi_t$ . Then, since empirically  $i_t^{(11)} - i_t^{(10)} \sim 0$  this implies  $\phi_t > 0$ . i.e., conditional on the current level of the term structure, surveys expected long-term rates to rise. Actually, it turned out that in our sample there was an unprecedented decline in long term rates and forecasters made consistently biased errors. This point highlights the link between subjective risk premia and expectational errors. Expectation errors are discussed in section IV.

Do forecasters believe bonds are genuinely hedges or is this an artefact of the sample period? Consider again figure 1. In both first half and second half of the sample (when interest rates were relatively flat) we observe that  $BRP_t < 0$  most of the time. Table A.6 confirms this point more formally by providing subsample statistics. Investigating this question further, figure A.5 displays stock-bond correlations ( $corr_{SB}$ ) computed as the rolling 200-day correlation between 10-year zero coupon bond returns and the corresponding major equity market indices in each country. The figure demonstrates that the well studied change from positive to negative  $corr_{SB}$  that first

occurred in the late 1990's in the U.S. is a common feature in the cross-section of G10 countries in our sample. This is an interesting observation since observing  $corr_{SB} < 0$  suggests bonds should also be hedges according to CAPM logic. Moreover, in the 1990s when  $corr_{SB} > 0$  we do, in fact, observe  $BRPs$  which are largely positive. Thus, the average sign of subjective  $BRP_t$ s is consistent with a risk based view of bond pricing.

Another notable feature of our subjective measures of bond risk premia is their co-movement. Figure 1 displays a clear systematic pattern across countries, which is confirmed by a very high average cross-country correlation equal to 53%, and all pairwise correlations are positive ranging from 19% between GBP and NOK, to 76% between EUR and AUD (Table A.3).

In the time-series expected excess bond returns appear counter-cyclical, with risk premia being high in bad states such as the early 2000s and during the 2008 financial crisis, while in more recent years the average risk premia are lower, mainly negative and less volatile. We study the cyclical properties of subjective bond risk premia by estimating pooled OLS regressions of  $BRP_t$ s on *subjective* expectations of 1-year growth rates in GDP, industrial production, consumption, and inflation (Figure 3). Table II reports the findings. A constant is included in regression but not reported and standard errors reported in  $(\cdot)$  parenthesis are computed using a Driscoll and Kraay (1998) estimator with 4-lags.<sup>5</sup>

[ INSERT FIGURE 3 AND TABLE II HERE ]

Considering specification (i) – (iii) we find that lower expected real growth rates are significantly linked to higher subjective bond risk premia. Specification (iv) shows a relatively weak relationship with expected inflation. Specification (v) shows a highly significant and positive link between subjective bond risk premia and the realized variance of bond returns (equation 1), which proxies for the current quantity of risk in bond markets.

Finally, specification (vi) studies the link between survey-implied bond risk premia and realized excess bond returns over the past year, which answers the question of whether subjective beliefs are extrapolative or not. The point estimates do not display evidence of extrapolation. In fact, we find the opposite: past bond returns and consensus expectations of future returns are negatively

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<sup>5</sup>In a panel setting,  $N \times T$  autocorrelated and spatial dependent observations contain less information than  $N \times T$  independent observations. Driscoll and Kraay (1998) propose a nonparametric covariance matrix estimator that generates consistent standard errors which are robust to heteroskedasticity and general forms of temporal and spatial dependence. The choice of 4-lags is inline with the automatic selection length as outlined in Hoechle (2007). Regression estimates in the remainder of this section are computed with the same estimator.

correlated, consistent with predictions from benchmark asset pricing models with rational pricing of risk. Indeed, rational asset pricing models generally predicts that low prices due to discount rate variation generate high expected returns going forward.

### B. Exchange Rate Risk Premia

We compute exchange rate risk premia using equation 8. Figure 2 displays the dynamics of the  $XRP_t$ s and panel (b) of table I reports summary statistics. They are time-varying and volatile relative to their mean, with standard deviations ranging between 2.48 (CAD) and 5.58 (NZD). The average  $XRP_t$  is negative for Switzerland and Japan, equal to -1.69% and -2.76%, consistent with the idea that these are ‘safe-haven’ or ‘investment currencies’ within the carry trade, and positive around 3.5% for Norway and Sweden. For the remaining countries, average average exchange rate risk premia are smaller ( $< 1.74\%$ ) but positive. All pairwise correlations between exchange rate risk premia are positive and their average is 50%. The  $XRP_t$  of Japan is less correlated to the remaining countries’  $XRP_t$ , in fact excluding Japan from the sample of countries the average pairwise correlation increases to around 63%. Interestingly,  $XRP_t$ s flip sign in a systematic fashion throughout the sample, being largely positive between 1995 – 2004 and oscillating between positive and negative values thereafter.

[ INSERT FIGURE 2 HERE ]

Next, we investigate the cyclicity of subjective exchange risk premia via pooled OLS regressions of  $XRP_t$  on *subjective* macro expectations, as we do directly above. However, for exchange rates, one should expect risk premia to be correlated with macroeconomic activity in both domestic and foreign markets. Therefore, table III shows the results of a panel regression of  $XRP_t$ s on differences in subjective macro expectations defined as the foreign expected growth rate minus the U.S. expected growth rate. These estimates demonstrate a clear pattern: a positive divergence in expected real growth rates is associated with an increase in  $XRP_t$ ; thus, differences in business cycle dynamics are correlated with subjective exchange rate risk premia.

Table III also shows a positive and significant link between  $XRP_t$  and risk neutral return variance implied by FX options.<sup>6</sup> Finally,  $XRP_t$ s display evidence of mean-reversion around

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<sup>6</sup>We also find a positive and significant link to realised exchange rate variance although the statistical significance is smaller.

past realised returns, which contrasts with extrapolative beliefs that has been proposed as an explanation for return predictability by the behavioural finance literature.

[ INSERT TABLE III HERE ]

### C. Predictability Regressions

We estimate a set of predictability regressions in order to characterise the informational content of surveys risk premia as signals about future realised returns, and contrast these to statistical expected return signals. The benchmark signals we consider are the interest rate differential and slope of the yield curve, which we denote<sup>7</sup>

$$IRD_t = i_t^{(1)} - i_t^{(1)*} \quad (10)$$

$$Slope_t \equiv i_t^{(10)} - i_t^{(1)}. \quad (11)$$

Figure A.7 plot time-series for the  $Slope_t$  and  $IRD_t$  variables, and Table A.8 reports summary statistics.

#### C.1. Bond Risk Premia

Table IV reports estimates from the pooled OLS predictability regressions

$$rx_{t+1}^{(11)} = a + b_1 Slope_t + b_2 BRP_t + \epsilon_{t+1}. \quad (12)$$

where standard errors reported in  $(\cdot)$  are computed using Driscoll and Kraay (1998) covariance matrix estimator with 12-lags. Within country, 1-year excess returns are computed with monthly overlapping observations which induces an MA(11) error structure in the residuals. Correcting for this we include 12-lags in the Driscoll and Kraay (1998) estimator which is consistent with the recommended bandwidth in the Newey and West (1987) estimator. As an alternative and for robustness in  $[\cdot]$  we report confidence intervals computed using the Wild cluster bootstrap of Cameron, Gelbach, and Miller (2008).

[ INSERT TABLE IV HERE ]

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<sup>7</sup>Results that follow are unaffected by using the 11-year minus 1-year slope of the yield curve.

Panel (a) of table IV shows that the slope of the yield curve significantly positively predicts future realized returns, as expected. More importantly, survey-based expected excess bond returns are also positively and highly significantly linked to future realized excess bond returns, consistent with the results of Buraschi, Piatti, and Whelan (2021) for the U.S but contrary to what Greenwood and Schleifer (2014) document for the equity market. A common null for rational expectations would be  $H_0 : b_2 = 1$ . The estimates imply that the one-sided upper confidence interval is  $0.52 + 1.72 \times 0.15 = 0.78$  which rejects but is not “a million miles away”. There are, of course, many well known reasons for which this is not an appropriate null hypothesis for testing rationality. More importantly, we infer that  $b_2 > 0$  with a high degree of statistical confidence. And interestingly, the predictive power of our survey-based measure does not disappear when adding the slope as an additional predictor, even if the estimated factor loading  $b_2$  drops from 0.52 to 0.30. This results suggests that survey forecasts contain valuable information to predict future bond returns, which is not completely spanned by the information in current interest rate term structure. This ‘lack of spanning’ also suggests that forecast errors are predictable by date  $t$  information since they do not subsume the predictive ability of the *Slope*. This suggests that bond market beliefs are not fully rational consistent with existing evidence (Cieslak, 2017). Section IV studies forecast errors and predictability in expectation errors.

### C.2. Exchange Rate Risk Premia

Beginning with Fama (1984) a vast literature has tested the UIP condition via a predictability regression of log spot exchange rate changes on lagged interest rate differentials (or forward-spot spreads). With the definition for  $IRD_t$  above, from equation (7) the zero currency excess return condition says  $IRD_t - \Delta x_{t+12} = 0$  which can be tested via the Fama regression

$$\Delta x_{t+1} = \alpha + \beta IRD_t + \epsilon_{t+1}. \quad (13)$$

The UIP condition predicts that  $\alpha = 0$  &  $\beta = 1$  so that earning positive carry from the perspective of a U.S. investor ( $IRD_t < 0$ ) is be offset by a capital loss (a foreign currency depreciation,  $\Delta x_{t+1} < 0$ ) when repatriating the initial investment. Subtracting  $i_t^{(1)} - i_t^{(1)*}$  from both sides of the

regression

$$\Delta x_{t+1} - IRD_t = \alpha + \underbrace{(\beta - 1)}_{b_1} IRD_t + \epsilon_{t+1}, \quad (14)$$

we see that considering a Fama regression with excess returns as a dependent variable, UIP implies a regression coefficient  $b_1 = 0$ . Testing the predictability of excess exchange rate returns, we estimate the following pooled OLS regressions

$$rx_{t+1}^{FX} = a + b_1 IRD_t + b_2 XRP_t + \epsilon_{t+1}, \quad (15)$$

Panel (b) of table IV shows that the coefficient for the usual UIP predictor, the interest rate differential, is significantly negative, consistent with the literature. The estimated  $b_1 = -1.56$  meaning that not only do U.S. investors in high interest rate currencies earn positive carry but they also earn a capital gain when closing out their positions.

As above, a natural null hypothesis for the  $XRP_t$  coefficient is  $H_0 : a = 0 \ \& \ b_2 = 1$ , i.e., if surveys were fully rational (in a full information sense) we should expect  $rx_{t+12}^{FX} = XRP_t + \epsilon_{t+12}$ . Testing this null, we do not reject at conventional levels based on Driscoll and Kraay (1998) standard errors. Moving to specification (iii) we find that both the interest rate differential and  $XRP_t$  are strongly statistically significant. In fact, the point estimate of coefficient  $b_2$  does not change much. Considering the measure's stability even when controlling for natural predictors, this result demonstrates strong informational content. Notably, the  $R^2$  of the single-variable regressions in columns (i) and (ii) almost add up to  $R^2$  of the multivariate regression in column (iii). In summary, survey forecast of exchange rate returns not only significantly positively predict future realized excess returns but they do not *only* use information completely spanned by current interest rate spreads. As with the bond predictability regression above, while positively correlated with future realised returns their errors are predictable by the current observables ( $IRD_t$  in this case) suggesting that beliefs are not fully rational, consistent with existing literature (e.g., Frankel and Froot, 1987)

The findings of this section demonstrate that the information contained in survey-implied subjective risk premia are an important proxy for the underlying true unobservable time-varying (bond and FX) risk premia. However, we note that in all regressions above subjective risk premia

are not completely spanned by date  $t$  observables. This result does not mean that forecasters ignore information in interest rate spreads. Indeed, they combine information in the term structures with other variables as well as their own judgement and intuition when they build projections. Identifying the exact information and models that survey forecasters use is not an easy task, if at all feasible, since they likely assign time-varying weights to alternative models. For example, they might rely fully on their own judgement one month and instead follow a standard slope predictor another month. Moreover, individual forecasters probably follow different rules, and observing only the consensus of an unbalanced panel does not allow us to recover these individual rules. Overall however, our results show that we can use the consensus survey forecast as an aggregate, observable proxy of these unobservable and time-varying predictive information models.

#### IV. Expectation Errors

We evaluate the forecast errors from surveys. We take consensus (arithmetic average) projections about a target variable  $y_{t+1}$  and compute forecast errors as follows:

$$y_{t+1} - y_t = E_t^C [y_{t+12} - y_t] + FE_{t,t+1} \quad (16)$$

Note that by construction the errors of these projections are in real-time (out-of-sample).

Table V provides insights into the properties of forecast errors, by showing summary statistics of 10-year yield and exchange rate expectation errors for all countries in the sample. In particular the table displays the two drivers of the RMSE, i.e. the mean and standard deviation of the errors, as well as the minimum and maximum.<sup>8</sup>

For interest rates, the mean expectational error is always negative, meaning that all forecasts over-predicted the level of future interest rates, consistent with a downward trend in rates during our sample that was unpredictable ex-ante. For exchange rates, the mean expectational error are close to zero, meaning that there is no systematic bias.

[ INSERT TABLE V HERE ]

Studying systematic patterns in forecast errors, figure 5 displays the time series dynamics of the 10-year yield forecast errors (left panel) and the exchange rate errors (right panel), for

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<sup>8</sup>We discuss the potential extent of information frictions at the level of the consensus forecast using forecast errors and forecast revisions in the framework of Coibion and Gorodnichenko (2015) in the OA.

all countries in our sample. Interestingly, forecast errors across countries are highly correlated. Moreover, errors are volatile around zero and display only mild persistence, as evidenced by the AR(1) coefficients sampled at annual horizons (final row of each panel in Table V).

[ INSERT FIGURE 5 HERE ]

### A. Forecast Error Predictability

Next, we ask whether the mild persistence observed in the autocorrelation of  $t + 12$  errors results in predictability by factors observable at date  $t$ . Such a question is important since forecast error predictability suggests that consensus beliefs (measured from surveys) are not incorporating all available information. Table VI shows individual country regressions of (a) 10-year yield or (b) foreign exchange forecast errors, at the annual horizon, on the level and slope of the yield curve and the 1-year interest rate differential, respectively.

$$FE_{t,t+1}^i = a^i + b^i X_t^i + \eta_{t+1}^i, \quad (17)$$

where for interest rate forecast errors  $X_t$  includes the  $Level_t = i_t^{(1)}$  and  $Slope_t = (i_t^{(10)} - i_t^{(1)})$  and for exchange rates  $IRD_t = (i_t^{(1)} - i_t^{*,(1)})$  and t-statistics, reported in parenthesis, are computed based on Newey-West standard errors computed with 12-lags.

Considering first panel (a) which displays interest rate forecast error predictability, we observe that the  $Level_t$  of the yield curve is significant at 5% for almost all of the countries in our panel, while the  $Slope_t$  of the yield curve is significantly linked to future forecast errors only for four of the countries. The point estimates on the  $Level_t$  are all positive, with the exception of GBP, meaning that a negative shock to the date  $t$  interest rate causes agents to under predict future interest rates, consistent with the idea that agents generally perceive rate shocks to be *more* persistent than they turns out to be ex-post.<sup>9</sup>

Consider now panel (b) which analyzes the predictability of exchange rate forecast errors. Exchange rate errors are also predictable by  $IRD_t$  with point estimates that are uniformly negative and statistically significant at 5% for almost all countries. For AUD and CHF the relationship between current interest rate differential and future forecast error is quite strong, with  $R^2$ 's of 26% and 30%, respectively, while for the other countries the predictability is limited. The negative sign

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<sup>9</sup>Piazzesi, Salomao, and Schneider, 2015 document a related result using BlueChip Financial Forecasts.



has the opposite interpretation than for interest rate errors: a negative shock to  $IRD_t$  causes agents to over predict future exchange rates, consistent with the idea that agents tend to perceive rate shocks to be *less* persistent than they turns out to be ex-post.

[ INSERT TABLE VI HERE ]

These findings suggests agents do not exploit all available information when forming their beliefs, which is inconsistent with the basic idea of rational expectations. However, to the best of our knowledge it remains an open question whether statistical predictability can be exploited to correct errors in real-time. On in the online appendix (Section A.3) we provide a partial answer to this question by designing an experiment which constructs fictitious expectations that correct for predictability in errors using date  $t$  observables. Summarising, we find that ‘uncorrected’ beliefs dominates their corrected counterparts in a mean-square-error sense, mainly in terms of variability, meaning that predictability in agents errors does not easily translate into forecast improvements.

## V. An Equilibrium Model with Subjective Beliefs

In this section, we ask whether an off-the-shelf asset pricing model adapted to incorporate subjective beliefs is able to match the salient features of (i) subjective risk premia; and (ii) expectation errors across sovereign bond and exchange rate markets. We will consider a 3-country world consisting of Australia, Switzerland and the U.S. and estimate the model via simulated methods of moments by exploiting beliefs from surveys and asset prices jointly. A full derivation is reported in the OA.

### A. Belief Distortion Example: Inflation

Consider an economy where the state-variables are adapted to the filtered probability space  $(\Omega, \mathcal{F}_{t \in T}, \mathbb{P})$ . The price level in this economy evolves according to

$$\frac{dQ_t}{Q_t} = i_t dt + \sigma_Q \sqrt{i_t} dW_t^{\mathbb{P}, Q}, \quad (18)$$

$$di_t = \kappa_i^{\mathbb{P}} (\theta_i^{\mathbb{P}} - i_t) dt + \sigma_i \sqrt{i_t} dW_t^{\mathbb{P}, i}, \quad (19)$$

where the correlation between shocks is  $\langle dW_t^{\mathbb{P}, Q}, dW_t^{\mathbb{P}, i} \rangle = \rho_{iQ} dt$ . The  $dQ_t$  process has a time-varying conditional mean component ( $i_t$ ) that follows a stationary CIR process. With a sufficiently

long history of data at hand an econometrician could estimate the parameters of this process from realisations versus conditional projections  $\left(\frac{dQ_t}{Q_t} - E_t^{\mathbb{P}}\left[\frac{dQ_t}{Q_t}\right]\right)$ , i.e., by observing forecast errors and updating their beliefs. Expected future inflation will be given by

$$E_t^{\mathbb{P}}[\ln Q_{t+\tau} - \ln Q_t] = \left(1 - \frac{1}{2}\sigma_Q^2\right)\left[\theta_i^{\mathbb{P}} \cdot \tau - \frac{1}{\kappa_i^{\mathbb{P}}}(i_t - \theta_i^{\mathbb{P}})(e^{-\kappa_i^{\mathbb{P}}\tau} - 1)\right]. \quad (20)$$

Now imagine an investor who only has a short sample of data available, or learns sub-optimally, or suffers from some behavioural biases. In this case, their beliefs will be formed under a subjective measure  $\mathbb{S}$ , which will not necessarily coincide with the physical (objective) measure  $\mathbb{P}$ . In this case their beliefs will be adapted to a different filtered probability space  $(\Omega, \mathcal{F}_{t \in T}, \mathbb{S})$  and subjective expected inflation will be given by

$$di_t = \kappa_i^{\mathbb{S}}(\theta_i^{\mathbb{S}} - i_t)dt + \sigma_i\sqrt{i_t}dW_t^{\mathbb{S},i}, \quad (21)$$

$$E_t^{\mathbb{S}}[\ln Q_{t+\tau} - \ln Q_t] = \left(1 - \frac{1}{2}\sigma_Q^2\right)\left[\theta_i^{\mathbb{S}} \cdot \tau - \frac{1}{\kappa_i^{\mathbb{S}}}(i_t - \theta_i^{\mathbb{S}})(e^{-\kappa_i^{\mathbb{S}}\tau} - 1)\right] \quad (22)$$

$$\neq E_t^{\mathbb{P}}[\ln Q_{t+\tau} - \ln Q_t] \quad (23)$$

More formally, this means that  $E_t^{\mathbb{P}}[i_s] = E_t^{\mathbb{S}}\left[\frac{d\mathbb{P}}{d\mathbb{S}}i_s\right]$  where from Girsanov<sup>10</sup>

$$\frac{d\mathbb{P}}{d\mathbb{S}} = M_t^{\mathbb{S},\mathbb{P}} = \exp\left(-\int_t^s \Delta_u dW_u^{\mathbb{P},i} - \frac{1}{2}\int_0^t \Delta_u^2 du\right), \quad (24)$$

$$dW_t^{\mathbb{P},i} = dW_t^{\mathbb{S},i} + \Delta_t^i dt \quad (25)$$

We call  $\Delta_t^i$  a BELIEF DISTORTION. Ignoring micro-foundations and for analytical simplicity, we assume  $\Delta_t^i = \phi_i\sqrt{i_t}$ . Then, we can relate the beliefs of the investor and the econometrician as follows

$$\theta_i^{\mathbb{P}} = \frac{\kappa_i^{\mathbb{S}}\theta_i^{\mathbb{S}}}{\kappa_i^{\mathbb{S}} + \phi_i\sigma_i}, \quad \kappa_i^{\mathbb{P}} = \kappa_i^{\mathbb{S}} + \phi_i\sigma_i. \quad (26)$$

So that if  $\phi_i = 0$  subjective real time beliefs coincide with the beliefs inferred from an econometrician with full information but, in general, they will be different. For a forecast horizon of 1-year, the forecast error that the agent makes is given by  $FE_{t,t+1}^Q = \ln Q_{t+1} - \ln Q_t - E^{\mathbb{S}}[\ln Q_{t+\tau} - \ln Q_t]$ .

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<sup>10</sup>We assume  $\mathbb{S}$  is absolutely continuous with respect to  $\mathbb{P}$ .

The unconditional average forecast error is given by

$$\begin{aligned}
E^{\mathbb{P}}[FE_{t,t+1}^Q] &= (1 - \frac{1}{2}\sigma_Q^2) (\theta_i^{\mathbb{P}} - \theta_i^{\mathbb{S}}) \left( 1 + \frac{e^{-\kappa_i^{\mathbb{S}}} - 1}{\kappa_i^{\mathbb{S}}} \right) \\
&= \theta_i^{\mathbb{S}} (1 - \frac{1}{2}\sigma_Q^2) \left( \frac{-\phi_i\sigma_i}{\kappa_i^{\mathbb{S}} + \phi_i\sigma_i} \right) \left( 1 + \frac{e^{-\kappa_i^{\mathbb{S}}} - 1}{\kappa_i^{\mathbb{S}}} \right)
\end{aligned} \tag{27}$$

and so  $\phi_i > 0$  implies negative average forecast errors and vice-versa, as long as the belief distortion is not too negative, i.e.  $\phi_i > \frac{-\kappa_i^{\mathbb{S}}}{\sigma_i}$ . At the same time a  $\phi_i > 0$  ( $\phi_i < 0$ ) lowers (raises) the subjectively perceived persistence of inflation shocks relative to physical beliefs.

The model above can be estimated via simulated method of moments (SMM) which is analogous to the generalized method of moments (GMM) estimator, but allows us to estimate the parameters of latent factors which are not directly observable. Section A.5 in the OA reports estimation details. The moments we target in estimation are the mean, standard deviation and monthly autocorrelation of 12-month realised inflation and 12-month subjective expected inflation from Consensus Economics. In addition, by combining realisations and expectations we obtain 12-month horizon forecast errors, displayed in figure A.13 in the OA, from which we compute two additional non-redundant moments, i.e. the standard deviation and autocorrelation of the errors.

Table VII reports parameter estimates, empirical moments, and model implied estimates alongside 95% confidence intervals for Australia. Tables A.15 and A.16 in the OA report estimates for Switzerland and the U.S. The estimates imply that the conditional mean of inflation is a low volatility ( $\sigma_i = 0.03$ ) and persistent ( $\kappa_i^{\mathbb{S}} = 0.07$ ) process and that there is a strong positive correlation ( $\rho_{iQ} = 0.85$ ) between shocks to price level and expected inflation. The subjectively perceived mean of the inflation process is  $\theta_i^{\mathbb{S}} = 2.6\%$  and we estimate a positive belief distortion,  $\phi_i = 0.36$ . Considering the empirical versus model implied moments we see the model is doing a good job at matching realisations, expectations and forecast error dynamics, and confidence intervals are relatively tight. Note that the mean forecast error in the data is slightly negative, which is matched exactly, since the positive distortion  $\phi_i$  implies that the inflation process has a higher persistence and a lower long term mean under the physical measure  $\mathbb{P}$  than under the subjective measure  $\mathbb{S}$ . More precisely, the implied physical unconditional mean is  $\theta_i^{\mathbb{P}} = 2.23\%$ , which is lower than the subjective unconditional mean  $\theta_i^{\mathbb{S}} = 2.60\%$ . With this example in mind, we next discuss how this idea can be applied to subjective asset pricing.

[ INSERT TABLE VII HERE ]

### B. Subjective Risk Premia

We now provide a decomposition mapping full information rational expectations risk pricing to subjective pricing of risk, considering the following variation on the homogeneous agent first order condition and assuming absence of arbitrage and complete markets. The nominal price of a  $T$  period bond is given by the inflation ( $Q_t/Q_T$ ) adjusted expected payoff discounted by the real SDF ( $\Lambda_t^r$ ). The nominal SDF is given by  $\Lambda_t = \Lambda_t^r Q_t^{-1}$  and for  $t < s < T$  bond prices satisfy

$$P_t^T = E_t^{\mathbb{P}} \left[ \frac{\Lambda_s}{\Lambda_t} P_s^{T-s} \right] \quad (28)$$

$$= E_t^{\mathbb{P}} \left[ e^{-\int_t^s r_u du} M_s^{\mathbb{P},\mathbb{Q}} P_s^{T-s} \right] \quad (29)$$

$$= E_t^{\mathbb{Q}} \left[ e^{-\int_t^s r_u du} P_s^{T-s} \right] \quad (30)$$

where  $\mathbb{Q}$  denotes the risk-neutral measure and  $M_t^{\mathbb{P},\mathbb{Q}}$  is the Radon-Nikodym derivative  $\frac{d\mathbb{Q}}{d\mathbb{P}}$ . Now consider the factorisation of the risk (Martingale) component of the physical SDF:  $M_t^{\mathbb{P},\mathbb{Q}} = M_t^{\mathbb{S},\mathbb{Q}} M_t^{\mathbb{P},\mathbb{S}}$ . where for an arbitrary Brownian shock, the changes of measure satisfy

$$dW^{\mathbb{Q}} = dW^{\mathbb{P}} + \Theta_t^{\mathbb{P}} dt \quad , \quad dW^{\mathbb{Q}} = dW^{\mathbb{S}} + \Theta_t^{\mathbb{S}} dt, \quad (31)$$

$$dW^{\mathbb{P}} = dW^{\mathbb{S}} + \Delta_t dt \quad \rightarrow \quad \Delta_t = \Theta_t^{\mathbb{S}} - \Theta_t^{\mathbb{P}}. \quad (32)$$

The first equation is standard, relating risk-adjusted and physical measures via physical risk prices ( $\Theta_t^{\mathbb{P}}$ ). The second equation is analogous but relates the risk adjusted measure to subjective risk prices ( $\Theta_t^{\mathbb{S}}$ ). Internal consistency and complete markets then reveals that the difference between subjective and physical risk prices is a BELIEF DISTORTION ( $\Delta_t$ ) relating physical and subjective measures.

Departing from the traditional view of rational expectations the nominal price of a  $T$  period

bond can equivalently be written as

$$P_t^T = E_t^{\mathbb{P}} \left[ e^{-\int_t^s r_u du} M_s^{\mathbb{S},\mathbb{Q}} M_s^{\mathbb{P},\mathbb{S}} P_s^{T-s} \right] \quad (33)$$

$$= E_t^{\mathbb{S}} \left[ e^{-\int_t^s r_u du} M_t^{\mathbb{S},\mathbb{Q}} P_s^{T-s} \right] \quad (34)$$

$$= E_t^{\mathbb{Q}} \left[ e^{-\int_t^s r_u du} P_s^{T-s} \right]. \quad (35)$$

From well known manipulations subjective risk premia are given by

$$E_t^{\mathbb{S}}[dR_t] - E_t^{\mathbb{Q}}[dR_t] = -E_t^{\mathbb{S}} \left[ \frac{d\Lambda_t}{\Lambda_t} \cdot \frac{dP_t^T}{P_t^T} \right] \quad (36)$$

but note that subjective risk premia are determined by the covariance of returns with the subjective SDF  $\Lambda_t$  whose shock sensitivities are given by subjective prices of risk  $\Theta_t^{\mathbb{S}}$  and evaluated under the subjective measure  $\mathbb{S}$ .

### C. Two Factor Production Economy

Consider a linear production economy where the growth of the capital stock is driven by two Cox, Ingersoll Jr, and Ross (1985) factors adapted to the filtered probability space  $(\Omega, \mathcal{F}_{t \in T}, \mathbb{S})$  and evolves as

$$\frac{dK_t}{K_t} = \left[ (g_t - \beta i_t) - \frac{C_t}{K_t} \right] dt + \sigma_{K,g} \sqrt{g_t} dW_t^{\mathbb{S},g} + \sigma_{K,i} \sqrt{i_t} dW_t^{\mathbb{S},i}; \quad K_0 \geq 0 \ \& \ \beta \geq 0 \quad (37)$$

$$dg_t = \kappa_g (\theta_g - g_t) dt + \sigma_g \sqrt{g_t} dW_t^{\mathbb{S},g}; \quad g_0 \geq 0, \quad (38)$$

$$di_t = \kappa_i (\theta_i - i_t) dt + \sigma_i \sqrt{i_t} dW_t^{\mathbb{S},i}; \quad i_0 \geq 0. \quad (39)$$

The hidden states are time-varying stochastic volatility processes and have opposite effects on the drift of  $dK_t$ ; they can be interpreted as “good” ( $g_t$ ) and “bad” ( $i_t$ ) volatility factors as in the “good” uncertainty “bad” uncertainty literature (see Segal, Shaliastovich, and Yaron, 2015, and the references therein).

While the  $i_t$  can be interpreted as expected instantaneous inflation, the  $g_t$  factor is purely latent and does not have a clear interpretation in terms of a macro economic variable. Instead, it captures residual capital growth beyond inflation. Since  $dW_t^{\mathbb{P},\mathbb{Q}}$  shocks are, in principle, observable, we assume their distribution is identical under  $\mathbb{S}$  and  $\mathbb{P}$ . The shock correlation between shocks to

the price level and subjective inflation is denoted  $\langle dW_t^Q, dW_t^{\mathbb{S},i} \rangle = \rho_{i,Q} dt$ .

The investor portfolio choice problem is

$$\sup_C E_t^{\mathbb{S}} \left[ \int_t^\infty e^{-\rho s} \frac{C_s^{1-\gamma}}{1-\gamma} ds \right] \quad (40)$$

In the log-utility case ( $\gamma = 1$ ) we can obtain closed form solutions to this program, while in the general case ( $\gamma \neq 1$ ) solutions are available in semi-closed form. For parsimony here we consider the log-utility case in which optimal consumption is given by  $C_t = \rho K_t$ . The investors nominal subjective SDF is given by  $\Lambda_t = \Lambda_t^r Q_t^{-1}$  where  $\Lambda_t^r = U_C(C_t, t)$  is the investors real subjective SDF. The diffusion for  $\Lambda_t$  is given by

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t dt - \Theta_t^g dW_t^{\mathbb{S},g} - \Theta_t^i dW_t^{\mathbb{S},i} - \Theta_t^Q dW_t^Q \quad (41)$$

$$r_t = \underbrace{\left(1 - \sigma_{K,g}^2\right)}_{r_g} g_t + \underbrace{\left(1 - \beta - \sigma_Q^2 - \sigma_{K,i}^2 - \rho_{i,Q} \sigma_{K,i} \sigma_Q\right)}_{r_{\mathbb{S},i}} i_t \quad (42)$$

$$\Theta_t^{\mathbb{S},g} = \sigma_{K,g} \sqrt{g_t} \quad , \quad \Theta_t^{\mathbb{S},i} = \sigma_{K,i} \sqrt{i_t} \quad , \quad \Theta_t^Q = \sigma_Q \sqrt{i_t} \quad (43)$$

The solution for bond prices is exponentially affine and given by

$$P_t^\tau = e^{-A_\tau^{\mathbb{S}} - B_\tau^{\mathbb{S},g} g_t - B_\tau^{\mathbb{S},i} i_t} \quad (44)$$

where the factors loadings are known in closed form and reported in the OA. Note that our complete markets assumption and a fully optimising representative investor implies the risk neutral measure is unique. Thus, for a given realisation of the state of nature all asset prices are singular meaning that the factor loadings under  $A_\tau^{\mathbb{S}}$  and  $A_\tau^{\mathbb{P}}$  are identical (we show this formally in the OA) and so we drop their superscripts. The term structure of interest rates is then given by  $y_t^{\mathbb{S}\tau} = a_\tau^{\mathbb{P}} + b_\tau^{\mathbb{P},g} g_t + b_\tau^{\mathbb{P},i} i_t$  where the yield factor loadings are equal to the bond pricing factor loadings multiplied by  $\tau^{-1}$ .

Given these solutions, time-varying nominal subjective bond risk premia on a  $T$  period bond are given by

$$BRP_t = -E_t^{\mathbb{S}} \left[ \frac{d\Lambda_t}{\Lambda_t} \cdot \frac{dP_t^T}{P_t^T} \right] \quad (45)$$

$$= -B_\tau^g \sigma_g \sigma_{K,g} g_t - B_\tau^i (\sigma_i \sigma_{K,i} + \sigma_i \sigma_Q \rho_{i,Q}) i_t. \quad (46)$$

Recently, efforts have been devoted to endogenizing belief distortions:  $\Delta_t$ . Micro-foundations for belief distortions have been proposed in settings where agents suffer behavioural biases or are subjective to information frictions biases (see Coibion and Gorodnichenko (2015) for an overview) or in settings where agents have preferences for statistical robustness (for recent examples, see Maenhout, Vedolin, and Xing, 2021 and Bhandari, Borovička, and Ho, 2022). Abstracting from a specific mechanism for subjective belief formation, we specify the following parametric specification for *priced* shocks

$$\Theta_t^{\mathbb{P},g} = \Theta_t^{\mathbb{S},g} - \underbrace{\phi_g \sqrt{g_t}}_{\Delta_t^g} = (\sigma_{K,g} - \phi_g) \sqrt{g_t} \quad (47)$$

$$\Theta_t^{\mathbb{P},i} = \Theta_t^{\mathbb{S},i} - \underbrace{\phi_i \sqrt{i_t}}_{\Delta_t^i} = (\sigma_{K,i} - \phi_i) \sqrt{i_t} \quad (48)$$

In our simulations, we use the physical dynamics of the latent processes (see Section A.4 for the expressions) to obtain the dynamics of the realized yields, and then compare them with their subjective conditional expectations to obtain the yield forecast errors. More precisely, for a forecast horizon of 1-year, the yield forecast error that the agent makes is given by

$$FE_{t,t+1}^y = y_{t+1}^{\$ \tau} - E_t^{\mathbb{S}}[y_{t+1}^{\$ \tau}]. \quad (49)$$

and the unconditional average forecast error as perceived by an econometrician is  $E_t^{\mathbb{P}}[FE_{t,t+1}^y]$ .

In order to solve for the exchange rate risk premium we need to make an assumption about the foreign countries. Let us assume there are  $N + 1$  consumption goods and  $N + 1$  countries: “home” country (the U.S.) and  $N$  “foreign” countries. As in Colacito and Croce (2011), we assume that each country behaves as in autarky, in both consumption and financial assets (total home bias) so that a representative investor within each country only consumes the good which they are endowed.

We assume foreign country dynamics have the same structure as in equation (A.15) but with different shocks and potentially different parameters. In particular, foreign country growth is also affected by two separate state variables. These are uncorrelated with each other but correlated with the corresponding shocks in the home country, with correlations  $\rho^g$  and  $\rho^i$ , respectively. Cross-country correlation among the state variables generates global and local factor pricing of

risk as in Lustig, Roussanov, and Verdelhan (2011). We also assume that the correlation between shocks in the two foreign countries is completely driven by their exposure to the home country shocks.<sup>11</sup>

Investors can trade in both domestic and foreign bond markets. Denote  $X_t$  the real exchange rate in the U.S good per the foreign good. When  $X_t$  goes up, the US dollar depreciates in real terms. If markets trade without frictions then the U.S. dollar price of the foreign bond ( $P_{t,\tau}^*$ ) is given by

$$\Lambda_t X_t P_{t,\tau}^* = E_t^{\mathbb{S}} \left[ \Lambda_{t+1} X_{t+1} P_{t+1,\tau-1}^* \right] \quad (50)$$

which implies that exchange rates are pinned down by the ratio of the SDFs  $X_t = \Lambda_t^* \Lambda_t^{-1}$ . and where the dynamics of the foreign SDF  $\Lambda_t^*$  follows exactly the same structure as the domestic SDF in Equation (41) but with the country-specific parameters and shocks.

The exchange rate risk premium is obtained as the negative of the covariance between the SDF and exchange rate changes, which is equal to the drift in the exchange rate dynamics plus the interest rate differential (equivalent to Equation (8)):

$$XRP_t = -\frac{1}{dt} Cov_t^{\mathbb{S}} \left( \frac{d\Lambda_t}{\Lambda_t}, \frac{dX_t}{X_t} \right) = \frac{1}{dt} Var_t^{\mathbb{S}} \left( \frac{d\Lambda_t}{\Lambda_t} \right) - \frac{1}{dt} Cov_t^{\mathbb{S}} \left( \frac{d\Lambda_t}{\Lambda_t}, \frac{d\Lambda_t^*}{\Lambda_t^*} \right) \quad (51)$$

$$= (r_t^* - r_t) + \frac{1}{dt} E_t^{\mathbb{S}} \left[ \frac{dX_t}{X_t} \right]. \quad (52)$$

which is spelled out explicitly in the OA.

#### D. Estimation Results

We estimate the model above using the simulated method of moments, considering a three-countries world, where the home country is the U.S. and the two foreign countries are Switzerland and Australia. The inflation parameters for all countries are fixed to the values estimated in Section V.A (see Tables VII, A.15 and A.16. All other parameters are estimated jointly<sup>12</sup> for a total of 22 parameters to estimate (Six for each country, plus two correlation parameters for each

<sup>11</sup>Note that the correlation between the domestic and foreign price level shocks, i.e.  $\rho^Q$ , is implied by the correlation between the inflation and price level shocks in each country and the correlation between inflation shocks in the two countries,  $\rho^Q = \rho_{i,Q} \rho_{i,Q}^*$ .

<sup>12</sup>Only parameter  $\sigma_{K,g}$  is fixed equal to 1 for all countries, based on preliminary estimates of single-country versions of our model for each country, which are unreported but available upon request.



of the foreign countries). In the estimation we use information from the term structures of yields in the three countries (mean, standard deviation and first order autocorrelation of the 3-month yield, as well as the mean 5 and 10-year yields), subjective bond risk premia and forecast errors (mean, standard deviation, skewness and autocorrelation, for all countries) and we complement these moments with information about the exchange rate risk premia in the foreign countries and correlations among bond and foreign exchange risk premia to pin down also the correlation parameters, for a total of 47 moments.

Table VIII reports parameter estimates alongside 95% confidence intervals, while Tables IX and X report empirical moments and model implied estimates, as well as 95% confidence intervals. The first thing to notice is that this simple model fits the unconditional mean and volatility of the bond and exchange rate risk premia remarkably well. In particular, the estimated average bond risk premia are negative, while the exchange rate risk premium is positive for AUD and negative for CHF, as in the data. The model also captures the correlations between risk premia, with a high estimated correlation between risk premia in the U.S and in the foreign countries. The main shortcoming of the model seems to be the fitting of the yield curves, with the model implying slightly downward sloping term structures for the U.S. and Australia, while the observed term structures are all slightly upward sloping.

Interestingly, the estimated belief distortions for the latent factor shocks ( $\phi_g$ ) are all very small, and exactly zero for the US, implying that the estimated belief distortion in the inflation processes is effectively enough to capture the properties of the subjective forecast errors on yields, which are slightly negative on average and volatile both in the model and in the data.

In terms of model parameters, we can see that shocks tend to be highly correlated across countries (see panel (d) of Table VIII). Inflation shocks in the US and Australia are particularly highly correlated, with an estimated  $\rho_{i2}$  of almost 60%. This means that both factors are more global than local, with inflation being particularly correlated across countries.

[ INSERT TABLES VIII, IX and X HERE ]

INTUITION: BELIEF DISTORTION. **DISCUSS**

INTUITION: SUBJECTIVE BOND RISK PREMIA. Shocks to  $g_t$  and  $i_t$  have opposing effects on the short term interest rate and therefore inherit positive ( $B_\tau^g > 0$ ) and negative ( $B_\tau^i < 0$ ) factor

loadings in the bond pricing function (equation 44 & figure 6). The subjective  $BRP_t$  in equation 46 is thus composed of two terms. The first term is negative since bonds hedge states of high marginal utility ( $dg_t \downarrow dK_t \downarrow dP_t^r \uparrow$ ). The second term is positive since bonds are risky bets on inflation ( $di_t \downarrow dK_t \downarrow dP_t^r \downarrow$ ). For all countries, SMM fits a negative average  $BRP_t$  but chooses the first term to dominate the second term; however, it also chooses a parameter set that allows that allows the  $BRP_t$  to flip sign. Considering again figure 1 we see that periods of conditionally negative bond risk premia are the ‘norm’ where bonds are acting as hedges against regular (good) growth rate shocks. Periods of positive bond risk premia, such as in the early 2000s or during the GFC, are states when bonds are perceived as risky bets due to expected inflation shocks when also generate long run negative productivity expectations ( $E_t^S[dK_t]$ ). **DISCUSS figure 7**

INTUITION: SUBJECTIVE FX RISK PREMIA. From equation 51 currency risk premia can be written as

$$XRP_t = \underbrace{(\Theta_t^{S,g})^2 + (\Theta_t^{S,i})^2 + (\Theta_t^{S,Q})^2}_{\text{USD SDF variance}} \quad (53)$$

$$- \underbrace{\Theta_t^{S,g}\Theta_t^{S,g,*}\rho_g + \Theta_t^{S,i}\Theta_t^{S,i,*}\rho_i + \Theta_t^{S,Q}\Theta_t^{S,Q,*}\rho_Q}_{\text{SDF covariances}} \quad (54)$$

**DISCUSS case of super high correlation in which sign is driven by the relative magnitude of the prices of risk. The intuition comes from thinking about how prices of risk amplify shocks to the short rate.**

[ INSERT FIGURES 6 and 7 HERE ]

## VI. Conclusion

This paper exploits survey data on bond yields and exchange rates to jointly estimate risk premia in the foreign exchange and fixed income markets for the G10 countries. Subjective expected excess returns are obtained directly from a panel of investor forecasts allowing us to measure model free real-time risk premia on bonds and currencies, and then combine them to study the risk premium on an economically important investment strategy that buys a foreign long-term bond and sells a long-term U.S bond.

We show that subjective risk premia are highly correlated across countries, volatile and tend to be countercyclical, consistent with leading asset pricing models. We also show that subjective risk premia are significantly positively linked to the realized volatility; therefore, are consistent with the basic idea of a risk-return tradeoff.

Importantly, we show that the subjective risk premia, in both fixed income and exchange rate markets, significantly positively predict future realized excess returns and the predictive power goes beyond that of standard predictors like the interest rate differential and the slope of the term structure. This finding suggests that survey forecasters use information beyond that spanned by current interest rate spreads when forming their expectations.

We argue that a significant link between subjective risk premia, subjective macro expectations and the quantity of risk, support asset pricing models that generate return predictability through cyclical variation in risk aversion, uncertainty, the likelihood of disasters or rational learning. Indeed, we conclude by deriving and estimating an asset pricing model in which return predictability arises through with time-variation in economic uncertainty. The model implied moments line up closely with the empirical survey based moments which demonstrates that subjective risk premia are well explained by a traditional model, irrespective of whether belief formation is rational or not.

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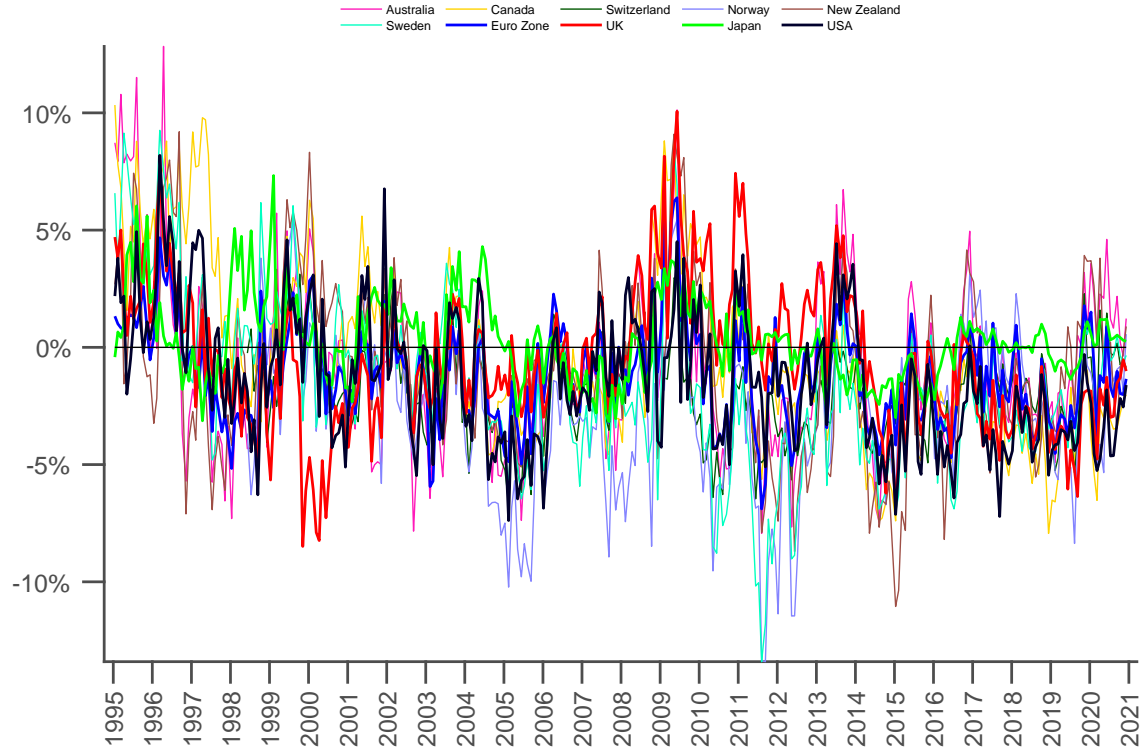
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## VII. Figures



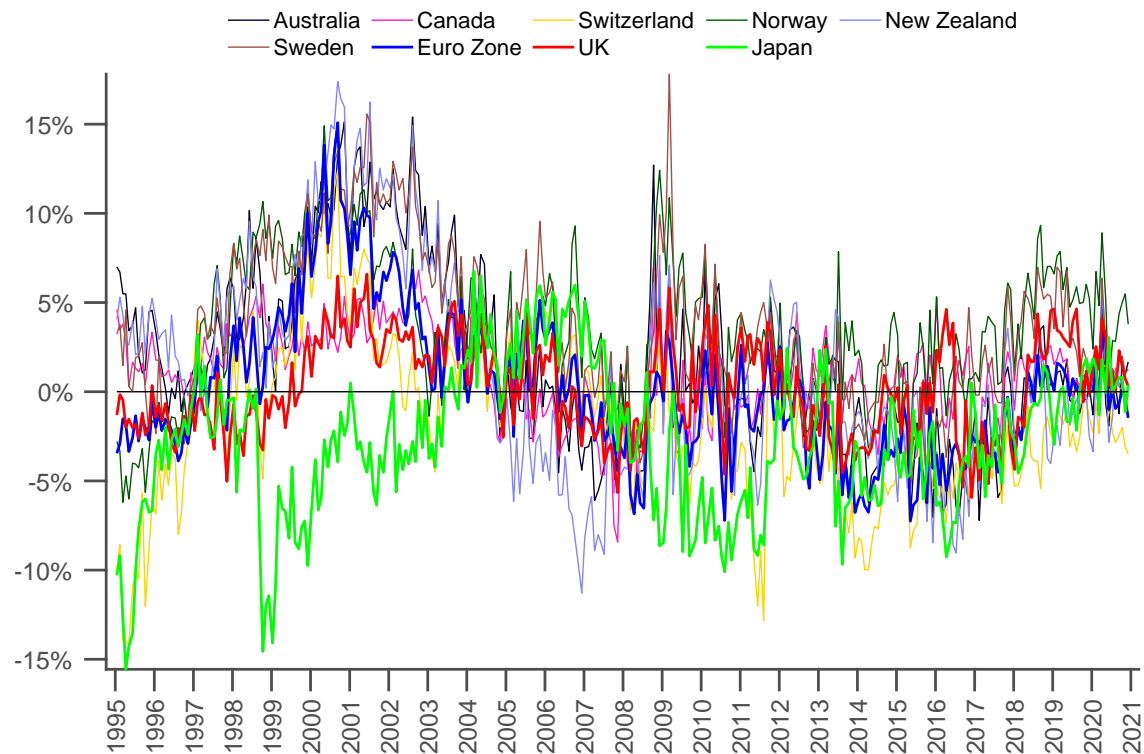
**Figure 1. Subjective Bond Risk Premia**

Figure displays real-time bond risk premia

$$BRP_t^{11} = E_t^S [rx_{t+1}^{11}] = E_t^S [p_{t+1}^{10}] - p_t^{(11)} - i_t^1$$

where  $p_t^n$ 's is the log zero coupon bond prices for maturity  $n$  and  $i_t^1$  is the continuously compounded one-year interest rate. The sample period is 1995.1 to 2020.12.



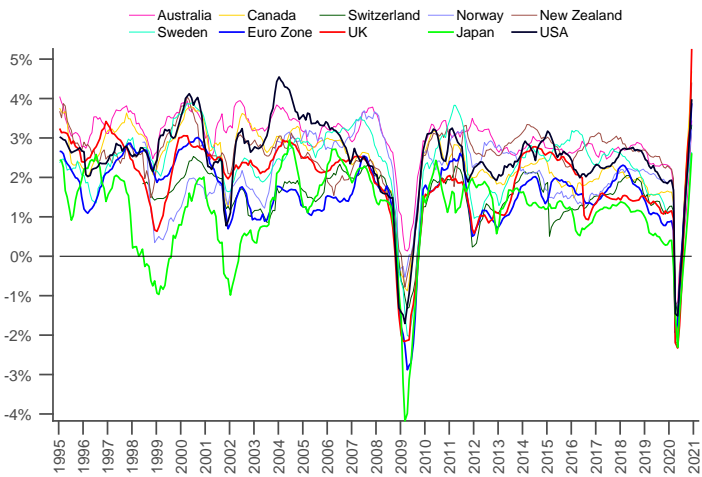


**Figure 2. Subjective Exchange Rate Risk Premia**

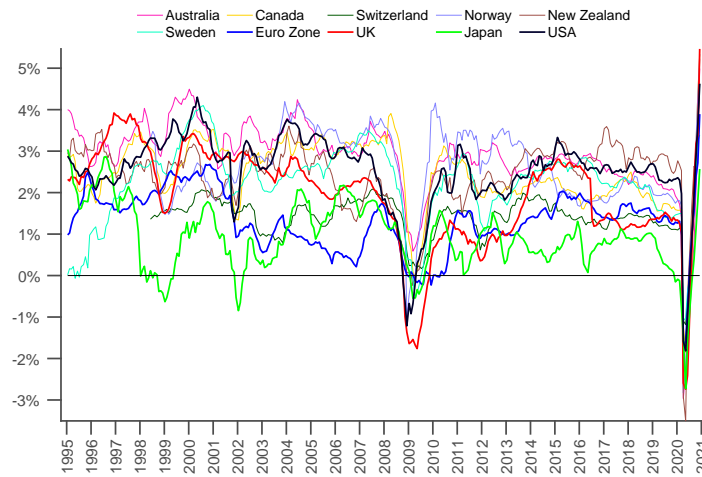
Figure displays real-time exchange rate risk premia

$$XRP_t = E_t^{\mathbb{S}} [rx_{t+1}^{FX}] = (i_t^{1,*} - i_t^1) + E_t^{\mathbb{S}} [\Delta x_{t+1}]$$

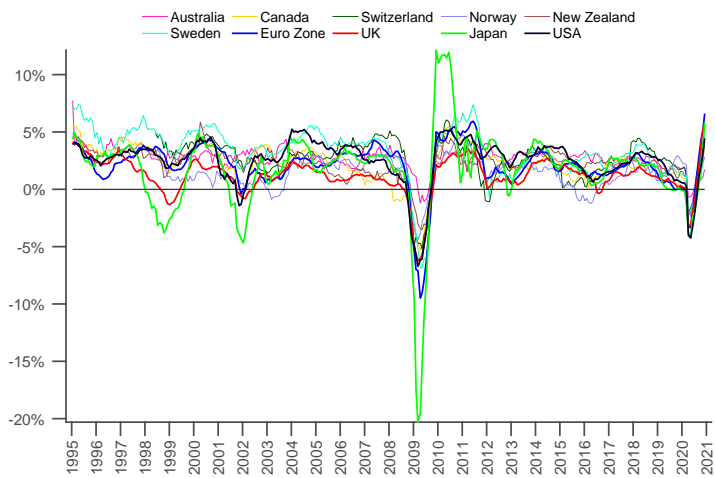
where the 1-year change in the log exchange rate is denoted  $\Delta x_{t+1}$ , and  $i_t^{(1)}$  is the continuously compounded one-year interest rate in the foreign (\*) versus domestic (U.S.) markets. The sample period is 1995.1 to 2020.12.



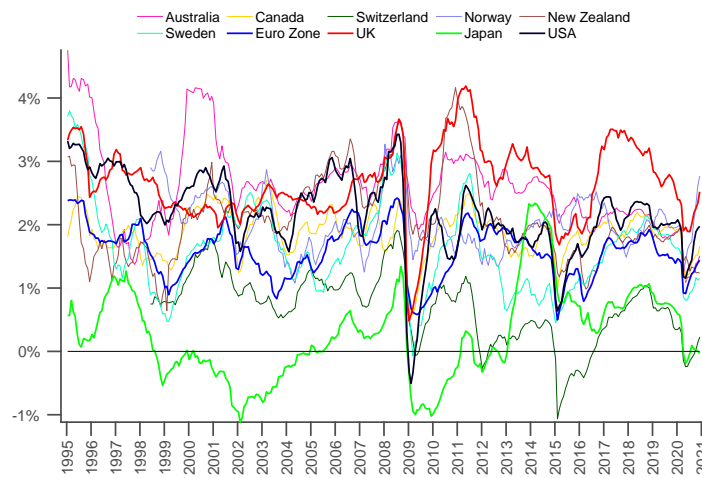
(a) GDP



(b) Consumption



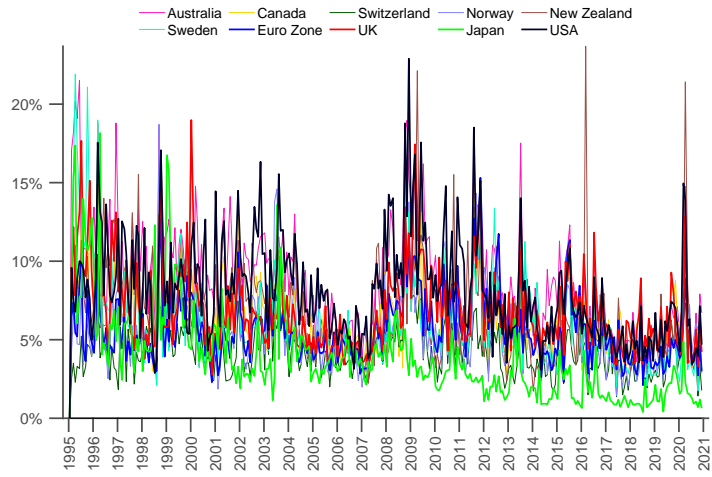
(c) Industrial Production



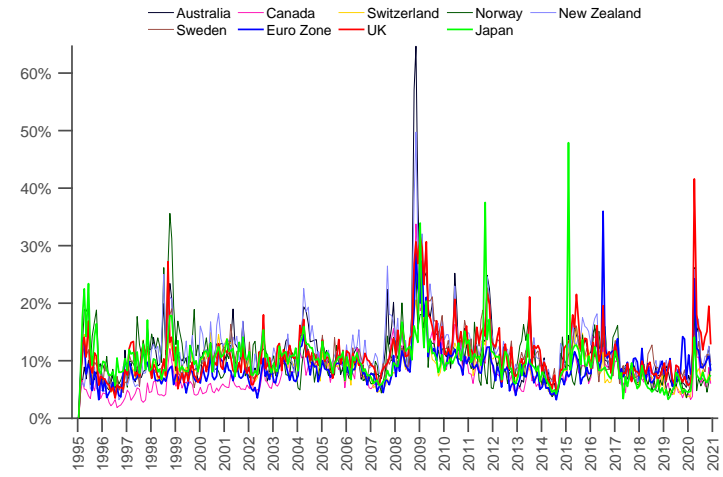
(d) Consumer Price Index

**Figure 3. Subjective Macro Expectations**

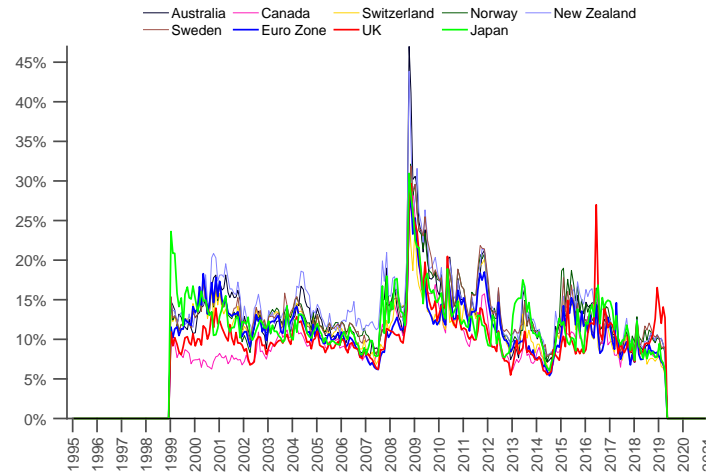
Figure displays subjective expectations of 12-month ahead GDP growth (%), real private consumption growth (%), real Industrial Production growth (%) and inflation (%) for AUD, CAD, CHF, NOK, NZD, SEK, JPY, EUR, GBP and USD. The sample period is 1995.1 to 2020.12.



(a) Bond returns



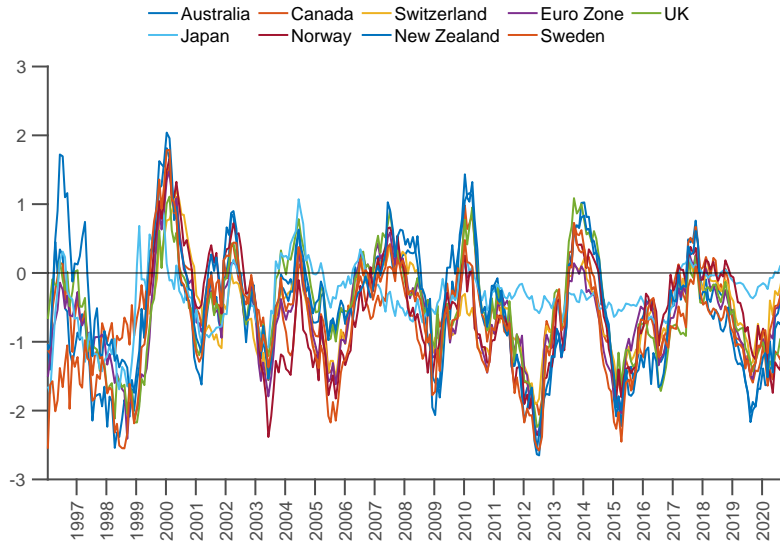
(b) Spot exchange rates



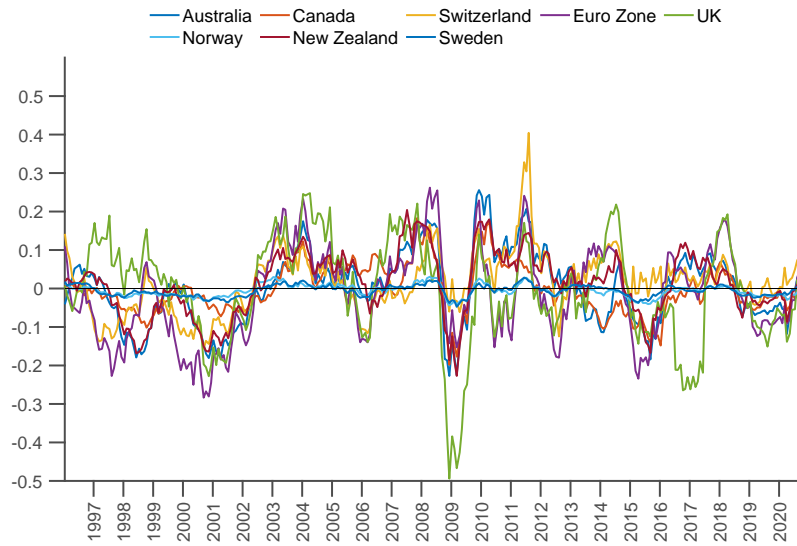
(c) Option-implied risk neutral exchange rate volatility

**Figure 4. Volatility of realised bond returns and realised and risk-neutral spot exchange rates**

Panel (a) displays the volatilities of ten year sovereign bond returns, panel (b) displays the volatilities of spot exchange rates changes and panel (c) displays option-implied risk neutral exchange rate volatilities for AUD, CAD, CHF, NOK, NZD, SEK, JPY, EUR, GBP (and USD for panel (a)). Volatilities in panels (a) and (b) are measured as the sum of squared differences of log prices in the 22 days preceding a sampled date. Dates are sampled as the survey dates of the Consensus Economics forecasts.



(a) IR



(b) XR

**Figure 5. Forecast Errors**

Forecast errors are calculated from

$$y_{t+1} - y_t = E_t^C [y_{t+1} - y_t] + \epsilon_{t+1}^S$$

and plotted for 10-year interest rates (IR, left panel) and exchange rates (XR, right panel) for the 1-year forecast horizon. Forecast errors are realised over the sample period 2001.1 - 2020.12. We don't plot the XR errors for the JPY due to the scale difference compared the remaining pairs.

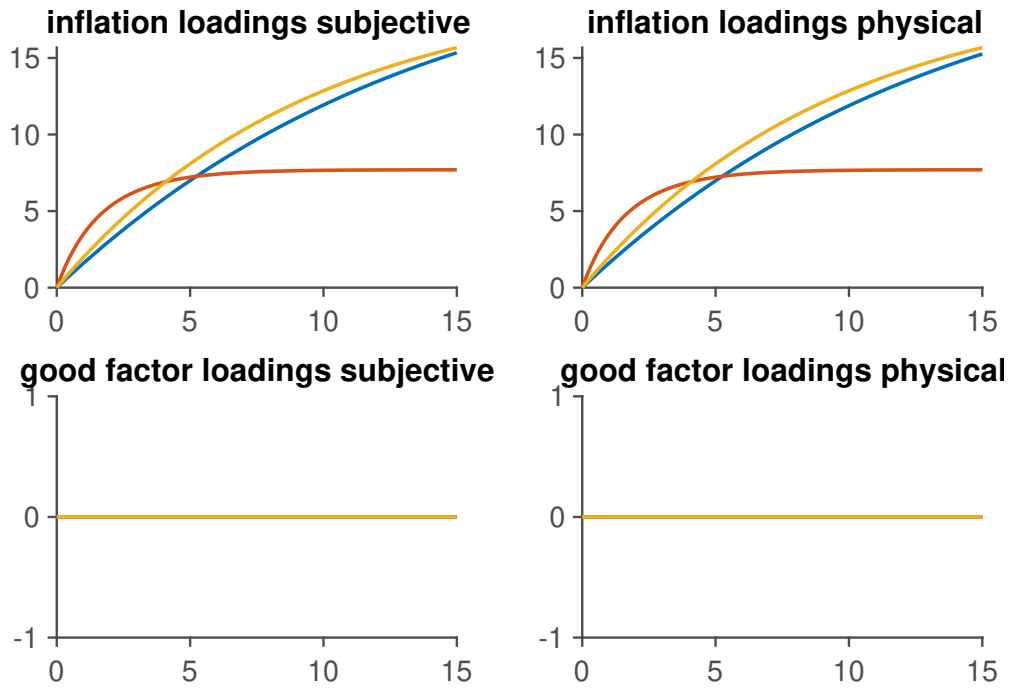


Figure 6. Factor Loadings

Factor Loadings.

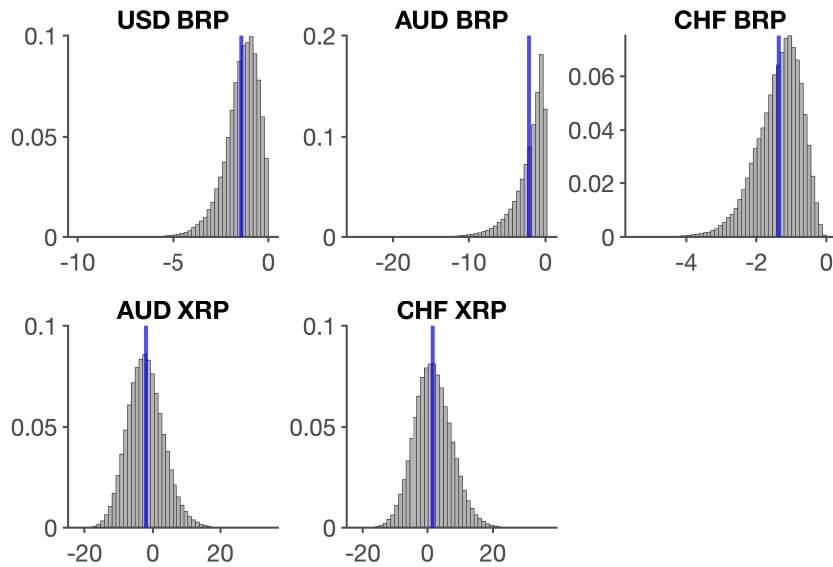


Figure 7. Model Implied Distribution of Subjective Risk Premia Factor Loadings.

## VIII. Tables

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	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
<b>Panel (a): BRP</b>										
Mean	-0.83	-0.43	-1.91	-1.18	-0.55	0.29	-2.72	-0.94	-1.56	-1.41
Std	3.45	3.78	1.00	2.14	3.03	1.74	3.34	3.58	3.58	2.85
Skew	0.85	0.72	0.11	0.47	0.43	0.73	-0.30	0.32	0.31	0.46
AR(1)	0.71	0.86	0.68	0.66	0.79	0.75	0.75	0.75	0.81	0.67
<b>Panel (b): XRP</b>										
Mean	1.74	1.03	-1.69	0.17	0.20	-2.76	3.55	1.14	3.48	
Std	4.91	2.48	4.48	4.04	2.64	4.12	3.00	5.58	4.32	
Skew	0.63	-0.33	0.11	0.84	0.06	-0.24	0.11	0.58	0.52	
AR(1)	0.87	0.70	0.85	0.88	0.73	0.83	0.81	0.88	0.85	

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**Table I. Descriptive Statistics for Subjective Risk Premia**

This table presents the means, standard deviations, skewness, and AR(1) coefficients for subjective exchange rate risk premia (*XRP*) and subjective bond risk premia (*BRP*) as defined in Equations 6 and 8, respectively. The sample period is 1995.1 to 2020.12.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)
$E[gdp]$	-0.55 [-0.65 -0.46]					
$E[ip]$		-0.20 [-0.25 -0.15]				
$E[cons]$			-0.57 [-0.67 -0.49]			
$E[cpi]$				0.02 [-0.08 0.13]		
$BondVar$					2.16 [1.75 2.61]	
$rx_{t-1,t}^{11}$						-0.09 [-0.11 -0.08]
$R^2(\%)$	3.56	1.72	3.78	0.00	5.85	5.11

**Table II. Explaining Subjective Bond Risk Premia**

This table reports estimates from pooled OLS regressions of the form

$$BRP_t = a + b^\top X_t + \epsilon_t.$$

where  $BRP_t$  is the survey-implied bond risk premium and  $X_t$  is a vector of explanatory variables containing the 12-month *expected* growth rate in *gdp*, industrial production *ip*, consumption *cons*, inflation *cpi*, 10-year bond return variance (*BondVar*), and the excess return on an 11-year bond realised between dates  $t - 1$  and  $t$ . A constant is included but not reported. We report 95% confidence intervals estimated through a circular block bootstrap in [.]. The bootstrap uses the optimal block length selection routine of Patton, Politis, and White (2009) and 1000 bootstrap samples. Corresponding standard errors, computed using the estimator of Driscoll and Kraay (1998), are reported in Table A.10. The sample period is 1995.1 to 2020.12.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)
$E[gdp^* - gdp]$	0.76 [-0.12 1.47]					
$E[ip^* - ip]$		0.39 [0.01 0.77]				
$E[con^* - con]$			0.72 [0.03 1.29]			
$E[cpi^* - cpi]$				0.89 [0.10 1.58]		
$FXVar$					0.78 [0.29 1.25]	
$rx_{t-1,t}^{FX}$						-0.09 [-0.13 -0.06]
$R^2(\%)$	2.17	1.86	2.40	3.65	3.58	4.81

**Table III. Explaining Subjective FX Risk Premia**

This table reports estimates from pooled OLS regressions of the form

$$XRP_t = a + b^\top X_t + \epsilon_t.$$

where  $XRP_t$  is the survey-implied currency risk premium and  $X_t$  is a vector of explanatory variables containing the 12-month *expected* growth rate in  $gdp$ , industrial production  $ip$ , consumption  $cons$ , inflation  $cpi$ , FX return variance ( $FXVar$ ), and the excess currency return realised between dates  $t-1$  and  $t$ . A constant is included but not reported. We report 95% confidence intervals estimated through a circular block bootstrap in  $[\cdot]$ . The bootstrap uses the optimal block length selection routine of Patton, Politis, and White (2009) and 1000 bootstrap samples. Corresponding standard errors, computed using the estimator of Driscoll and Kraay (1998), are reported in Table [A.11](#). The sample period is 1995.1 to 2020.12.



<b>Panel (a): Bond Return</b>			
	(i)	(ii)	(iii)
<i>Slope</i>	2.81 [2.28 3.36]		2.49 [1.95 3.06]
<i>BRP</i>		0.52 [0.37 0.67]	0.30 [0.13 0.46]
<i>R</i> <sup>2</sup> (%)	13.85	4.96	14.79
<b>Panel (b): FX Return</b>			
	(i)	(ii)	(iii)
<i>IRD</i>	-1.56 [-2.05 -1.00]		-1.41 [-1.87 -0.91]
<i>XRP</i>		0.55 [0.29 0.81]	0.45 [0.23 0.70]
<i>R</i> <sup>2</sup> (%)	8.89	5.43	12.54

**Table IV. Bond & FX Return Panel Predictability Regressions**

This table reports estimates from POOLED OLS regressions of the form

$$rx_{t,t+1}^{(11)} = a + b_1 Slope_t + b_2 BRP_t + \epsilon_{t,t+1} \quad \text{and} \quad rx_{t,t+1}^{FX} = a + b_1 IRD_t + b_2 XRP_t + \epsilon_{t,t+1}$$

where the dependent variables are the one-year excess return on an 11-year bond and the one-year currency excess return in panels (a) and (b), respectively.  $Slope_t = (i_t^{(10)} - i_t^{(1)})$  is the slope of the domestic yield curve,  $BRP_t$  is the survey-implied bond risk premium,  $IRD_t = (i_t^{(1)} - i_t^{*(1)})$  is the one-year interest rate differential, and  $XRP_t$  is the survey-implied currency risk premium. A constant is included but not reported. We report 95% confidence intervals estimated through a circular block bootstrap in [.]. The bootstrap uses the optimal block length selection routine of Patton, Politis, and White (2009) and 1000 bootstrap samples. Corresponding standard errors, computed using the estimator of Driscoll and Kraay (1998), are reported in Table A.12. The sample period is 1995.1 to 2020.12.

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Panel (a): IR	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
	<u>Surveys</u>									
Mean	-0.57	-0.55	-0.52	-0.64	-0.52	-0.33	-0.55	-0.45	-0.78	-0.60
Std	0.89	0.65	0.58	0.67	0.71	0.47	0.78	0.87	0.81	0.77
Min	-2.65	-2.06	-1.90	-2.41	-2.24	-1.75	-2.38	-2.58	-2.58	-2.37
Max	2.04	1.55	1.24	1.48	1.11	1.07	1.66	1.81	1.79	1.80
AR(1)	-0.18	-0.03	-0.08	-0.18	-0.10	-0.05	0.07	-0.10	-0.12	-0.23

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Panel (b): XR	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
	<u>Surveys</u>								
Mean	0.00	0.00	0.02	-0.01	0.00	0.00	-0.01	0.01	-0.01
Std	0.10	0.06	0.08	0.12	0.13	0.00	0.02	0.08	0.01
Min	-0.23	-0.20	-0.16	-0.28	-0.49	0.00	-0.04	-0.23	-0.05
Max	0.26	0.18	0.40	0.26	0.25	0.00	0.03	0.20	0.03
AR(1)	0.07	0.08	0.16	0.01	-0.05	0.23	0.01	0.02	-0.05

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**Table V. Summary Statistics: Expectation Errors**

This table shows the mean, standard deviation, minimum and maximum of the forecast errors of survey forecasts. It also shows, at an annualised horizon, the 12 month autocorrelation. For each country in our sample, statistics are reported for long-term (10Y) interest rates, and for spot exchange rates w.r.t the US in panels (a) and (b), respectively. Realised forecast errors are computed for the sample period 2000.1 to 2020.12.

	Pooled		Pooled
$Level_t$	0.08 [0.02 0.15]	$IRD_t$	-0.95 [-1.35 -0.46]
$Slope_t$	0.08 [-0.02 0.15]		
$a$	-0.01 [-0.01 -0.01]	$a$	0.00 [-0.02 0.01]
$R^2(\%)$	5.71	$R^2(\%)$	5.73
(a) Interest Rates		(b) Ex- change Rates	

**Table VI. Forecast Error Predictability**

This table shows POOLED OLS regressions of either (a) 10-year yield or (b) foreign exchange forecast errors, at the annual horizon, on either the slope of the yield curve or the 1-year interest rate differential

$$FE_{t,t+1}^i = a^i + b^i X_t^i + \eta_{t+1}^i,$$

where for interest rate forecast errors  $X_t$  includes the  $Level_t = i_t^{(1)}$  and  $Slope_t = (i_t^{(10)} - i_t^{(1)})$  and for exchange rates  $IRD_t = (i_t^{(1)} - i_t^{*,(1)})$ .

We report 95% confidence intervals estimated through a circular block bootstrap in [-]. The bootstrap uses the optimal block length selection routine of Patton, Politis, and White (2009) and 1000 bootstrap samples. Corresponding standard errors, computed using the estimator of Driscoll and Kraay (1998), are reported in Table A.13. Individual country-by-country regressions are presented in Table A.14.

The sample period 1995.1 to 2020.12

<b>Panel (a)</b>				
Parameter	Estimate	Lower	Upper	
$\sigma_Q$	0.07	0.07	0.07	
$\sigma_i$	0.03	0.03	0.03	
$\kappa_i^S$	0.07	0.05	0.09	
$\theta_i^S$	0.03	0.03	0.03	
$\phi_i$	0.36	0.27	0.45	
$\rho_{iQ}$	0.85	0.74	0.97	

---

<b>Panel (b)</b>				
Moment	Data	Model	Lower	Upper
mean realised inflation	2.37	2.37	2.27	2.47
std realised inflation	1.24	1.24	1.20	1.28
AR(1) realised inflation	0.93	0.93	0.93	0.93
mean expected inflation	2.59	2.59	2.51	2.67
std expected inflation	0.68	0.68	0.65	0.71
AR(1) expected inflation	0.98	0.96	0.96	0.97
mean forecast error	-0.22	-0.22	-0.24	-0.20
std forecast error	1.21	1.21	1.18	1.24
AR(1) forecast error	0.92	0.90	0.90	0.90

**Table VII. AUD Inflation Parameter Estimates**

This table reports estimation results for the Australian subjective inflation process

$$\frac{dQ_t}{Q_t} = i_t dt + \sigma_Q \sqrt{i_t} dW_t^{\mathbb{P},Q},$$

$$di_t = \kappa_i^S (\theta_i^S - i_t) dt + \sigma_i \sqrt{i_t} dW_t^{\mathbb{S},i}.$$

where the correlation between shocks is  $\langle dW_t^{\mathbb{P},Q}, dW_t^{\mathbb{S},i} \rangle = \rho_{iQ} dt$ . The model is estimated via simulated method of moments, targeting the mean, standard deviation and monthly autocorrelation of 12-month realised inflation and 12-month subjective expected inflation from consensus economics. In addition, by combining realisations and expectations we obtain 12-month horizon forecast errors from which we compute two additional non-redundant moments - the standard deviation and autocorrelation - of the errors. Section A.5 in the OA reports estimation details. Panel (a) reports point estimates alongside 95% confidence intervals. Panel (b) reports moments in the data and model implied moments alongside 95% confidence intervals.

Parameter	$\sigma_{Ki}$	$\beta$	$\kappa_g$	$\theta_g$	$\sigma_g$	$\phi_g$
<b>Panel (a) USD</b>						
Point	1.67	-3.57	0.74	2.14	0.07	0.00
Lower	1.66	-3.68	-3.19	0.66	-0.09	0.00
Upper	1.68	-3.46	4.67	3.61	0.23	0.00
<b>Panel (b) CHF</b>						
Point	5.77	-37.59	0.71	20.45	0.14	0.06
Lower	5.56	-40.06	-12.02	15.39	-0.82	0.06
Upper	5.98	-35.11	13.44	25.52	1.10	0.06
<b>Panel (c) AUD</b>						
Point	1.15	-2.47	0.70	25.70	0.23	-0.08
Lower	1.13	-2.59	-3.34	20.57	-0.20	-0.08
Upper	1.16	-2.35	4.73	30.84	0.67	-0.08
<b>Panel (d) Shock correlations</b>						
Parameter	$\rho_{x1}$	$\rho_{i1}$	$\rho_{x2}$	$\rho_{i2}$		
Point	0.33	0.35	0.29	0.59		
Lower	0.25	0.30	0.17	0.53		
Upper	0.41	0.40	0.41	0.66		

**Table VIII. Simulated Method of Moments: Parameters**

This table reports parameter estimates for the production economy presented in section V. For the U.S. (USD), Switzerland (CHF) and Australia (AUD) we report points estimates alongside 95% confidence intervals.

	Data	Model	Lower	Upper	Data	Model	Lower	Upper	Data	Model	Lower	Upper
	<b>Panel (a) USD</b>				<b>Panel (b) CHF</b>				<b>Panel (c) AUD</b>			
mean $y_t^{3m}$	2.33	3.62	3.43	3.80	0.82	1.15	1.13	1.17	4.13	4.85	4.68	5.02
std $\Delta y_t^{3m}$	0.21	0.19	0.18	0.20	0.19	0.42	0.41	0.43	0.25	0.28	0.27	0.30
AR(1) $\Delta y_t^{3m}$	0.19	-0.01	-0.01	-0.01	0.19	-0.05	-0.05	-0.05	0.05	-0.01	-0.01	-0.01
mean $y_t^{60m}$	3.37	3.23	3.07	3.40	1.42	1.60	1.59	1.61	4.65	4.57	4.42	4.73
mean $y_t^{120m}$	3.98	2.89	2.75	3.03	1.96	1.73	1.72	1.74	4.96	4.37	4.22	4.52
	<b>Panel (d) USD</b>				<b>Panel (e) CHF</b>				<b>Panel (f) AUD</b>			
mean $BRP$	-1.41	-1.45	-1.53	-1.37	-1.91	-2.06	-2.15	-1.98	-0.83	-1.42	-1.48	-1.35
std $BRP$	2.85	0.40	0.38	0.42	2.00	1.68	1.61	1.75	3.45	0.38	0.37	0.40
skew $BRP$	0.46	-0.32	-0.32	-0.32	0.11	-1.13	-1.13	-1.13	0.85	-0.20	-0.20	-0.20
AR(1) $BRP$	0.67	0.96	0.96	0.96	0.68	0.94	0.94	0.94	0.71	0.96	0.96	0.96
	<b>Panel (g) USD</b>				<b>Panel (h) CHF</b>				<b>Panel (i) AUD</b>			
mean $FE^{10}$	-0.60	-0.13	-0.13	-0.12	-0.52	-0.34	-0.34	-0.34	-0.57	-0.47	-0.49	-0.46
std $FE^{10}$	0.77	0.47	0.44	0.49	0.58	0.31	0.30	0.31	0.89	0.61	0.59	0.63
AR(1) $FE^{10}$	-0.23	-0.06	-0.06	-0.06	-0.08	0.11	0.11	0.11	-0.18	0.00	0.00	0.00

**Table IX. Simulated Method of Moments: Moments I**

This table reports moment estimates for the production economy presented in section V. For the U.S. (USD), Switzerland (CHF) and Australia (AUD) we report point estimates alongside 95% confidence intervals.

---

	Data	Model	Lower	Upper
<b>Panel (a): CHF/USD</b>				
mean $XRP$	-1.69	-1.69	-1.94	-1.44
std $XRP$	4.48	4.52	4.38	4.67
Skew $XRP$	0.11	0.03	0.00	0.06
AR(1) $XRP$	0.85	0.93	0.92	0.93
<b>Panel (b): AUD/USD</b>				
mean $XRP$	1.74	1.74	1.50	1.98
std $XRP$	4.91	4.91	4.83	5.00
Skew $XRP$	0.63	0.10	0.08	0.12
AR(1) $XRP$	0.87	0.92	0.92	0.93
<b>Panel (c): correlations</b>				
$BRP^{US}, BRP^{CHF}$	0.54	0.21	0.18	0.24
$BRP^{US}, BRP^{AUD}$	0.61	0.52	0.46	0.58
$XRP^{CHF}, XRP^{AUD}$	0.55	0.47	0.44	0.50

---

**Table X. Simulated Method of Moments: Moments II**

This table reports moment estimates for the production economy presented in section V. For the U.S. (USD), Switzerland (CHF) and Australia (AUD) we report point estimates alongside 95% confidence intervals.

---

<b>Panel (a) USD</b>								
	ag0	ag1	sigmag	rhogc	phig	av0	av1	sigmav
Parameters	0.0001	-0.0208	0.0236	0.9816	0.8155	0.0013	-0.2031	0.0456
Lower	0.0001	-0.0213	0.0235	0.9631	0.6367	0.0011	-0.2339	0.0430
Upper	0.0001	-0.0204	0.0238	1.0001	0.9943	0.0014	-0.1722	0.0481

<b>Panel (b) CHF</b>								
	ag0f1	ag1f1	sigmagf1	rhogcf1	phigf1	av0f1	av1f1	sigmavf1
Parameters	0.0008	-0.1631	0.0185	0.7915	1.3919	0.0078	-0.3073	0.0889
Lower	0.0008	-0.1699	0.0176	-3.2518	-5.7812	-0.0309	-0.3812	-0.1348
Upper	0.0009	-0.1564	0.0194	4.8348	8.5650	0.0465	-0.2334	0.3126

<b>Panel (c) AUD</b>								
	ag0f2	ag1f2	sigmagf2	rhogcf2	phigf2	av0f2	av1f2	sigmavf2
Parameters	0.0016	-0.1844	0.0484	0.6386	1.6019	0.0038	-0.3047	0.0919
Lower	0.0015	-0.1989	0.0451	0.0485	0.1421	0.0004	-0.4058	0.0499
Upper	0.0018	-0.1698	0.0516	1.2288	3.0617	0.0071	-0.2035	0.1340

---

<b>Panel (d) Shock correlations</b>					
	rhoc1	rhoc2	rhoq1	rhoq2	gamma
Parameters	0.7710	0.7992	0.1495	0.1654	3.5425
Lower	-1.2065	0.4254	-19.6162	-8.1982	3.4190
Upper	2.7485	1.1729	19.9151	8.5290	3.6660

---

**Table XI. NEW Simulated Method of Moments: Parameters**

This table reports parameter estimates for the CRRA time-varying model. For the U.S. (USD), Switzerland (CHF) and Australia (AUD) we report points estimates alongside 95% confidence intervals.



	Data	Model	Lower	Upper	Data	Model	Lower	Upper	Data	Model	Lower	Upper
	<b>Panel (a) USD</b>				<b>Panel (b) CHF</b>				<b>Panel (c) AUD</b>			
mean $y_t^{3m}$	2.3300	3.3506	3.2395	3.4617	0.8200	1.3060	1.2713	1.3407	4.1300	4.2350	4.0161	4.4540
std $\Delta y_t^{3m}$	0.2100	0.2327	0.2273	0.2380	0.1900	0.1812	0.1796	0.1828	0.2500	0.3453	0.3345	0.3560
AR(1) $\Delta y_t^{3m}$	0.1900	-0.0086	-0.0163	-0.0009	0.1900	-0.0496	-0.0526	-0.0466	0.0500	0.0019	-0.0126	0.0164
mean $y_t^{60m}$	3.3700	3.3265	3.2144	3.4387	1.4200	1.5007	1.4777	1.5237	4.6500	4.6482	4.4609	4.8355
mean $y_t^{120m}$	3.9800	3.3317	3.2185	3.4449	1.9600	1.6394	1.6228	1.6561	4.9600	4.8833	4.7071	5.0595
	<b>Panel (d) USD</b>				<b>Panel (e) CHF</b>				<b>Panel (f) AUD</b>			
mean $BRP$	-1.4100	-2.0074	-2.1005	-1.9143	-1.9100	-2.2843	-2.3722	-2.1964	-0.8300	-2.1662	-2.2912	-2.0411
std $BRP$	2.8500	1.2732	1.2208	1.3256	2.0000	1.3234	1.2789	1.3679	3.4500	1.7892	1.7013	1.8771
skew $BRP$	0.4600	-0.8590	-0.9041	-0.8138	0.1100	-0.6340	-0.6697	-0.5982	0.8500	-1.0077	-1.0580	-0.9574
AR(1) $BRP$	0.6700	0.9529	0.9510	0.9547	0.6800	0.9435	0.9381	0.9489	0.7100	0.9443	0.9371	0.9515
	<b>Panel (g) USD</b>				<b>Panel (h) CHF</b>				<b>Panel (i) AUD</b>			
mean $FE^{10}$	-0.6000	-0.2241	-0.2658	-0.1824	-0.5200	-0.5131	-0.5364	-0.4897	-0.5700	-0.5773	-0.6166	-0.5379
std $FE^{10}$	0.7700	0.6217	0.5997	0.6437	0.5800	0.2849	0.2783	0.2915	0.8900	0.7581	0.7390	0.7772
AR(1) $FE^{10}$	-0.2300	0.3617	0.3304	0.3930	-0.0800	0.1676	0.1421	0.1931	-0.1800	0.0489	0.0175	0.0803

**Table XII. NEW Simulated Method of Moments: Moments I**

This table reports parameter estimates for the CRRA time-varying model. For the U.S. (USD), Switzerland (CHF) and Australia (AUD) we report points estimates alongside 95% confidence intervals.

---

	Data	Model	Lower	Upper
<b>Panel (a): CHF/USD</b>				
mean $XRP$	-1.6900	-1.6970	-1.8945	-1.4995
std $XRP$	4.4800	4.5162	4.3496	4.6828
Skew $XRP$	0.1100	0.3660	0.3212	0.4109
Skew $XRP$	0.8500	0.9471	0.9431	0.9510
<b>Panel (b): AUD/USD</b>				
mean $XRP$	1.7400	1.7842	1.5163	2.0520
std $XRP$	4.9100	5.0139	4.8186	5.2093
Skew $XRP$	0.6300	0.5386	0.4940	0.5833
Skew $XRP$	0.8700	0.9443	0.9397	0.9490
<b>Panel (c): correlations</b>				
$BRP^{US}, BRP^{CHF}$	0.5426	0.0234	0.0226	0.0242
$BRP^{US}, BRP^{AUD}$	0.6136	-0.0549	-0.0569	-0.0528
$XRP^{CHF}, XRP^{AUD}$	0.5525	0.2833	0.2626	0.3040

---

**Table XIII. NEW Simulated Method of Moments: Moments II**

This table reports parameter estimates for the CRRA time-varying model. For the U.S. (USD), Switzerland (CHF) and Australia (AUD) we report point estimates alongside 95% confidence intervals.

# Subjective Risk Premia in Bond and FX Markets

## ONLINE APPENDIX

This online appendix is not intended for publication. Section A.1 contains additional details on the construction of our survey dataset. Section A.2 contains supplementary results to Section III in the main body, such as a principal component analysis of the constructed risk premia and comparison to statistical benchmark measures. Section A.3 provides supplementary material to Section IV in the main body, such as an analysis of belief formation and rationality. Section A.4 reports the derivations of the subjective beliefs equilibrium model discussed in Section V. Section A.5 provides details of the simulated method of moments estimation approach. Sections A.6 and A.7 contains supplementary tables and figures to the main body of paper.

### A.1. Data Appendix

Integral part of this paper is the measurement of subjective beliefs. We use the Consensus Economics dataset to get these beliefs for a set of professional forecasters such as banks, funds, and economic advisors. In particular, we use the Consensus Forecasts - G7 & Western Europe and Asia Pacific Consensus Forecasts datasets. These two datasets contain a wide cross-section of countries - presented in Table A.1.

While a large cross-section of countries available, the number of forecasters participating for less developing economies is significantly smaller than the number of forecasters for developed economies. Furthermore, FX data is not available at all for some currencies and realised data is sparse for the smaller economies. As such we restrict our attention to the universe of countries outlined in Section I to maintain a satisfactory number of observations for each date.

Included in the two datasets are expectations about several macroeconomic variables, interest rates, and exchange rates. While we know the identity of forecasters predicting macroeconomic variables and interest rates, Consensus Economics only reports a consensus estimate for exchange rates. Figure A.1 displays the number of forecasters that submitted estimates of future 10-year yields for each of the ten countries selected.

[ INSERT TABLE A.1 AND FIGURE A.1 HERE ]

#### A. Interest Rate Expectations

Consensus Economics asks its panellists to provide estimates of “yields on 10 year government bonds”, without specifying what type of yield. However, it is generally understood that they are providing estimates of the on-the-run bond yield to maturity, which is effectively a par yield forecast. Since we only have two maturities available, we cannot bootstrap zero coupon bond yield estimates from the par yields provided. Therefore, in the main body of the text, we treat par yield forecasts as zero coupon forecasts. Moreover, the compounding frequency of the yields provided is also not explicitly stated, so we assume they are continuously compounded, i.e. log yields. This appendix shows the robustness of our results with respect to these assumptions, by comparing empirically par yields and zero-coupon bond yields for 10-year government bonds, as well as yields and bond returns based on different compounding frequency assumptions.

Panel A of Table A.2 displays the mean and standard deviation for (1) US 10-year par yields obtained from the Fed (H15), (2) US 10-year zero yields obtained from Bloomberg (BB), and (3) their differences. Figure A.2 shows the time series of the same par (H15) and zero (BB) yields, as well as their difference. We can see that the two series are extremely close and their difference is close to zero and insignificant. A similar picture arises when looking at the bond returns implied by par and zero yields.

Panel B of Table A.2 contains similar summary statistics for US 10-year zero log-yields obtained from Bloomberg assuming different compounding frequencies of the raw data. Again, both mean and standard deviation of the yields are extremely similar (see also Figure A.3).

[ INSERT TABLE A.2 AND FIGURE A.2 HERE ]

Summarising, we show that par yields and zero-coupon bond yields for 10-year government bonds are empirically very close and that the compounding frequency has little impact on the bond yields, so our results would be practically unchanged if we assumed that yields are annually or semi-annually compounded instead of continuously compounded, and they are robust to our assumption that survey forecasters provide zero yields.

### B. Macro Economic Expectations

In addition to interest rate and foreign exchange forecasts CE covers a large set of macro-economic variables. We focus on real GDP growth, industrial production growth, personal consumption growth, and the rate of unemployment. A complication with the survey projections is that respondents are asked to report expectations over the current and the next calendar year (except for interest rates, which are constant maturity forecasts); thus, the dataset represents a set of variable maturity events. For example, in July 2003 each contributor to the survey made a forecast for the percentage change in GDP for the remaining two quarters of 2003 (6 months ahead), and an average percentage change for 2004 (18 months ahead). The December 2003 issue contains forecasts for the remaining period of 2003 (1 month ahead) and an average for 2004 (13 months ahead). The moving forecast horizon induces a seasonal pattern in the survey. We compute an implied constant maturity forecast for each individual forecaster as in Buraschi and Whelan (2022) and Fendel, Lis, and Rülke (2011). Let  $j$  be the month of the year, so that  $j = 1$  for January and  $j = 1, 2..12$ . A constant maturity expectation is formed taking as weight  $(1 - \frac{j}{12})$ , for the short term projection (the remaining forecast for the same year), and  $\frac{j}{12}$ , for the long-term projection (the forecast for the following year). Figure A.4 illustrates the weighting procedure visually.

[ INSERT FIGURE A.4 HERE ]

## A.2. Supplementary material to Section III

### A. Factor Models

How many factors are needed to explain the cross-sectional variation in subjective risk premia? The answer to this question speaks to whether a risk based interpretation of subjective beliefs is supported in the data. To answer this question, we follow Lustig, Roussanov, and Verdelhan

(2011) and compute factors  $PC_t$  (principle components) from an eigenvalue decomposition of the covariance matrix of subjective exchange rate risk premia.

Time-series of factors are displayed in figure A.6 and variance explained by each factor is reported in table A.7. We find that 64% of the variation in subjective  $XRP$  is explained by a single factor (PC1), which as usual is a level factor. Estimating factors from portfolio returns sorted on interest rate differentials, as a proxy for expected returns, Lustig, Roussanov, and Verdelhan (2011) call PC1 the ‘dollar risk factor’.<sup>13</sup> Currencies are sorted via their exposure to  $PC2$ , which explains 14% of the variation in subjective  $XRP$ , and their loadings plotted in panel (a) of figure A.6.  $PC2$  is clearly a ‘slope’ factor and has a straightforward economic interpretation inline with the risk based view of Lustig, Roussanov, and Verdelhan (2011): the factor loads negatively on the JPY and CHF and positively on AUD and NZD and so generates a return spread between investment currencies and funding currencies. The factor itself became increasingly negative between 2003 and 2007 (see Panel (b) of Figure A.6) implying that agents believed the carry trade was falling apart but increased from 2007-2012 implying that subjectively they were forecasting increasing carry trade returns. Thus, complimenting the arguments Lustig, Roussanov, and Verdelhan (2011), we show that there are common factors in subjective currency returns which supports a risk-based view of subjective asset pricing.

[ INSERT FIGURE A.6 AND TABLE A.7 HERE]

### B. Comparison with statistical models

We now compare the dynamics of our survey-based bond risk premia ( $BRP$ ) and exchange rate risk premia ( $XRP$ ) with standard statistical models. Projection-based bond risk premia are obtained by regressing realised excess returns at time  $t+1$  (one year holding period) on the slope of each yield curve at time  $t$ , defined as the spread between the 10-year and the 1-year yield. For exchange rate risk premia, we use the forecasts implied by 1-year interest rate differentials. Statistical forecasts are computed in sample so are subject to a look-ahead bias, contrary to the survey-based forecasts which are real-time. However, we do not attempt to compute projection-based estimates in real time, i.e. out-of-sample, as our goal here is just to compare the time series dynamics of the survey and statistical-based foreign bond risk premia, not their predictive power.

Table A.5 reports summary statistics for risk premia based on statistical projections. Comparing summary statistics of for subjective versus statistical projections reveals strong differences between the two sets of premia. For example, bond risk premia using statistical models are uniformly positive and do not switch sign, contrary to subjective  $BRP$  which are negative on average but frequently flip sign. The ranking of the country average exchange rate risk premia is clearly different than survey-based  $XRP$ . In particular, the negative exchange rate risk premia of SEK and NOK appear inconsistent with the standard intuition behind carry trade strategies.

A shared feature of the two sets of premia is their high cross-sectional correlation (see Tables A.3 and A.4), which in the case of the projection-based premia is driven by the high cross-sectional correlation of the predictors, i.e. yield curve slopes and interest rate differentials, across countries. Given such large cross-sectional correlation, to facilitate the comparison between our survey-based measures and statistical models, Figure A.10 displays time series of equally-weighted averages for each country’s subjective risk premia against the same time series obtained with statistical models.

<sup>13</sup>We note that the JPY has a zero loading on  $PC1$

The dynamics of the statistical model based premia are clearly very different in both panel (a) (bonds) and panel (b) (exchange rates) from the corresponding dynamics of the survey-based premia. In fact, their correlation is even slightly negative. In the section C, we evaluate the information content of statistical versus survey-based risk forecasts for future *realised* returns.

[ INSERT TABLE A.5 AND FIGURE A.10 HERE ]

[ INSERT TABLES A.3 AND A.4 HERE ]

### A.3. Supplementary material to Section IV

#### A. How Rational are Survey Forecasts?

The main goal of this paper is to exploit surveys to obtain direct, forward-looking measures of risk premia in foreign exchange and fixed income markets. Exploiting surveys Section V derives and estimates a subjective general equilibrium model in which the representative agents first order condition is formed under a subjective probability measure  $\mathbb{S} \neq \mathbb{P}$ . Thus; we allow for the possibility that investors make forecast errors but we take beliefs as given and model the belief wedge between  $\mathbb{S}$  and  $\mathbb{P}$  exogenously.

In other words, we are silent on the micro-foundations for deviations from full information rationality. However, for the interested reader we do provide an analysis on the extent to which subjective *consensus* beliefs about financial variables display evidence of behavioural biases and/or information frictions.

#### B. Over-reaction (under-reaction) to news

An important stream of the literature investigate economies with rational but sticky-information (Mankiw and Reis, 2002) and with rational but noisy-information (Woodford, 2002, Sims, 2003, and Mackowiak and Wiederholt, 2009). In the first case, agents update their information sets infrequently as a result of a fixed costs of information acquisition and the degree of information rigidity is the probability of not acquiring new information each period. When agents are subject to noisy information, they rationally update their beliefs but, since they can never fully observe the true state, they use an optimal signal-extraction filter. If one were to aggregate these expectations, Coibion and Gorodnichenko (2015) (CG) show that average forecast errors are predictable by forecast revisions. Considering inflation expectations from the Survey of Professional Forecasters CG find that consensus forecast revisions positively predict forecast errors which in their framework corresponds to under-reaction due to information frictions. Exploiting the Consensus Economics dataset Bordalo, Gennaioli, Ma, and Shleifer (2020) confirms the CG findings of under-reaction at the consensus level for macroeconomic series.<sup>14</sup>

While Singleton (2021) argues that predictability of the forecast errors is not necessarily evidence of irrationality, as even a purely Bayesian agent would exhibit serial dependence of the errors if factors are not fully observable, we still use the CG framework to examine the potential

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<sup>14</sup>Bordalo, Gennaioli, Ma, and Shleifer (2020) also study CG regressions at the individual forecast level and show that they typically overreaction which can be explained by a deviation from full information rationality due to what they call ‘diagnostic expectations’. Note that Bordalo, Gennaioli, Ma, and Shleifer (2020) do not use an annual forecasting horizon but a quarterly one.

extent of information frictions at the level of the consensus forecast for the financial variables under consideration in this paper. Namely, we estimate regressions of forecast errors on forecast revisions

$$FE_{t,t+1}^i = a^i + b^i FR_{t-1,t}^i + \eta_{t+1}^i. \quad (\text{A.1})$$

The rational expectations hypothesis predicts that forecast errors should be unpredictable conditional on the publicly available information filtration which includes forecast revisions; thus predicting  $b^i = 0$ . Instead if forecasters under-react to information, because for example they are inattentive or hold adaptive expectations, we expect  $b^i > 0$ . Similarly, when forecasters over-react to information, because for example they are overly optimistic or pessimistic, we should expect  $b^i < 0$ .

[ INSERT TABLE A.9 HERE ]

Table A.9 presents parameter estimates with Newey-West adjusted t-statistics. First, consider panel (a) which displays estimates for 10-year interest rate survey forecasts. None of the point estimates for  $b$  are statistically different from zero and are generally small in economic terms. The estimated intercepts are, however, uniformly negative and statistically significant consistent with the bias in interest rate forecast errors in Table V. Second, consider panel (b) which displays estimates for exchange rate survey forecasts. The results here are mixed: while almost all point estimates are positive only two out of nine of the point estimates (CAD and GBP) for  $b$  are at least marginally statistically significant consistent with the CG findings of under-reaction.

### C. Forecast Error Predictability

Section IV in the main body of the paper studies whether mild persistence observed in the autocorrelation of  $t \rightarrow t + 12$  errors results in predictability by observables at date  $t$ . The regression results show that interest rate and exchange rate expectation errors are predictable by interest rate levels and spreads. These findings suggests agents do not exploit all available information when forming their beliefs, which is inconsistent with the basic idea of rational expectations. However, to the best of our knowledge it remains an open question whether statistical predictability can be exploited to correct errors in real-time. In order to address this question we investigate the economic significance of the predictable component of forecast errors.

### D. Economic Significance of Deviation from Rational Expectations

To study the economic significance of behavioural components in agents expectations, we design an experiment in which we construct fictitious expectations by correcting the predictable errors using information available in date  $t$  observables. In this real-time experiment, we initialise a rolling regression with a window of  $w$ -years of data and recursively estimate a projection of realised errors on yields or interest rate differentials. The loadings available in the forecast error regression at date  $t$  can only be learned from errors realized one year earlier. These loadings are then applied

to date  $t$  observables in order to build a ‘corrected’ beliefs from the following system

$$\widehat{FE}_{t-1,t}^n = \hat{\alpha}_t + \hat{\beta}_t^\top X_{t-1} \quad (\text{A.2})$$

$$\xi_t = \hat{\alpha}_t + \hat{\beta}_t^\top X_t \quad (\text{A.3})$$

$$\widehat{Y}_t = E_t^{\mathbb{S}}[Y_{t+1}] + \xi_t \quad (\text{A.4})$$

The subscript  $t$  in the parameters  $\hat{\alpha}_t$  and  $\hat{\beta}_t^\top$  indicate that the correction is restricted to use only real-time information which is available at time  $t$ . The predictable component of forecast error is estimated using a rolling window to replicate real-life conditions of a trader. In unreported results we also consider expanding windows after a 5-year initial burn in period; the main message that follows is quantitatively similar.

Panel (a) of figure A.11 compares the change in the *RMSE*’s (y-axis) implied by both the original forecasts and the corrected forecasts for various rolling window lengths (x-axis). We find that, although the initial regressions indicate the existence of predictability in the forecast errors, the *RMSE* of the corrected beliefs are unambiguously *higher* than the uncorrected ones. For instance, using a rolling window of 5 years in the estimation of the correction parameters, the *RMSE* increase by around 96% for the 10-year bond. This shows that the expectations extracted from surveys cannot be easily improved using market based state-dependent information. In panel (b) of figure A.11 we show that if one were to correct interest rate expectations for a constant bias, obtained from the (ex-post) mean of the forecast errors, the *RMSE* of the forecast would decrease by about 11%. However, we note that this is due to the bias in forecasts coming from over-prediction in sample rather than agents omitting useful information from the term structure. The result of replicating this experiment for exchange rate expectations yields similar results and is visualised in figure A.12.

Summarising, the findings of this section show that ‘uncorrected’ beliefs dominates their corrected counterparts in a mean-square-error sense, mainly in terms of variability, meaning that predictability in agents errors does not easily translate into forecast improvements. This provides a possible explanation for why subjective expectation can be persistently different from the null implied by rational expectations.

[ INSERT FIGURE A.11 AND A.12 HERE ]

#### E. Note on the Driscoll and Kraay (1998) estimator

In Section III, we discuss tables that show evidence about the explainability and predictability of survey-implied risk premia. For better readability, we choose to report point estimates as well as 95% confidence intervals. These intervals are estimated using a circular block bootstrap that uses the optimal block length selection routine of Patton, Politis, and White (2009) and 1000 bootstrap samples.

As a robustness check, we also present equivalent tables with standard errors instead of confidence intervals.

As shown in Tables A.3 and A.4, our panel dataset is characterised by significant cross-sectional correlation. We choose the Driscoll and Kraay (1998) estimator to account for this cross-sectional dependence in the estimation. Hoechle (2007) provides a discussion of the inner workings of



the estimator and includes a STATA program called *xtscc* to run the estimation. In short, the estimator of Driscoll and Kraay (1998) applies a correction similar to Newey and West (1987) but adds robustness to cross-sectionally clustered standard errors. By combining these two properties, it yields a method to obtain standard errors that are not only robust to autocorrelation and heteroskedasticity but also consistent in settings of cross-sectional dependence. Another appealing property of the estimator is that it can, with a minor adjustment, handle unbalanced samples by applying the Newey and West (1987) correction at every time  $t$  to the moment conditions of individuals  $N(t)$ .

In terms of hypothesis testing, the Tables A.10, A.11 and A.12 yield the same results as their counterparts (Tables II, III and IV) in the main body of the paper.

## A.4. Production Economy Derivations

### A. Inflation Expectations

Consider an economy where the state-variables are adapted to the filtered probability space  $(\Omega, \mathcal{F}_{t \in T}, \mathbb{P})$ . The price level in this economy evolves according to

$$\frac{dQ_t}{Q_t} = i_t dt + \sigma_Q \sqrt{i_t} dW_t^{\mathbb{P}, Q}, \quad (\text{A.5})$$

$$di_t = \kappa_i (\theta_i - i_t) dt + \sigma_i \sqrt{i_t} dW_t^{\mathbb{P}, i}, \quad (\text{A.6})$$

With a sufficiently long history of data at hand an econometrician can estimate the parameters of this process from realisations versus conditional projections  $\left( \frac{dQ_t}{Q_t} - E_t^{\mathbb{P}} \left[ \frac{dQ_t}{Q_t} \right] \right)$ , i.e., from observing their forecast errors and updating their beliefs. Beliefs about expected future inflation will then be given by

$$\frac{Q_{t+\tau} - Q_t}{Q_t} \approx \ln Q_{t+\tau} - \ln Q_t \quad (\text{A.7})$$

$$Q_{t+\tau} = Q_t e^{\int_t^{t+\tau} [i_u - \frac{1}{2} \sigma_Q^2 i_u] du} e^{\sigma_Q \int_t^{t+\tau} \sqrt{i_u} dW_u^{\mathbb{P}, Q}} \quad (\text{A.8})$$

$$Q_{t+\tau} = Q_t e^{(1 - \frac{1}{2} \sigma_Q^2) \int_t^{t+\tau} i_u du} e^{\sigma_Q \int_t^{t+\tau} \sqrt{i_u} dW_u^{\mathbb{P}, Q}} \quad (\text{A.9})$$

$$\ln Q_{t+\tau} - \ln Q_t = (1 - \frac{1}{2} \sigma_Q^2) \int_t^{t+\tau} i_u du + \sigma_Q \int_t^{t+\tau} \sqrt{i_u} dW_u^{\mathbb{P}, Q} \quad (\text{A.10})$$

$$E_t^{\mathbb{P}} [\ln Q_{t+\tau} - \ln Q_t] = (1 - \frac{1}{2} \sigma_Q^2) E_t^{\mathbb{P}} \left[ \int_t^{t+\tau} i_u du \right] + \sigma_Q E_t^{\mathbb{P}} \left[ \int_t^{t+\tau} \sqrt{i_u} dW_u^{\mathbb{P}, Q} \right], \quad (\text{A.11})$$

$$E_t^{\mathbb{P}} [i_u] = \theta_i + (i_t - \theta_i) e^{-\kappa_i (u-t)} \quad u > t \quad (\text{A.12})$$

$$\int_t^{t+\tau} E_t^{\mathbb{P}} [\theta_i + (i_t - \theta_i) e^{-\kappa_i (u-t)}] du = \theta_i \cdot \tau - \frac{1}{\kappa_i} (i_t - \theta_i) (e^{-\kappa_i \tau} - 1) \quad (\text{A.13})$$

$$E_t^{\mathbb{P}} [\ln Q_{t+\tau} - \ln Q_t] = (1 - \frac{1}{2} \sigma_Q^2) \left[ \theta_i \cdot \tau - \frac{1}{\kappa_i} (i_t - \theta_i) (e^{-\kappa_i \tau} - 1) \right] \quad (\text{A.14})$$

Forecasts under the subjective measure have the same form but with different parameters.

### B. Dynamics

Consider a ‘Solow’ type production economy where the growth of the capital stock is driven by two Cox, Ingersoll Jr, and Ross (1985) factors adapted to the filtered probability space  $(\Omega, \mathcal{F}_{t \in T}, \mathbb{S})$  and evolve as

$$\frac{dK_t}{K_t} = \left[ \left( g_t - \beta i_t \right) - \frac{C_t}{K_t} \right] dt + \sigma_{K,g} \sqrt{g_t} dW_t^{\mathbb{S},g} + \sigma_{K,i} \sqrt{i_t} dW_t^{\mathbb{S},i}; \quad K_0 \geq 0 \ \& \ \beta \geq 0 \quad (\text{A.15})$$

$$dg_t = \kappa_g^{\mathbb{S}} (\theta_g^{\mathbb{S}} - g_t) dt + \sigma_g \sqrt{g_t} dW_t^{\mathbb{S},g}; \quad g_0 \geq 0, \quad (\text{A.16})$$

$$di_t = \kappa_i^{\mathbb{S}} (\theta_i^{\mathbb{S}} - i_t) dt + \sigma_i \sqrt{i_t} dW_t^{\mathbb{S},i}; \quad i_0 \geq 0. \quad (\text{A.17})$$

The *latent (unobservable)* states are time-varying and have opposite effects on the drift of  $dK_t$ .

### C. Preferences

The log-utility investor problem is reported in the main body and the HJB derived from this control problem is given by:

$$\max_{C^* C} [\mathcal{D}J(K_t, g_t, i_t) + U(C, t)] = 0 \quad (\text{A.18})$$

which spelt out explicitly is given by

$$\begin{aligned} \max_{C^* C} & \left[ J_t + J_K (K_t (g_t - \beta i_t) - C_t) + J_g (\kappa_g^{\mathbb{S}} \theta_g^{\mathbb{S}} - \kappa_g^{\mathbb{S}} g_t) + J_i (\kappa_i^{\mathbb{S}} \theta_i^{\mathbb{S}} - \kappa_i^{\mathbb{S}} i_t) \right. \\ & + \frac{1}{2} J_{KK} (K_t^2 \sigma_{K,g}^2 g_t + K_t^2 \sigma_{K,i}^2 i_t) + \frac{1}{2} J_{gg} \sigma_g^2 g_t + \frac{1}{2} J_{ii} \sigma_i^2 i_t + J_{Kg} K_t \sigma_g \sigma_{K,g} g_t + J_{Ki} K_t \sigma_i \sigma_{K,i} i_t \\ & \left. + e^{-\rho t} \log(C_t) \right] = 0 \end{aligned} \quad (\text{A.19})$$

Taking first order conditions w.r.t consumption we obtain:  $C_t^* = \frac{e^{-\rho t}}{J_K}$ . Now conjecture

$$J(K_t, g_t, i_t, t) = A_t^1 \log(K_t) + A_t^2 g_t + A_t^3 i_t + A_t^4 \quad (\text{A.20})$$

with terminal boundary conditions  $A_{1,T} = A_{2,T} = A_{3,T} = A_{4,T} = 0$ . Optimal consumption satisfies

$$C_t^* = \frac{e^{-\rho t} K_t}{A_{1,t}} \quad (\text{A.21})$$

Taking partials and substituting back into the HJB we obtain

$$\begin{aligned} & \frac{\partial A_{1,t}}{\partial t} \left( \log(C_t^*) + \log(A_{1,t}) \right) + \frac{\partial A_{2,t}}{\partial t} g_t + \frac{\partial A_{3,t}}{\partial t} i_t + \frac{\partial A_{4,t}}{\partial t} \\ & + \frac{A_{1,t}}{K_t} (K_t (g_t - \beta i_t) - C_t^*) + A_{2,t} (\kappa_g^{\mathbb{S}} \theta_g^{\mathbb{S}} - \kappa_g^{\mathbb{S}} g_t) + A_{3,t} (\kappa_i^{\mathbb{S}} \theta_i^{\mathbb{S}} - \kappa_i^{\mathbb{S}} i_t) \\ & + \frac{1}{2} \left( \frac{-A_{1,t}}{K_t^2} \right) (K_t^2 \sigma_{K,g}^2 g_t + K_t^2 \sigma_{K,i}^2 i_t) + e^{-\rho t} \log(C_t^*) \Big] = 0 \end{aligned} \quad (\text{A.22})$$

Simplifying and collecting terms we have

$$\frac{\partial A_{1,t}}{\partial t} \log(A_{1,t}) + \frac{\partial A_{4,t}}{\partial t} - e^{-\rho t} + A_{2,t} \kappa_g^{\mathbb{S}} \theta_g^{\mathbb{S}} + A_{3,t} \kappa_i^{\mathbb{S}} \theta_i^{\mathbb{S}} \quad (\text{A.23})$$

$$+ \left[ \frac{\partial A_{2,t}}{\partial t} - A_{2,t} \kappa_g^{\mathbb{S}} + A_{1,t} \left( 1 - \frac{\sigma_{K,g}^2}{2} \right) \right] g_t \quad (\text{A.24})$$

$$+ \left[ \frac{\partial A_{3,t}}{\partial t} - A_{3,t} \kappa_i^{\mathbb{S}} - A_{1,t} \left( 1 + \frac{\sigma_{K,i}^2}{2} \right) \right] i_t \quad (\text{A.25})$$

$$+ \left[ \frac{\partial A_t^1}{\partial t} + e^{-\rho t} \right] \log(C_t^*) = 0 \quad (\text{A.26})$$

which must hold for all realisations of the state variables and so separating variables. So

$$\frac{\partial A_{1,t}}{\partial t} = -e^{-\rho t} \quad (\text{A.27})$$

$$\frac{\partial A_{2,t}}{\partial t} = A_{2,t} \kappa_g^{\mathbb{S}} - A_{1,t} \left( 1 - \frac{\sigma_K^2}{2} \right) \quad (\text{A.28})$$

$$\frac{\partial A_{3,t}}{\partial t} = A_{3,t} \kappa_i^{\mathbb{S}} + A_{1,t} \left( 1 + \frac{\sigma_{K,i}^2}{2} \right) \quad (\text{A.29})$$

$$\frac{\partial A_{4,t}}{\partial t} = -\frac{\partial A_{1,t}}{\partial t} \log(A_{1,t}) + e^{-\rho t} - A_{2,t} \kappa_g^{\mathbb{S}} \theta_g^{\mathbb{S}} - A_{3,t} \kappa_i^{\mathbb{S}} \theta_i^{\mathbb{S}} \quad (\text{A.30})$$

Integrating and imposing the boundary condition we obtain

$$A_{1,t} = \int_t^T e^{-\rho t} dt = \frac{e^{-\rho t} - e^{-\rho T}}{\rho} \quad (\text{A.31})$$

Now we can compute the optimal consumption of the capital stock as

$$C_t^* = \frac{K_t \rho}{e^{\rho t} (e^{-\rho t} - e^{-\rho T})} = \frac{\rho K_t}{1 - e^{-\rho(T-t)}}$$

#### D. The Subjective SDF

The investors SDF in the economy is given by

$$\Lambda_t = U_C(C_t^*, t) = e^{-\rho t} (C_t^*)^{-1} = \left( \frac{e^{-\rho t} - e^{-\rho T}}{\rho} \right) K_t^{-1} \quad (\text{A.32})$$

and applying Itô's lemma we obtain its diffusion

$$d\Lambda_t = -\frac{e^{-\rho t}}{K_t} dt - \Lambda_t \left( \frac{dK_t}{K_t} \right) + \Lambda_t \left\langle \frac{dK_t}{K_t}, \frac{dK_t}{K_t} \right\rangle \quad (\text{A.33})$$

$$\frac{d\Lambda_t}{\Lambda_t} = -\left( \frac{e^{-\rho t}}{K_t} \right) \left( \frac{\rho K_t}{e^{-\rho t} - e^{-\rho T}} \right) dt - \left( \frac{dK_t}{K_t} \right) + \left\langle \frac{dK_t}{K_t}, \frac{dK_t}{K_t} \right\rangle \quad (\text{A.34})$$

$$= -\rho \left( \frac{e^{-\rho t}}{e^{-\rho t} - e^{-\rho T}} \right) dt - \left( \frac{dK_t}{K_t} \right) + \left\langle \frac{dK_t}{K_t}, \frac{dK_t}{K_t} \right\rangle. \quad (\text{A.35})$$

Write  $dK_t/K_t$  in terms of the constant optimal consumption-to-capital stock ratio

$$\frac{dK_t}{K_t} = \left[ (g_t - \beta i_t) - \frac{\rho}{1 - e^{-\rho(T-t)}} \right] dt + \sigma_{K,g} \sqrt{g_t} dW_t^{\mathbb{S},g} + \sigma_{K,i} \sqrt{i_t} dW_t^{\mathbb{S},i}. \quad (\text{A.36})$$

So then substituting in  $dK_t/K_t$  we obtain

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t^R dt - \Theta_t^{\mathbb{S},g} dW_t^{\mathbb{S},g} - \Theta_t^{\mathbb{S},i} dW_t^{\mathbb{S},i} \quad (\text{A.37})$$

$$r_t^R = \underbrace{(1 - \sigma_{K,g}^2)}_{r_g} g_t + \underbrace{(-\beta - \sigma_{K,i}^2)}_{r_i} i_t \quad (\text{A.38})$$

$$= r_g^R \cdot g_t + r_i^R \cdot i_t \quad (\text{A.39})$$

$$\Theta_t^{\mathbb{S},g} = \sigma_{K,g} \sqrt{g_t} \quad (\text{A.40})$$

$$\Theta_t^{\mathbb{S},i} = \sigma_{K,i} \sqrt{i_t} \quad (\text{A.41})$$

which makes clear that the subjective instantaneous price of risk for both  $dW_t^{\mathbb{S},g}$  and  $dW_t^{\mathbb{S},i}$  shocks is *positive* meaning that positive innovations (which raise  $dK_t$  instantaneously) lower marginal utility  $\rightarrow$  these are good states of nature. However, the shocks have opposite effects on future growth rates which is reflected through opposing loadings on the short rate.

### E. Nominal Bond Pricing

The price level in this economy evolves according to

$$\frac{dQ_t}{Q_t} = i_t dt + \sigma_Q \sqrt{i_t} dW_t^Q, \quad (\text{A.42})$$

$$(\text{A.43})$$

with the process for  $i_t$  (expected inflation) given above and where the correlation between shocks is  $\langle dW_t^{\mathbb{S},Q}, dW_t^{\mathbb{S},i} \rangle = \rho_{iQ} dt$ . Note that since  $dW_t^Q$  is observable its distribution is identical under both  $\mathbb{S}$  and  $\mathbb{P}$  measures and so we drop the superscript. The price at time  $t$  of a default free

nominal zero-coupon bond maturity at time  $T = t + \tau$  is given by

$$P_t^\tau = E_t^{\mathbb{S}} \left[ \frac{\Lambda_T^r Q_t}{\Lambda_t^r Q_T} \right] = E_t^{\mathbb{S}} \left[ \frac{\Lambda_T}{\Lambda_t} \right] \quad (\text{A.44})$$

$$= E_t^{\mathbb{S}} \left[ e^{-\int_t^T r_s ds} \frac{d\mathbb{Q}}{d\mathbb{S}} \right] = E_t^{\mathbb{Q}} \left[ e^{-\int_t^T r_s ds} \right] \quad (\text{A.45})$$

where we have defined the nominal SDF  $\Lambda_t = \Lambda_t^r Q_t^{-1}$ . The diffusion for  $\Lambda_t$  given by

$$\frac{d\Lambda_t}{\Lambda_t} = \frac{d\Lambda_t^r}{\Lambda_t^r} - \frac{dQ_t}{Q_t} + \frac{1}{2}(2) \left\langle \frac{dQ_t}{Q_t}, \frac{dQ_t}{Q_t} \right\rangle - \left\langle \frac{d\Lambda_t^r}{\Lambda_t^r}, \frac{dQ_t}{Q_t} \right\rangle \quad (\text{A.46})$$

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t dt - \Theta_t^{\mathbb{S},g} dW_t^{\mathbb{S},g} - \Theta_t^{\mathbb{S},i} dW_t^{\mathbb{S},i} - \Theta_t^{\mathbb{Q}} dW_t^{\mathbb{Q}} \quad (\text{A.47})$$

$$r_t = r_t^R + i_t - \sigma_Q^2 i_t - \sigma_{K,i} \sigma_Q \rho_{i,Q} i_t \quad (\text{A.48})$$

$$= \underbrace{\left(1 - \sigma_{K,g}^2\right) g_t}_{r_g} + \underbrace{\left(1 - \beta - \sigma_Q^2 - \sigma_{K,i}^2 - \rho_{i,Q} \sigma_{K,i} \sigma_Q\right) i_t}_{r_{\mathbb{S}i}} \quad (\text{A.49})$$

$$= r_g \cdot g_t + r_i \cdot i_t \quad (\text{A.50})$$

$$\Theta_t^{\mathbb{S},g} = \sigma_{K,g} \sqrt{g_t} \quad (\text{A.51})$$

$$\Theta_t^{\mathbb{S},i} = \sigma_{K,i} \sqrt{i_t} \quad (\text{A.52})$$

$$\Theta_t^{\mathbb{Q}} = \sigma_Q \sqrt{i_t} \quad (\text{A.53})$$

From Girsanov the changes of measure for the state variables  $z_t = \{g_t, i_t\}$  are

$$W_t^{z,\mathbb{Q}} = W_t^z + \int_0^t \Theta_t^{\mathbb{S},z} du \quad (\text{A.54})$$

which are Brownian motions under the  $\mathbb{Q}$ -measure. It follows that

$$dz_t = \kappa_z^{\mathbb{S}} (\theta_z^{\mathbb{S}} - z_t) dt + \sigma_z \sqrt{z_t} dW_t^{\mathbb{S},z} \quad (\text{A.55})$$

$$= (\kappa_z^{\mathbb{S}} \theta_z^{\mathbb{S}} - \kappa_z^{\mathbb{S}} z_t - \sigma_z \sqrt{z_t} \Theta_t^{\mathbb{S},z}) dt + \sigma_z \sqrt{z_t} dW_t^{\mathbb{Q},z} \quad (\text{A.56})$$

$$= (\kappa_z^{\mathbb{S}} \theta_z^{\mathbb{S}} - (\kappa_z^{\mathbb{S}} + \sigma_z \sigma_{K,z}) z_t) dt + \sigma_z \sqrt{z_t} dW_t^{\mathbb{Q},z} \quad (\text{A.57})$$

$$= \kappa_z^{\mathbb{Q}} (\theta_z^{\mathbb{Q}} - z_t) dt + \sigma_z \sqrt{z_t} dW_t^{\mathbb{Q},z} \quad (\text{A.58})$$

where we have defined risk-neutral long run mean and persistence parameters

$$\theta_z^{\mathbb{Q}} = \frac{\kappa_z^{\mathbb{S}} \theta_z^{\mathbb{S}}}{\kappa_z^{\mathbb{S}} + \sigma_z \sigma_{K,z}} \quad (\text{A.59})$$

$$\kappa_z^{\mathbb{Q}} = \kappa_z^{\mathbb{S}} + \sigma_z \sigma_{K,z}. \quad (\text{A.60})$$

From discounted Feynman-Kac the bond pricing function solves

$$\mathcal{D}P_t^\tau = r_t P_t^\tau + \frac{\partial P}{\partial \tau} \quad (\text{A.61})$$

Guess the solution takes the form

$$P_t^\tau = e^{-A_\tau^\mathbb{S} - B_\tau^{\mathbb{S},g} g_t - B_\tau^{\mathbb{S},i} i_t} \quad (\text{A.62})$$

Take partial derivatives, substituting into the pricing PDE, and after some algebra we obtain

$$\frac{\partial A_\tau}{\partial \tau} = B_\tau^x \kappa_g^\mathbb{Q} \theta_g^\mathbb{Q} + B_\tau^i \kappa_i^\mathbb{Q} \theta_i^\mathbb{Q} \quad (\text{A.63})$$

$$\frac{\partial B_\tau^{\mathbb{S},g}}{\partial \tau} = -\frac{\sigma_g^2}{2} (B_\tau^x)^2 - \kappa_g^\mathbb{Q} B_\tau^x + r_g \quad (\text{A.64})$$

$$\frac{\partial B_\tau^i}{\partial \tau} = -\frac{\sigma_i^2}{2} (B_\tau^i)^2 - \kappa_i^\mathbb{Q} B_\tau^i + r_i \quad (\text{A.65})$$

The factor loadings in the bond pricing solution for  $z = \{x, i\}$  are given by

$$B_\tau^z = \frac{r_z (e^{d^z \tau} - 1)}{d^z - \frac{1}{2} (\kappa_z^\mathbb{Q} + d^z) (1 - e^{d^z \tau})}, \quad (\text{A.66})$$

$$d^z = \sqrt{(\kappa_z^\mathbb{Q})^2 + 2\sigma_z^2 r_z} \quad (\text{A.67})$$

and  $A_\tau$  follows by direct integration and imposing the boundary condition  $A_0 = 0$ .

Given these solutions, time-varying nominal subjective bond risk premia on a  $T$  period bond are given by

$$BRP_t = -E_t^\mathbb{S} \left[ \frac{d\Lambda_t}{\Lambda_t} \cdot \frac{dP_t^T}{P_t^T} \right] \quad (\text{A.68})$$

$$= -B_\tau^x \sigma_g \sigma_{K,g} g_t - B_\tau^i (\sigma_i \sigma_{K,i} + \sigma_i \sigma_Q \rho_{i,Q}) i_t. \quad (\text{A.69})$$

#### F. The Physical (Econometricians) Measure

Here, we use superscripts  $\mathbb{P}$  to indicate the physical SDF that an econometrician would infer if they had full information. More formally, the relationship between the econometricians measure and the subjective measure is given by  $E_t^\mathbb{P} [z_s] = E_t^\mathbb{S} \left[ \frac{d\mathbb{P}}{d\mathbb{S}} z_s \right]$  where from Girsanov<sup>15</sup>

$$\frac{d\mathbb{P}}{d\mathbb{S}} = M_t^{\mathbb{S},\mathbb{P}} = \exp \left( - \int_t^s \Delta_u dW_u^{\mathbb{P},z} - \frac{1}{2} \int_0^t \Delta_u^2 du \right), \quad (\text{A.70})$$

$$dW_t^{\mathbb{P},z} = dW_t^{\mathbb{S},z} + \Delta_t^z dt \quad (\text{A.71})$$

<sup>15</sup>We assume  $\mathbb{S}$  is absolutely continuous with respect to  $\mathbb{P}$ .

We call  $\Delta_t^z$  a BELIEF DISTORTION. Now we have two changes of measure which satisfy

$$dW^{\mathbb{Q}} = dW^{\mathbb{P},z} + \Theta_t^{\mathbb{P},z} dt \quad , \quad dW^{\mathbb{Q},z} = dW^{\mathbb{S},z} + \Theta_t^{\mathbb{S},z} dt, \quad (\text{A.72})$$

$$dW^{\mathbb{P},z} = dW^{\mathbb{S},z} + \Delta_t^z dt \quad \rightarrow \quad \Delta_t^z = \Theta_t^{\mathbb{S},z} - \Theta_t^{\mathbb{P},z} \quad (\text{A.73})$$

Abstracting from a specific mechanism for subjective belief formation, we specify the following parametric specification for priced shocks:  $\Delta_t^z = \phi_z \sqrt{z_t}$ :

$$\Theta_t^{\mathbb{P},g} = \Theta_t^{\mathbb{S},g} - \underbrace{\phi_g \sqrt{g_t}}_{\Delta_t^g} = (\sigma_{K,g} - \phi_g) \sqrt{g_t} \quad (\text{A.74})$$

$$\Theta_t^{\mathbb{P},i} = \Theta_t^{\mathbb{S},i} - \underbrace{\phi_i \sqrt{i_t}}_{\Delta_t^i} = (\sigma_{K,i} - \phi_i) \sqrt{i_t} \quad (\text{A.75})$$

It follows that

$$dz_t = \kappa_z^{\mathbb{S}} (\theta_z^{\mathbb{S}} - z_t) dt + \sigma_g \sqrt{z_t} dW_t^{\mathbb{S},z} \quad (\text{A.76})$$

$$= (\kappa_z^{\mathbb{S}} \theta_z^{\mathbb{S}} - \kappa_z^{\mathbb{S}} z_t - \sigma_z \sqrt{z_t} \Delta_t^z) dt + \sigma_z \sqrt{z_t} dW_t^{\mathbb{P},z} \quad (\text{A.77})$$

$$= (\kappa_z^{\mathbb{S}} \theta_z^{\mathbb{S}} - (\kappa_z^{\mathbb{S}} + \sigma_z \phi_z z_t)) dt + \sigma_z \sqrt{z_t} dW_t^{\mathbb{P},z} \quad (\text{A.78})$$

$$= \kappa_z^{\mathbb{P}} (\theta_z^{\mathbb{P}} - z_t) dt + \sigma_z \sqrt{z_t} dW_t^{\mathbb{P},z} \quad (\text{A.79})$$

where we have defined risk-neutral long run mean and persistence parameters

$$\theta_z^{\mathbb{P}} = \frac{\kappa_z^{\mathbb{S}} \theta_z^{\mathbb{S}}}{\kappa_z^{\mathbb{S}} + \sigma_z \phi_z}, \quad (\text{A.80})$$

$$\kappa_z^{\mathbb{P}} = \kappa_z^{\mathbb{S}} + \sigma_z \phi_z. \quad (\text{A.81})$$

The econometricians SDF is given by

$$\frac{d\Lambda_t^E}{\Lambda_t^E} = -r_t dt - \Theta_t^{\mathbb{P},g} dW_t^{\mathbb{P},g} - \Theta_t^{\mathbb{P},i} dW_t^{\mathbb{S},i} - \Theta_t^{\mathbb{Q}} dW_t^{\mathbb{Q}} \quad (\text{A.82})$$

$$r_t = r_g \cdot g_t + r_i \cdot i_t \quad \dots \text{ as above} \quad (\text{A.83})$$

$$\Theta_t^{\mathbb{P},g} = (\sigma_{K,g} - \phi_g) \sqrt{g_t} \quad (\text{A.84})$$

$$\Theta_t^{\mathbb{P},i} = (\sigma_{K,i} - \phi_i) \sqrt{i_t} \quad (\text{A.85})$$

$$\Theta_t^{\mathbb{Q}} = \sigma_Q \sqrt{i_t} \quad (\text{A.86})$$

Repeating the bond pricing exercise above, this time changing the measure from  $\mathbb{P}$  to  $\mathbb{Q}$  this time we adjust by  $\Theta_t^{\mathbb{P},z}$ , as opposed to  $\Theta_t^{\mathbb{S},z}$ . To convince ourselves that the SDF decomposition above

is internally consistent let's change the measure for  $\mathbb{P}$  to  $\mathbb{Q}$ .

$$dz_t = \kappa_z^{\mathbb{P}}(\theta_z^{\mathbb{P}} - z_t)dt + \sigma_z \sqrt{z_t} dW_t^{\mathbb{P},z} \quad (\text{A.87})$$

$$= (\kappa_z^{\mathbb{P}}\theta_z^{\mathbb{P}} - \kappa_z^{\mathbb{P}}z_t - \sigma_z \sqrt{z_t} \Theta_t^{\mathbb{P},z})dt + \sigma_z \sqrt{z_t} dW_t^{\mathbb{Q},z} \quad (\text{A.88})$$

$$= (\kappa_z^{\mathbb{P}}\theta_z^{\mathbb{P}} - (\kappa_z^{\mathbb{P}} + \sigma_z(\sigma_{K,z} - \phi_z))z_t)dt + \sigma_z \sqrt{z_t} dW_t^{\mathbb{Q},z} \quad (\text{A.89})$$

$$= \kappa_z^{\mathbb{Q}}(\theta_z^{\mathbb{Q}} - z_t)dt + \sigma_z \sqrt{z_t} dW_t^{\mathbb{Q},z} \quad (\text{A.90})$$

where

$$\theta_z^{\mathbb{Q}} = \frac{\kappa_z^{\mathbb{P}}\theta_z^{\mathbb{P}}}{\kappa_z^{\mathbb{P}} + \sigma_z(\sigma_{K,z} - \phi_z)}, \quad (\text{A.91})$$

$$\kappa_z^{\mathbb{Q}} = \kappa_z^{\mathbb{P}} + \sigma_z(\sigma_{K,z} - \phi_z). \quad (\text{A.92})$$

Comparing this with above

$$\theta_z^{\mathbb{Q}} = \frac{\kappa_z^{\mathbb{S}}\theta_z^{\mathbb{S}}}{\kappa_z^{\mathbb{S}} + \sigma_z\sigma_{K,z}}, \quad (\text{A.93})$$

$$\kappa_z^{\mathbb{Q}} = \kappa_z^{\mathbb{S}} + \sigma_z\sigma_{K,z}, \quad (\text{A.94})$$

and

$$\theta_z^{\mathbb{P}} = \frac{\kappa_z^{\mathbb{S}}\theta_z^{\mathbb{S}}}{\kappa_z^{\mathbb{S}} + \sigma_z\phi_z}, \quad (\text{A.95})$$

$$\kappa_z^{\mathbb{P}} = \kappa_z^{\mathbb{S}} + \sigma_z\phi_z. \quad (\text{A.96})$$

So equating equation A.92 with A.94 we obtain A.96. The numerators of equations A.91 and A.93 must therefore be the same:

$$\kappa_z^{\mathbb{P}}\theta_z^{\mathbb{P}} = \kappa_z^{\mathbb{S}}\theta_z^{\mathbb{S}} \quad (\text{A.97})$$

$$(\kappa_z^{\mathbb{S}} + \sigma_z\phi_z) \frac{\kappa_z^{\mathbb{S}}\theta_z^{\mathbb{S}}}{\kappa_z^{\mathbb{S}} + \sigma_z\phi_z} = \kappa_z^{\mathbb{S}}\theta_z^{\mathbb{S}} \quad (\text{A.98})$$

$$\kappa_z^{\mathbb{P}}\theta_z^{\mathbb{P}} = \kappa_z^{\mathbb{P}}\theta_z^{\mathbb{P}} \quad (\text{A.99})$$

as claimed. This shows there is only one risk neutral measure in our model and thus the market is complete. Finally, bond prices are given by the same form

$$P_t^\tau = e^{-A_t^\mathbb{P} - B_t^{\mathbb{P},g} g_t - B_t^{\mathbb{P},i} i_t} \quad (\text{A.100})$$



Following the same steps as above we arrive at the following ODEs for

$$\frac{\partial A_\tau^{\mathbb{P}}}{\partial \tau} = B_\tau^{\mathbb{P},g} \kappa_g^{\mathbb{Q}} \theta_g^{\mathbb{Q}} + B_\tau^{\mathbb{P},i} \kappa_i^{\mathbb{Q}} \theta_i^{\mathbb{Q}} \quad (\text{A.101})$$

$$\frac{\partial B_\tau^{\mathbb{P},g}}{\partial \tau} = -\frac{\sigma_g^2}{2} (B_\tau^{\mathbb{P},g})^2 - \kappa_g^{\mathbb{Q}} B_\tau^{\mathbb{P},g} + r_g \quad (\text{A.102})$$

$$\frac{\partial B_\tau^{\mathbb{P},i}}{\partial \tau} = -\frac{\sigma_i^2}{2} (B_\tau^{\mathbb{P},i})^2 - \kappa_i^{\mathbb{Q}} B_\tau^{\mathbb{P},i} + r_i \quad (\text{A.103})$$

which rigorously shows that we have a unique bond pricing function identical to the one arising from the subjective beliefs first order condition, i.e.,

$$A_\tau^{\mathbb{P}} = A_\tau \quad (\text{A.104})$$

$$B_\tau^{\mathbb{P},z} = B_\tau^z \quad (\text{A.105})$$

## A.5. Simulated Method of Moments

We estimate the model via simulated method of moments (SMM) which is analogous to the generalized method of moments (GMM) estimator, but allows us to estimate the parameters even if latent volatility factors are not directly observable. Moreover, SMM avoids difficulties of computing analytical moment conditions which given the number of moment conditions is a tedious procedure.

We collect the structural parameters of interest in a  $q \times 1$  parameter vector  $\beta$ . Given  $\beta$ , we simulate the counterpart  $\tilde{x}(\beta)$  of the observed data sample  $x$  using the model specification. Given a simulation of length  $\tau \times T$  where  $\tau > 1$  we obtain the  $p > q$  vector of moments  $M(\tilde{x}(\beta))$ . Under the assumption that the model  $\tilde{x}(\beta)$  is correctly specified and  $\beta_0$  is the true structural parameter vector, the moment conditions  $M(\tilde{x}(\beta_0))$  converge asymptotically to the sample moments  $M(x)$ . Thus, SMM proceeds in a similar fashion to GMM by choosing  $\hat{\beta}$  to minimize the weighted sum of squared moments errors  $G_T(\hat{\beta}) = [M(\tilde{x}(\hat{\beta})) - M(x)]$

$$\hat{\beta} = \min_{\hat{\beta}} G_T^\top W G_T \quad (\text{A.106})$$

where  $W$  is a  $q \times q$  positive definite weighting matrix. As in GMM, the optimal weighting matrix  $W$  is obtained from the inverse of the covariance matrix of the data moments. Under the regularity conditions set out in Duffie and Singleton (1993),  $\hat{\beta}$  is a consistent estimate of  $\beta_0$ . Moreover, when using the optimal weighting matrix  $W^*$ , the asymptotic distribution of the parameters is given by

$$\sqrt{T}(\hat{\beta} - \beta_0) \sim \mathcal{N}[0, (1 + 1/\tau)V^{-1}], \quad (\text{A.107})$$

$$\text{where } V = d^\top W^* d \quad \text{and} \quad d = E \left[ \frac{\partial M}{\partial \beta} \right]. \quad (\text{A.108})$$

To assess the model specification, we use the Chi-square test statistic proposed by Lee and Ingram

(1991), which is the SMM analogue of the Hansen (1982)  $J_T$ -statistic:

$$J_T = T(1 + 1/\tau) G_T^\top W^* G_T \xrightarrow{d} \chi_{(p-q)}^2. \quad (\text{A.109})$$

We choose a set of moment conditions  $p > q$  so that the system is over-identified.

Diffusions are discretized using a Milstein scheme (Kloeden and Platen (2013)). Our sample moments are estimated with  $T = 20$  years of data, we set  $\tau = 50$  and discard the first 5-years of each path to avoid sensitivity to initial conditions. Random number streams are held constant for each simulation path to avoid introducing sampling error. Moments  $\widehat{M}_S$  are computed on data sampled at monthly frequency consistent with the sampling frequency of the empirical moment vector  $M_D$ .  $\widehat{S}$  is computed using 2000 replications and a Newey and West (1987) with lag length  $K = 12$ , and the Jacobian  $d$  is computed using 100 replications in line with the recommendations of Gouriéroux and Monfort (2000).

## A.6. Tables

Asia Pacific	G7 & Western Europe	Latin America
Australia*	United States*	Argentina
China	Japan*	Brazil
Hong Kong	Germany	Chile
India	France	Mexico
Indonesia	United Kingdom*	Venezuela
Japan	Italy	Colombia
Malaysia	Canada*	Peru
New Zealand*	Euro zone*	Bolivia
Philippines	Netherlands	Costa Rica
Singapore	Norway*	Dominican Republic
South Korea	Spain	Ecuador
Taiwan	Sweden*	El Salvador
Thailand	Switzerland*	Guatemala
Bangladesh	Austria	Honduras
Pakistan	Belgium	Nicaragua
Sri Lanka	Denmark	Panama
Vietnam	Egypt	Paraguay
Myanmar	Finland	Uruguay
	Greece	
	Ireland	
	Israel	
	Nigeria	
	Portugal	
	Saudi Arabia	
	South Africa	

**Table A.1. All countries included in the Consensus Economics datasets.** Highlighted countries (\*) are selected to be part of this paper’s main scope as they offer a rich cross-section of contributors.

Panel (a): Summary Statistics	Yields		Returns	
	Mean	Std Dev	Mean	Std Dev
Par	3.87	1.64	4.53	1.49
Zero	3.98	1.67	4.79	1.55
Difference	-0.13	0.09	-0.28	0.21

Panel (b): Frequency Comparison	Yields	
	Mean	Std Dev
Continuous	3.98	1.67
Annualy	4.07	1.74
Semi-Annual	4.02	1.70

**Table A.2. Summary Statistics: Par Yields and Zero Yields**

This table shows the mean and standard deviation for (1) US 10-year par yields obtained from the Fed, (2) US 10-year zero yields obtained from Bloomberg, and (3) their differences. Panel (b) contains similar summary statistics for US 10-year zero log-yields obtained from Bloomberg assuming different compounding frequencies of the raw data. Sample period is monthly observations from between 01/1995 and 12/2020.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
<b>Panel (a): <i>BRP</i></b>										
AUD	1.00	0.54	0.66	0.76	0.44	0.31	0.65	0.75	0.61	0.62
CAD	0.54	1.00	0.48	0.68	0.52	0.52	0.45	0.66	0.53	0.69
CHF	0.66	0.48	1.00	0.72	0.21	0.36	0.66	0.59	0.66	0.54
EUR	0.76	0.68	0.72	1.00	0.54	0.37	0.65	0.68	0.69	0.69
GBP	0.44	0.52	0.21	0.54	1.00	0.28	0.19	0.34	0.21	0.56
JPY	0.31	0.52	0.36	0.37	0.28	1.00	0.32	0.36	0.41	0.32
NOK	0.65	0.45	0.66	0.65	0.19	0.32	1.00	0.59	0.73	0.46
NZD	0.75	0.66	0.59	0.68	0.34	0.36	0.59	1.00	0.65	0.62
SEK	0.61	0.53	0.66	0.69	0.21	0.41	0.73	0.65	1.00	0.49
USD	0.62	0.69	0.54	0.69	0.56	0.32	0.46	0.62	0.49	1.00
<b>Panel (b): <i>XRP</i></b>										
AUD	1.00	0.72	0.55	0.78	0.53	-0.07	0.54	0.91	0.78	
CAD	0.72	1.00	0.30	0.53	0.40	-0.17	0.47	0.71	0.58	
CHF	0.55	0.30	1.00	0.84	0.50	0.41	0.72	0.49	0.69	
EUR	0.78	0.53	0.84	1.00	0.67	0.15	0.76	0.73	0.88	
GBP	0.53	0.40	0.50	0.67	1.00	0.12	0.55	0.46	0.62	
JPY	-0.07	-0.17	0.41	0.15	0.12	1.00	0.06	-0.17	0.01	
NOK	0.54	0.47	0.72	0.76	0.55	0.06	1.00	0.48	0.77	
NZD	0.91	0.71	0.49	0.73	0.46	-0.17	0.48	1.00	0.69	
SEK	0.78	0.58	0.69	0.88	0.62	0.01	0.77	0.69	1.00	

**Table A.3. Cross Country Correlations of Survey Premia**

This table shows the correlation coefficients between the measures of subjective risk premia, as defined in section II in the main body of the paper. Based on monthly observations from between 1995.1 to 2020.12.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
<b>Panel (a): <math>BRP_{proj}</math></b>										
AUD	1.00	0.60	0.52	0.69	0.47	0.23	0.42	0.46	0.60	0.36
CAD	0.60	1.00	0.65	0.77	0.62	0.34	0.52	0.43	0.68	0.75
CHF	0.52	0.65	1.00	0.81	0.25	0.56	0.61	0.10	0.62	0.31
EUR	0.69	0.77	0.81	1.00	0.60	0.55	0.72	0.34	0.75	0.44
GBP	0.47	0.62	0.25	0.60	1.00	-0.10	0.52	0.60	0.46	0.72
JPY	0.23	0.34	0.56	0.55	-0.10	1.00	0.24	-0.29	0.28	-0.14
NOK	0.42	0.52	0.61	0.72	0.52	0.24	1.00	0.11	0.73	0.30
NZD	0.46	0.43	0.10	0.34	0.60	-0.29	0.11	1.00	0.40	0.61
SEK	0.60	0.68	0.62	0.75	0.46	0.28	0.73	0.40	1.00	0.36
USD	0.36	0.75	0.31	0.44	0.72	-0.14	0.30	0.61	0.36	1.00
<b>Panel (b): <math>XRP_{proj}</math></b>										
AUD	1.00	0.73	0.81	0.84	0.55	0.63	0.60	0.82	0.76	
CAD	0.73	1.00	0.69	0.73	0.44	0.57	0.71	0.56	0.79	
CHF	0.81	0.69	1.00	0.89	0.36	0.83	0.76	0.61	0.60	
EUR	0.84	0.73	0.89	1.00	0.69	0.60	0.86	0.78	0.76	
GBP	0.55	0.44	0.36	0.69	1.00	0.02	0.55	0.78	0.69	
JPY	0.63	0.57	0.83	0.60	0.02	1.00	0.46	0.36	0.32	
NOK	0.60	0.71	0.76	0.86	0.55	0.46	1.00	0.51	0.69	
NZD	0.82	0.56	0.61	0.78	0.78	0.36	0.51	1.00	0.76	
SEK	0.76	0.79	0.60	0.76	0.69	0.32	0.69	0.76	1.00	

**Table A.4. Cross Country Correlations of Projected Premia**

This table shows the correlation coefficients between the measures of subjective risk premia, as defined in section II in the main body of the paper. Based on monthly observations from between 1995.1 to 2020.12.

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	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
<b>Panel (a): BRP projected</b>										
Mean	4.80	5.16	3.97	5.20	4.75	3.59	3.64	3.52	6.19	4.65
Std	2.99	3.21	1.88	2.98	2.82	2.57	3.44	2.49	3.77	1.86
Skew	0.04	0.40	0.29	0.27	0.24	0.44	-0.33	0.10	0.56	0.15
AR(1)	0.93	0.97	0.96	0.96	0.98	0.97	0.96	0.97	0.96	0.98
<b>Panel (b): XRP projected</b>										
Mean	1.43	0.29	-0.68	-1.32	-0.31	-3.00	-0.89	2.19	-1.20	
Std	4.19	1.15	4.24	3.53	1.40	3.18	3.37	3.79	3.65	
Skew	0.06	-0.11	0.02	0.13	0.25	-0.33	0.58	-0.42	0.29	
AR(1)	0.98	0.97	0.99	0.99	0.98	0.99	0.98	0.98	0.98	

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**Table A.5. Descriptive Statistics for Projected Risk Premia**

This table presents the means, standard deviations, skewness, and AR(1) coefficients for exchange rate risk premia (*XRP*) and bond risk premia (*BRP*) where conditional expectations defined in Equations 6 and 8 are replaced by projections. These predictions are obtained by regressing realized ex-post excess returns on the slope of the yield curve (for bond risk premia) and the interest rate differential between the foreign country and the United States (for exchange rate risk premia). The sample period is 1995.1 to 2020.12.

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	USD
Panel (a): 1995.1 to 2007.12.										
Mean	-0.82	0.91	-1.61	-1.02	-0.86	0.58	-2.67	-0.29	-0.04	-0.73
Std	3.77	3.73	2.04	2.03	2.66	2.02	3.22	3.26	3.33	2.96
Skew	1.08	0.62	0.07	0.10	-0.03	0.58	-0.09	0.66	0.51	0.28
AR(1)	0.72	0.83	0.63	0.66	0.74	0.72	0.73	0.70	0.80	0.66
Panel (b): 2008.1 to 2020.12.										
Mean	-0.85	-1.76	-2.13	-1.33	-0.24	-0.02	-2.78	-1.60	-3.08	-2.07
Std	3.10	3.34	1.93	2.23	3.33	1.34	3.43	3.76	3.13	2.56
Skew	0.41	0.96	0.10	0.77	0.56	0.37	-0.41	0.25	0.11	0.55
AR(1)	0.71	0.85	0.71	0.66	0.81	0.79	0.75	0.78	0.73	0.65

**Table A.6. Sub Sample Summary Statistics  $BRP$**

This table presents the means, standard deviations, skewness and AR(1) coefficients for subjective bond risk premia ( $BRPs$ ) for the sample periods 1995.1 to 2007.12. (panel A) and 2008.1 to 2020.12. (panel B).

	PC1	PC2	PC3
BRP	58.75	11.76	8.42
XRP	63.96	14.34	6.55

**Table A.7. PCA Analysis: Variance Explained (%)**

Risk premium factors  $PC_t$  are formed from an eigenvalue decomposition of the covariance matrix of subjective risk premia  $var(RP_t) = QDQ^\top$ . This table displays the fraction of subjective risk premium variance due to the  $n$ 'th factor which is computed from  $D(n, n) / \sum_n D(n, n)$ . Factors (principle components) are computed from linear combinations (rotations) of the input series via  $PC_t = RP_t Q$ . The sample period 1995.1 to 2020.12



	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK
<u>Panel (a): <math>\Delta x_{t+1}</math></u>									
Mean	0.09	0.85	-0.11	0.93	-0.26	-0.37	2.92	-0.99	3.70
Std	5.33	2.50	5.08	4.35	2.81	4.81	4.25	5.85	4.72
AR(1)	0.88	0.70	0.88	0.89	0.75	0.87	0.83	0.89	0.87
<u>Panel (b): <math>IRD</math></u>									
Mean	-1.64	-0.18	1.59	0.76	-0.47	2.39	-0.63	-2.13	0.22
Std	1.66	0.86	1.53	1.37	1.14	2.06	1.76	1.56	1.70
AR(1)	0.98	0.97	0.99	0.99	0.98	0.99	0.98	0.98	0.98
<u>Panel (c): <math>rx_{t+1}^{(11)}</math></u>									
Mean	4.80	5.16	3.97	5.20	4.75	3.59	3.64	3.52	6.19
Std	8.94	6.65	5.80	6.69	7.33	4.46	7.20	8.29	8.68
AR(1)	0.90	0.88	0.91	0.89	0.90	0.87	0.88	0.90	0.90
<u>Panel (d): Slope</u>									
Mean	0.78	1.26	0.98	1.35	0.93	1.09	0.80	0.55	1.27
Std	0.65	0.96	0.64	0.83	1.21	0.70	1.03	1.12	0.77
AR(1)	0.93	0.98	0.96	0.97	0.98	0.97	0.96	0.97	0.96

**Table A.8. Descriptive Statistics Predictive Regressions**

This table presents the means, standard deviations, and skewness for log spot rate changes ( $\Delta x_{t+1}$ ), interest rate differentials ( $IRD_t = (i_t^{(1)} - i_t^{*,(1)})$ ), realised excess bond returns ( $rx_{t+1}^{(11)}$ ), and the slope of the yield curve ( $i_t^{*,(10)} - i_t^{*,(1)}$ ). The sample period is 1995.1 to 2020.12.

	AUD	CAD	CHF	EUR	JPY	GBP	NZD	NOK	SEK	USD
<i>FR</i>	0.41 (0.84)	0.01 (0.03)	0.10 (0.24)	0.03 (0.06)	0.36 (0.90)	0.05 (0.16)	-0.01 (-0.04)	0.32 (0.74)	0.25 (0.61)	0.08 (0.22)
<i>a</i>	-0.01 (-4.00)	-0.01 (-5.21)	-0.01 (-5.07)	-0.01 (-5.99)	-0.01 (-4.52)	-0.00 (-4.54)	-0.01 (-3.95)	-0.00 (-3.17)	-0.01 (-5.89)	-0.01 (-4.97)
<i>R</i> <sup>2</sup> (%)	0.53	0.00	0.04	0.00	0.50	0.01	0.00	0.34	0.26	0.03

(a) Interest Rates

	AUD	CAD	CHF	EUR	JPY	GBP	NZD	NOK	SEK
<i>FR</i>	0.53 (0.88)	0.98 (1.73)	-0.12 (-0.40)	0.09 (0.20)	0.21 (0.52)	1.13 (2.07)	0.15 (0.34)	0.82 (1.37)	0.18 (0.39)
<i>a</i>	0.00 (0.16)	-0.00 (-0.44)	0.01 (1.13)	-0.01 (-0.77)	-0.00 (-0.20)	0.00 (0.04)	-0.01 (-2.39)	0.01 (0.79)	-0.01 (-2.54)
<i>R</i> <sup>2</sup> (%)	0.63	2.33	0.06	0.03	0.17	3.25	0.07	1.50	0.10

(b) Exchange Rates

**Table A.9. Information Frictions: Coibion and Gorodnichenko (2015)**

This table shows individual country regressions of either (a) 10-year yields or (b) foreign exchange forecast errors, at the annual horizon, on forecast revisions defined as the monthly change in the respective forecasts

$$FE_{t,t+12}^i = a^i + b^i FR_{t-12,t}^i + \eta_{t+12}^i,$$

where t-statistics, reported in parentheses, are computed with a Newey-West standard errors computed with 12-lags. The sample period 1995.1 to 2020.12

	(i)	(ii)	(iii)	(iv)	(v)	(vi)
$E[gdp]$	-0.55 [-0.65 -0.46]					
$E[ip]$		-0.20 [-0.25 -0.15]				
$E[cons]$			-0.57 [-0.67 -0.49]			
$E[cpi]$				0.02 [-0.08 0.13]		
$BondVar$					2.16 [1.75 2.61]	
$rx_{t-1,t}^{11}$						-0.09 [-0.11 -0.08]
$R^2(\%)$	3.56	1.72	3.78	0.00	5.85	5.11

**Table A.10. Explaining Subjective Bond Risk Premia**

This table reports estimates from pooled OLS regressions of the form

$$BRP_t = a + b^\top X_t + \epsilon_t.$$

where  $BRP_t$  is the survey-implied bond risk premium and  $X_t$  is a vector of explanatory variables containing the 12-month *expected* growth rate in  $gdp$ , industrial production  $ip$ , consumption  $cons$ , inflation  $cpi$ , 10-year bond return variance ( $BondVar$ ), and the excess return on an 11-year bond realised between dates  $t - 1$  and  $t$ . A constant is included but not reported. Standard errors reported in  $(\cdot)$  parenthesis are computed using a Driscoll and Kraay (1998) estimator with 4-lags. The sample period is 1995.1 to 2020.12.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)
$E[gdp^* - gdp]$	0.76 [-0.12 1.47]					
$E[ip^* - ip]$		0.39 [0.01 0.77]				
$E[con^* - con]$			0.72 [0.03 1.29]			
$E[cpi^* - cpi]$				0.89 [0.10 1.58]		
$FXVar$					0.78 [0.29 1.25]	
$rx_{t-1,t}^{FX}$						-0.09 [-0.13 -0.06]
$R^2(\%)$	2.17	1.86	2.40	3.65	3.58	4.81

**Table A.11. Explaining Subjective FX Risk Premia**

This table reports estimates from pooled OLS regressions of the form

$$XRP_t = a + b^\top X_t + \epsilon_t.$$

where  $XRP_t$  is the survey-implied currency risk premium and  $X_t$  is a vector of explanatory variables containing the 12-month *expected* growth rate in  $gdp$ , industrial production  $ip$ , consumption  $cons$ , inflation  $cpi$ , FX return variance ( $FXVar$ ), and the excess currency return realised between dates  $t-1$  and  $t$ . A constant is included but not reported. Standard errors reported in  $(\cdot)$  parenthesis are computed using a Driscoll and Kraay (1998) estimator with 4-lags. The sample period is 1995.1 to 2020.12.

<b>Panel (a): Bond Return</b>			
	(i)	(ii)	(iii)
<i>Slope</i>	2.81 (0.53)		2.49 (0.58)
<i>BRP</i>		0.52 (0.15)	0.30 (0.17)
<i>R</i> <sup>2</sup> (%)	13.85	4.96	14.79
<b>Panel (b): FX Return</b>			
	(i)	(ii)	(iii)
<i>IRD</i>	-1.56 (0.60)		-1.41 (0.56)
<i>XRP</i>		0.55 (0.29)	0.45 (0.27)
<i>R</i> <sup>2</sup> (%)	8.89	5.43	12.54

**Table A.12. Bond & FX Return Panel Predictability Regressions**

This table reports estimates from POOLED OLS regressions of the form

$$rx_{t,t+1}^{(11)} = a + b_1 Slope_t + b_2 BRP_t + \epsilon_{t,t+1} \quad \text{and} \quad rx_{t,t+1}^{FX} = a + b_1 IRD_t + b_2 XRP_t + \epsilon_{t,t+1}$$

where the dependent variables are the one-year excess return on an 11-year bond and the one-year currency excess return in panels (a) and (b), respectively.  $Slope_t = (i_t^{(10)} - i_t^{(1)})$  is the slope of the domestic yield curve,  $BRP_t$  is the survey-implied bond risk premium,  $IRD_t = (i_t^{(1)} - i_t^{*,(1)})$  is the one-year interest rate differential, and  $XRP_t$  is the survey-implied currency risk premium. A constant is included but not reported. Standard errors reported in (·) parentheses are computed using a Driscoll and Kraay (1998) estimator with 12-lags. The sample period is 1995.1 to 2020.12.

	Pooled		Pooled
$Level_t$	0.08 (0.03)	$IRD_t$	-0.95 (0.21)
$Slope_t$	0.08 (0.05)		
$a$	-0.01 (0.00)	$a$	0.00 (0.01)
$R^2(\%)$	5.71	$R^2(\%)$	5.73
(a) Inter- est Rates		(b) Exchange Rates	

**Table A.13. Forecast Error Predictability**

This table shows POOLED OLS regressions of either (a) 10-year yield or (b) foreign exchange forecast errors, at the annual horizon, on either the slope of the yield curve or the 1-year interest rate differential

$$FE_{t,t+1}^i = a^i + b^i X_t^i + \eta_{t+1}^i,$$

where for interest rate forecast errors  $X_t$  includes the  $Level_t = i_t^{(1)}$  and  $Slope_t = (i_t^{(10)} - i_t^{(1)})$  and for exchange rates  $IRD_t = (i_t^{(1)} - i_t^{*,(1)})$ .

Standard errors reported in  $(\cdot)$  parentheses are computed using a Driscoll and Kraay (1998) estimator with 12-lags.

The sample period 1995.1 to 2020.12

	AUD	CAD	CHF	EUR	JPY	GBP	NZD	NOK	SEK	USD
$Level_t$	0.14 (1.97)	0.13 (2.26)	0.17 (2.62)	0.09 (1.74)	0.12 (2.12)	-0.85 (-3.16)	0.17 (2.28)	0.28 (4.69)	0.03 (0.51)	0.25 (3.92)
$Slope_t$	0.17 (0.85)	0.12 (1.27)	-0.04 (-0.25)	-0.02 (-0.18)	0.23 (2.09)	0.26 (2.25)	0.08 (0.48)	0.43 (3.49)	0.06 (0.38)	0.31 (2.62)
$a$	-0.01 (-3.34)	-0.01 (-4.48)	-0.01 (-3.68)	-0.01 (-3.75)	-0.01 (-4.14)	-0.00 (-3.96)	-0.01 (-3.36)	-0.02 (-5.82)	-0.01 (-3.34)	-0.02 (-5.25)
$R^2(\%)$	8.94	12.57	17.20	6.75	11.80	16.29	17.51	34.99	0.95	26.02

(a) Interest Rates

	AUD	CAD	CHF	EUR	JPY	GBP	NZD	NOK	SEK
$IRD_t$	-3.09 (-4.20)	-0.88 (-0.74)	-2.99 (-4.86)	-3.14 (-2.57)	-4.03 (-2.46)	-0.02 (-2.01)	-0.22 (-1.70)	-1.91 (-2.47)	-0.35 (-2.65)
$a$	-0.05 (-2.74)	-0.01 (-0.52)	0.06 (4.48)	0.01 (0.43)	-0.02 (-1.07)	0.00 (1.51)	-0.01 (-2.90)	-0.03 (-1.43)	-0.00 (-2.09)
$R^2(\%)$	30.13	1.36	33.37	13.91	12.65	10.33	6.44	13.28	14.20

(b) Exchange Rates

**Table A.14. Forecast Error Predictability: Individual Country Regressions**

This table shows individual country regressions of either (a) 10-year yield or (b) foreign exchange forecast errors, at the annual horizon, on either the slope of the yield curve or the 1-year interest rate differential

$$FE_{t,t+1}^i = a^i + b^i X_t^i + \eta_{t+1}^i,$$

where for interest rate forecast errors  $X_t$  includes the  $Level_t = i_t^{(1)}$  and  $Slope_t = (i_t^{(10)} - i_t^{(1)})$  and for exchange rates  $IRD_t = (i_t^{(1)} - i_t^{*,(1)})$  and t-statistics, reported in parenthesis, are computed with a Newey-West standard errors computed with 12-lags. The sample period 1995.1 to 2020.12

Panel A				
Parameter	Estimate	Lower	Upper	
$\sigma_Q$	0.17	0.17	0.17	
$\sigma_i$	0.07	0.07	0.07	
$\kappa_i^S$	0.36	0.36	0.36	
$\theta_i^S$	0.01	0.01	0.01	
$\phi_i$	10.04	10.04	10.04	
$\rho_{iQ}$	0.38	0.38	0.38	

Panel B				
Moment	Data	Model	Lower	Upper
mean realised inflation	0.45	0.47	0.45	0.48
std realised inflation	0.85	0.85	0.84	0.86
AR(1) realised inflation	0.95	0.91	0.90	0.92
mean expected inflation	0.69	0.72	0.71	0.73
std expected inflation	0.55	0.48	0.47	0.49
AR(1) expected inflation	0.98	0.94	0.93	0.95
mean forecast error	-0.24	-0.26	-0.27	-0.24
std forecast error	0.96	0.96	0.95	0.97
AR(1) forecast error	0.94	0.90	0.89	0.92

**Table A.15. CHF Inflation Parameter Estimates**

This table reports estimation results for the Swiss subjective inflation process

$$\frac{dQ_t}{Q_t} = i_t dt + \sigma_Q \sqrt{i_t} dW_t^{\mathbb{P},Q},$$

$$di_t = \kappa_i^S (\theta_i^S - i_t) dt + \sigma_i \sqrt{i_t} dW_t^{\mathbb{S},i}.$$

where the correlation between shocks is  $\langle dW_t^{\mathbb{P},Q}, dW_t^{\mathbb{P},i} \rangle = \rho_{iQ} dt$ . The model is estimated via simulated method of moments targeting the mean, standard deviation and monthly autocorrelation of 12-month realised inflation and 12-month subjective expected inflation from consensus economics. In addition, by combining realisations and expectations we obtain 12-month horizon forecast errors from which we compute two additional non-redundant moments - the standard deviation and autocorrelation - of the errors. Section A.5 in the OA reports estimation details. Panel (a) reports point estimates alongside 95% confidence intervals. Panel (b) reports moments in the data and model implied moments alongside 95% confidence intervals.



<b>Panel A</b>				
Parameter	Estimate	Lower	Upper	
$\sigma_Q$	0.09	0.08	0.09	
$\sigma_i$	0.03	0.03	0.03	
$\kappa_i^S$	0.02	0.00	0.03	
$\theta_i^S$	0.02	0.02	0.02	
$\phi_i$	0.03	0.00	0.06	
$\rho_{iQ}$	0.97	0.87	1.07	

<b>Panel B</b>				
Moment	Data	Model	Lower	Upper
mean realised inflation	2.25	2.25	2.10	2.39
std realised inflation	1.28	1.36	1.31	1.41
AR(1) realised inflation	0.94	0.92	0.92	0.92
mean expected inflation	2.18	2.18	2.07	2.30
std expected inflation	0.63	0.61	0.58	0.65
AR(1) expected inflation	0.96	0.97	0.97	0.97
mean forecast error	0.07	0.07	0.03	0.10
std forecast error	1.45	1.38	1.33	1.43
AR(1) forecast error	0.94	0.90	0.90	0.90

**Table A.16. USD Inflation Parameter Estimates**

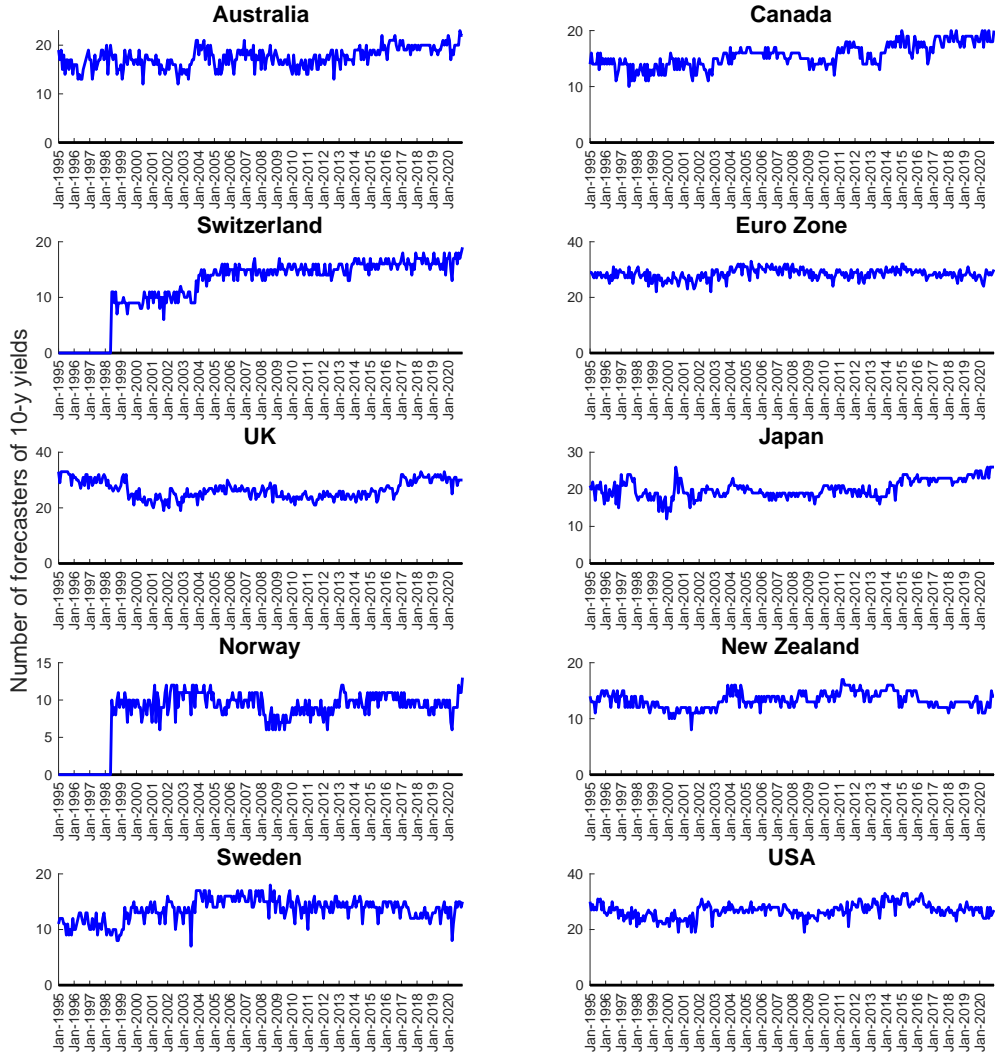
This table reports estimation results for the U.S. subjective inflation process

$$\frac{dQ_t}{Q_t} = i_t dt + \sigma_Q \sqrt{i_t} dW_t^{\mathbb{P},Q},$$

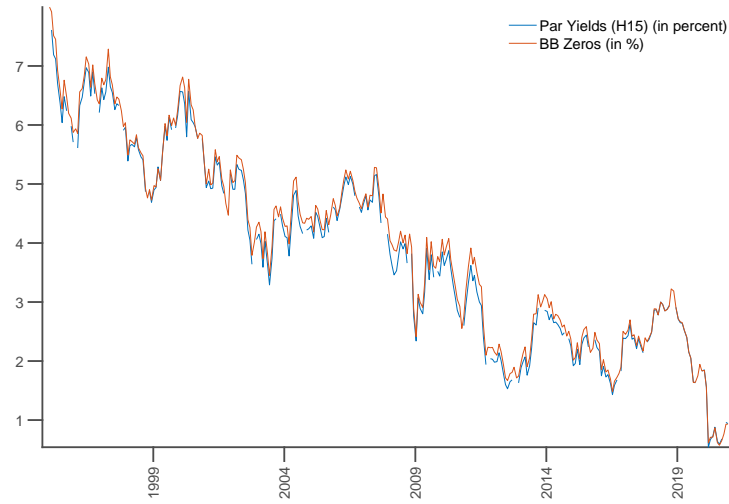
$$di_t = \kappa_i^S (\theta_i^S - i_t) dt + \sigma_i \sqrt{i_t} dW_t^{\mathbb{S},i}.$$

where the correlation between shocks is  $\langle dW_t^{\mathbb{P},Q}, dW_t^{\mathbb{P},i} \rangle = \rho_{iQ} dt$ . The model is estimated via simulated method of moments targeting the mean, standard deviation and monthly autocorrelation of 12-month realised inflation and 12-month subjective expected inflation from consensus economics. In addition, by combining realisations and expectations we obtain 12-month horizon forecast errors from which we compute two additional non-redundant moments - the standard deviation and autocorrelation - of the errors. Section A.5 in the OA reports estimation details. Panel (a) reports point estimates alongside 95% confidence intervals. Panel (b) reports moments in `thg1data` and model implied moments alongside 95% confidence intervals.

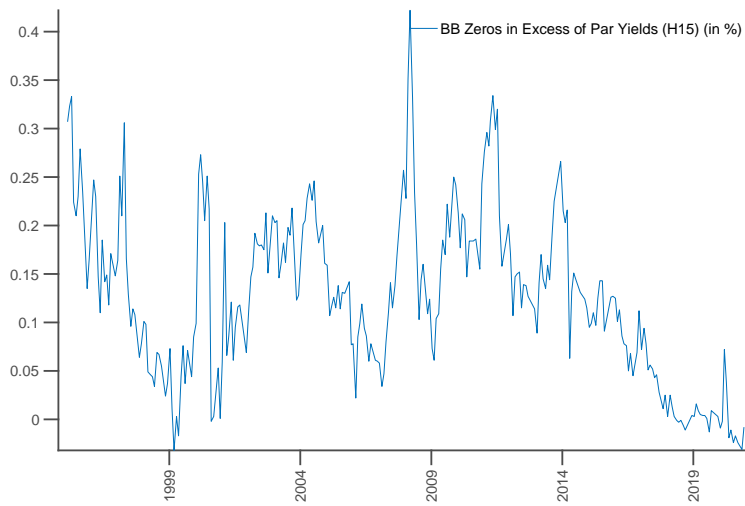
## A.7. Figures



**Figure A.1. Number of long-term bond yield forecasters by country**  
 This figure displays number of forecasters predicting 10-year as part of the Consensus Economics surveys for the given country. The sample period is 1995.1 - 2020.12.



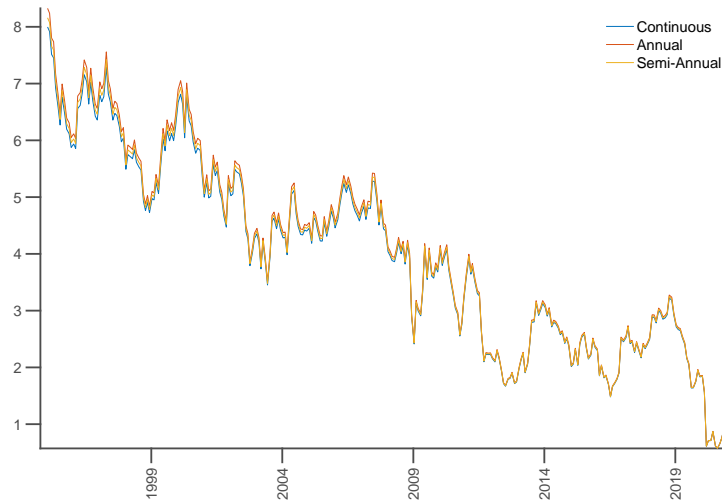
(a)



(b)

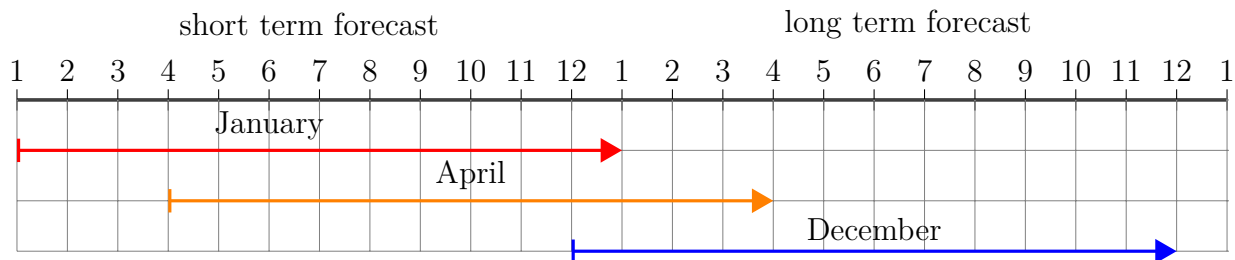
**Figure A.2. Comparison of H15 and BB yields**

The above figures show the time series of US 10-year par yields obtained from the Fed and US 10-year zero yields obtained from Bloomberg (Figure a) as well as the difference between the two series (Figure b). Data is available for the sample period 2000.1 to 2020.12.



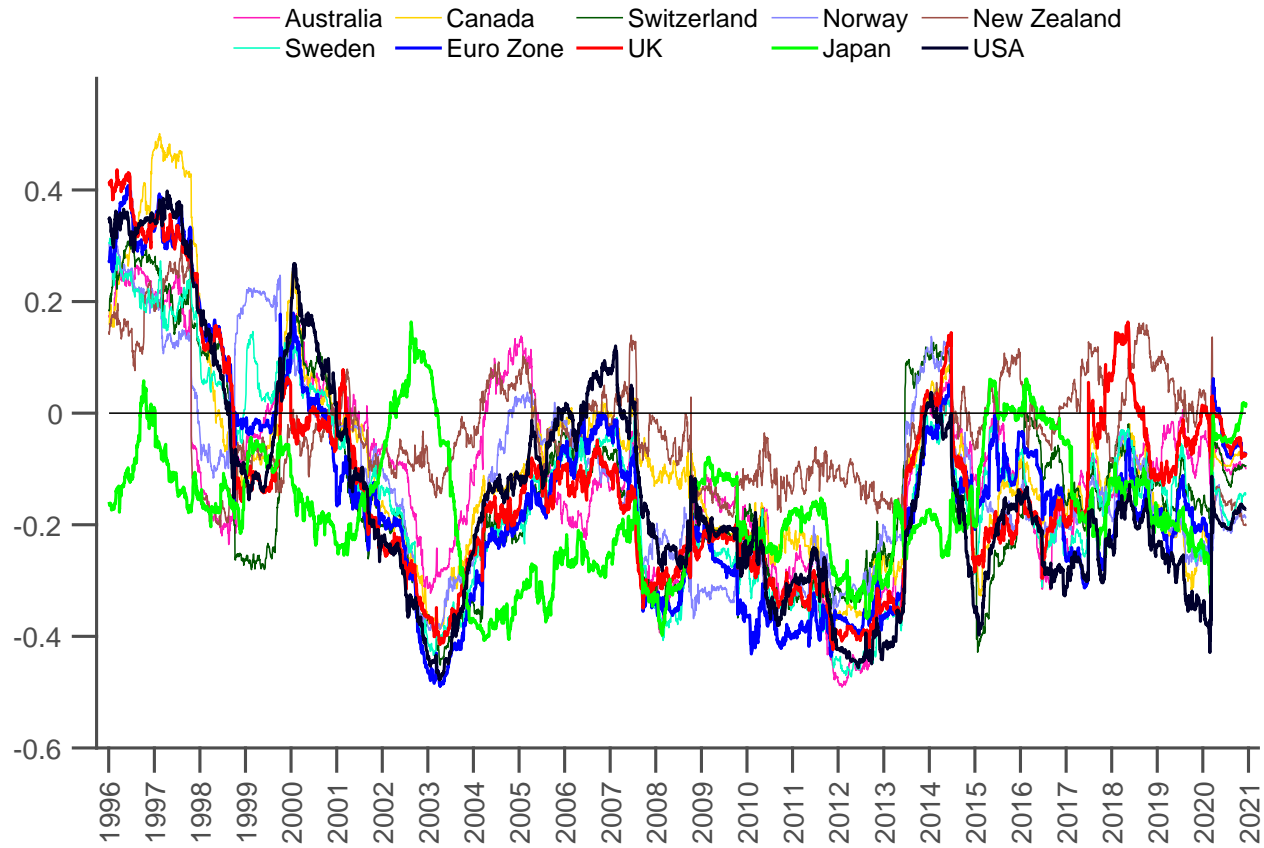
**Figure A.3. Comparison of compounding frequencies**

The above figure shows the time series of US 10-year zero log yields obtained from Bloomberg that have been generated assuming continuous, annual, and semi-annual compounding. Data is available for the sample period 2000.1 to 2020.12.



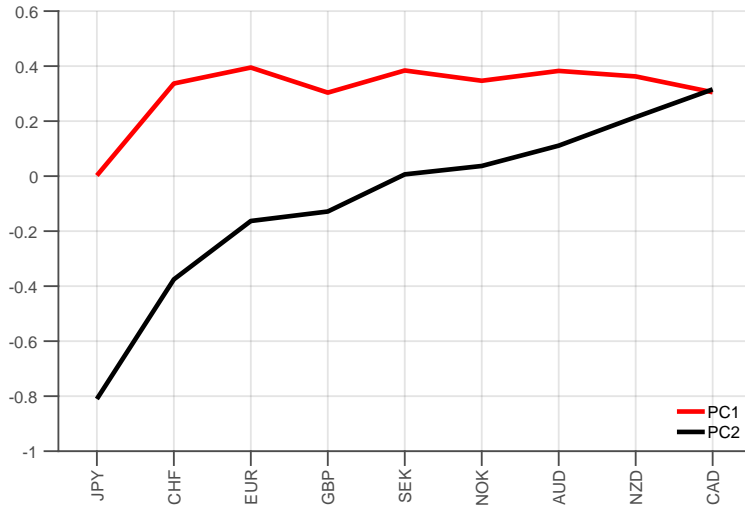
**Figure A.4. Constant Maturity Macro Expectations**

Figure displays a visual explanation to the construction of the constant maturity proxy. Let  $j$  be the month of the year, so that  $j = 1$  for January and  $j = 1, 2, \dots, 12$ . A constant maturity expectation is formed taking as weight  $(1 - \frac{j}{12})$ , for the short term projection (the remaining forecast for the same year), and  $\frac{j}{12}$ , for the long-term projection (the forecast for the following year).

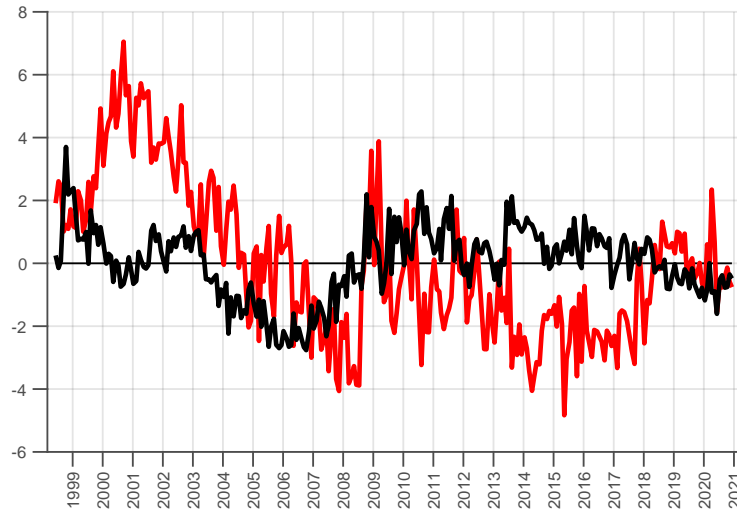


**Figure A.5. Stock-Bond Correlation**

Figure displays the time series of rolling window correlations between ten-year log bond returns and log equity returns for a given country. We use the respective country-specific benchmark equity indices. The rolling window length is 222 daily observations. The sample period is 1995.1 to 2020.12.



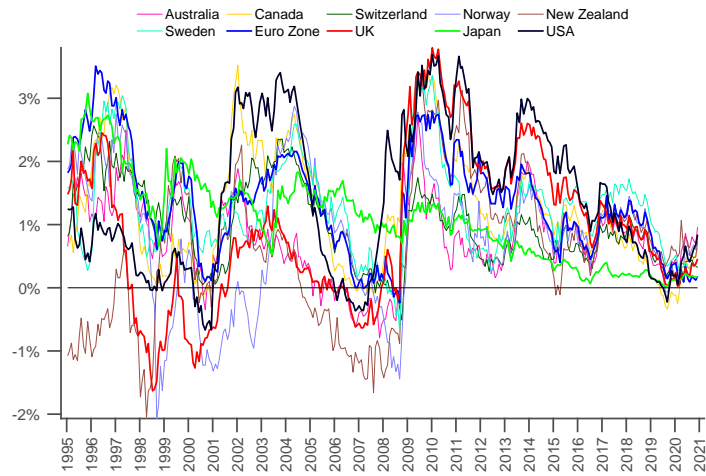
(a) Factor Loadings



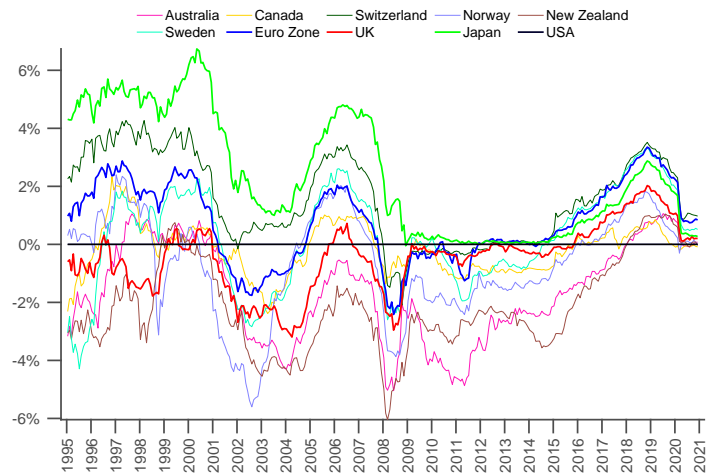
(b) Factor Time Series

### Figure A.6. PCAs of Subjective Exchange Rate Risk Premia

Subjective exchange rate risk premia factors  $PC_t$  are formed from an eigenvalue decomposition of the covariance matrix of  $var(XRP_t) = QDQ^\top$ . The variance due to the  $n$ 'th factor is computed from  $D(n, n) / \sum_n D(n, n)$  which is displayed in the online appendix. Principle components (PCs) are computed from the rotation  $PC_t = XRP_t Q$ . Panel (a) displays the factor loadings (the columns of  $Q$ ) and panel (b) displays the dynamics of PC1 and PC2. The sample period is 1995.1 to 2020.12.



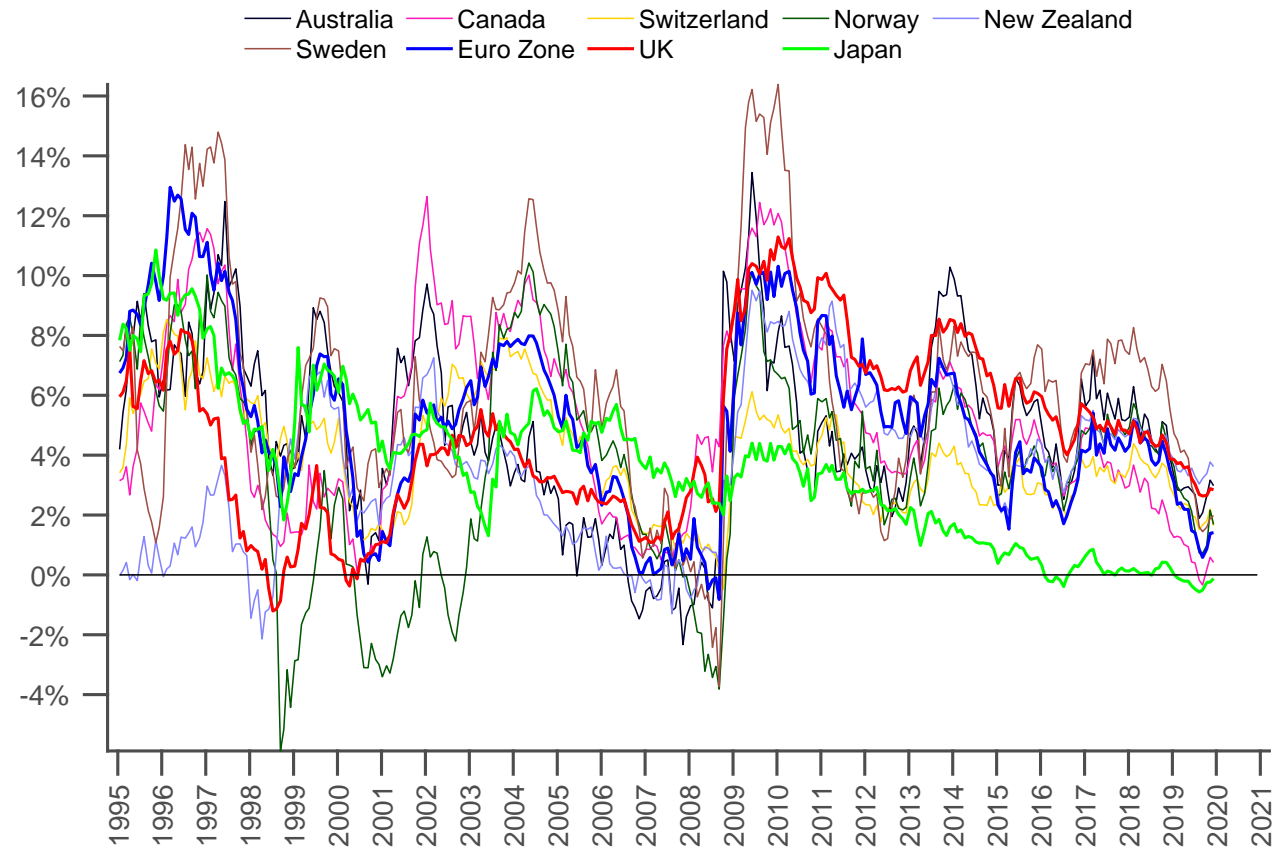
(a) Slopes



(b) IRDs

**Figure A.7. Interest Rate Spreads**

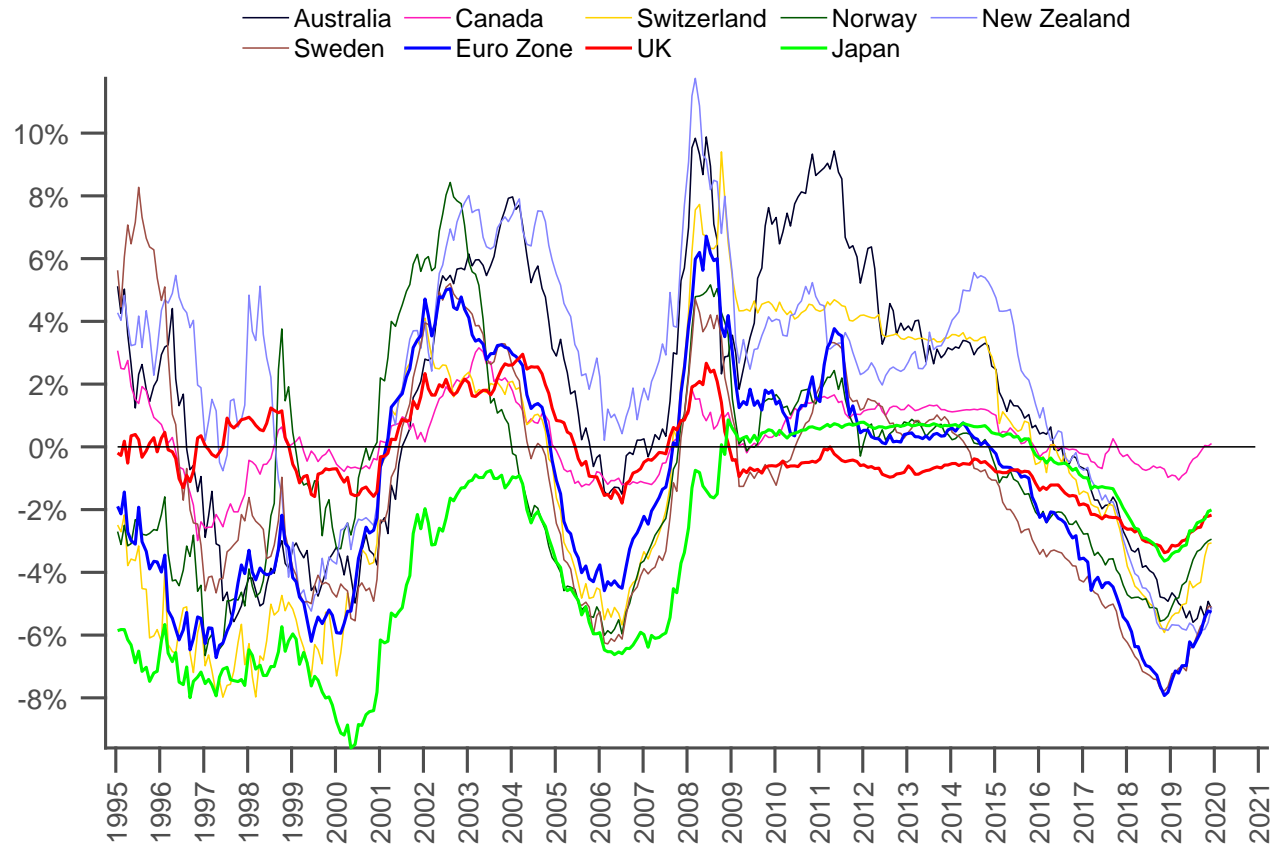
Figure displays term structure slopes (panel a) and 1-year interest rate differentials (panel b) for AUD, CAD, CHF, NOK, NZD, SEK, JPY, EUR, GBP and USD. The slope of the yield curve is defined as the difference between the respective country’s ten year bond yield and its one year bond yield. The sample period is 1995.1 to 2020.12.



**Figure A.8. Projected Bond Risk Premia**

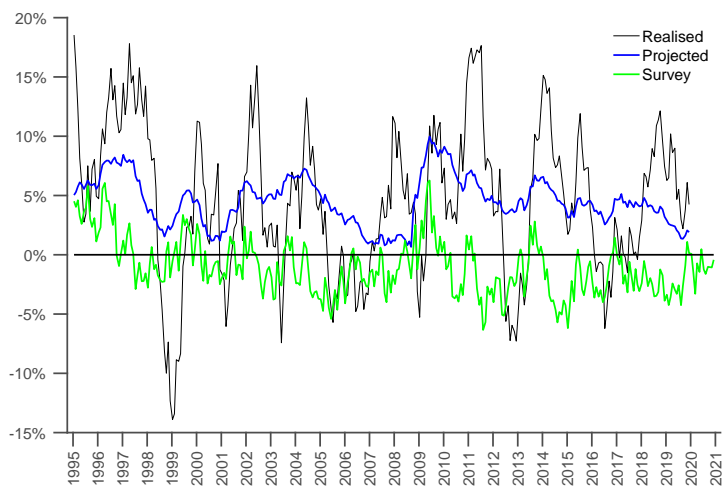
Figure displays projected bond risk premia for AUD, CAD, CHF, NOK, NZD, SEK, JPY, EUR, and GBP. The projections for the bond risk premium are obtained by regressing realised ex-post premia on the slope of the yield curve and then forming 12-month ahead projections. Sample period is monthly observations from between 12/1995 and 12/2019.



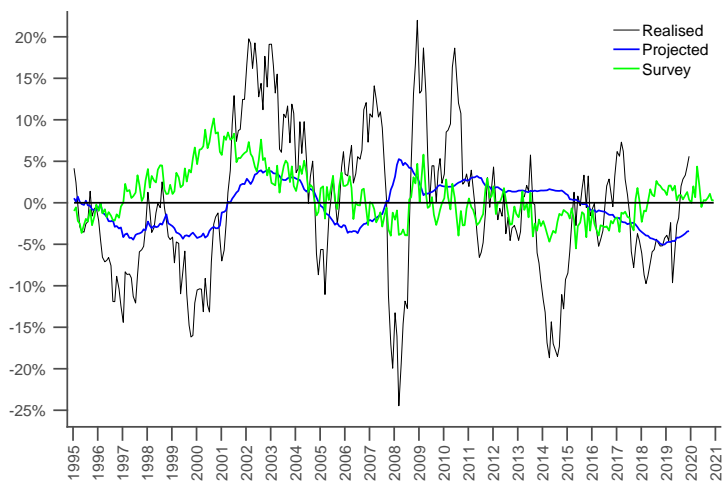


**Figure A.9. Projected Exchange Rate Risk Premia**

Figure displays subjective exchange rate risk premia for AUD, CAD, CHF, NOK, NZD, SEK, JPY, EUR, and GBP. The projection for the bond risk premium are obtained by regressing realised ex-post premia on the interest rate differential between the foreign country and the United States and then forming 12-month ahead projections. The sample period is 1995.1 to 2020.12.



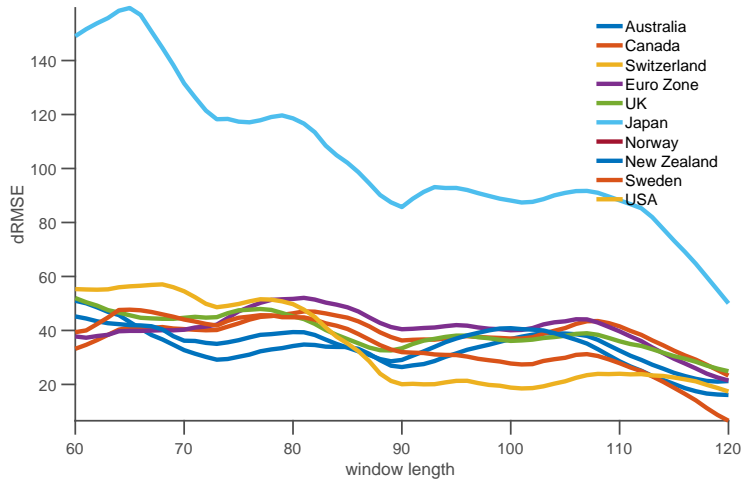
(a)



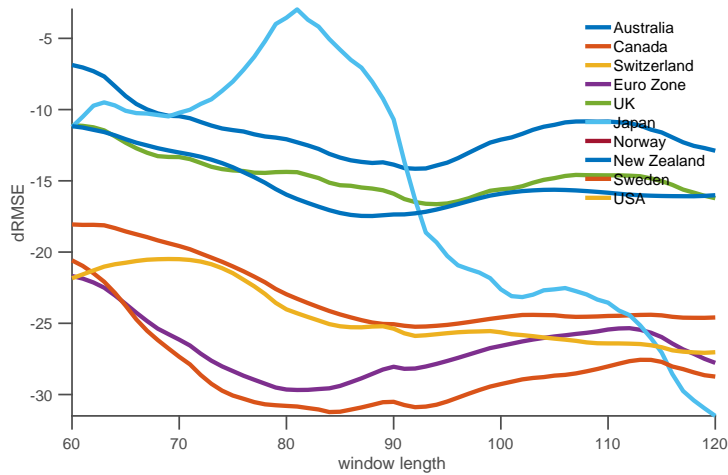
(b)

**Figure A.10. Survey vs Projected Risk Premia**

Figure displays subjective risk premia as an equally-weighted average across countries. Blue lines are the average projected risk premia while green lines are average survey-implied risk premia. In panel A projections for bond risk premia are obtained by regressing realised excess returns on the slope of the yield curve. In panel B projections for exchange rate risk premia are obtained by regressing realised excess returns on the interest rate differential between the foreign country. The sample period is 1995.1 to 2020.12 for survey forecasts and 1995.1 to 2019.12 for projection-based forecasts.



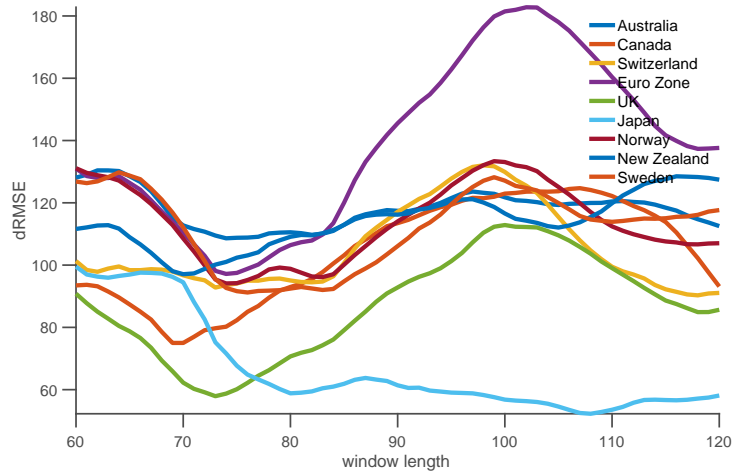
(a)



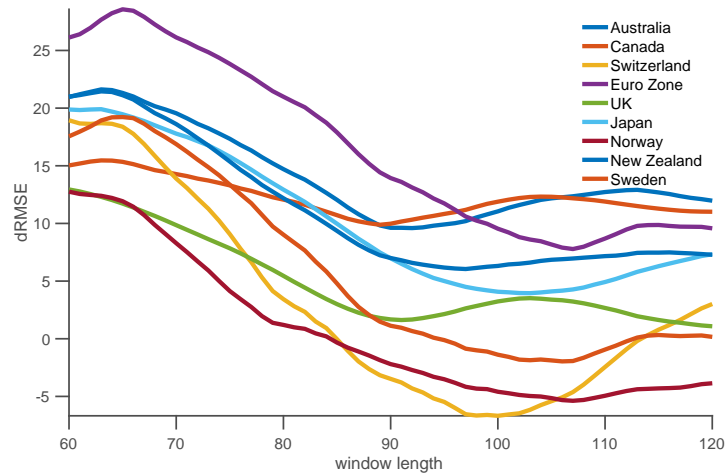
(b)

**Figure A.11. Correcting Errors:  $\Delta$  RMSE in the Fixed Income market**

Figure displays the exploitation of predictable forecast errors using Equation A.4. Panel A shows the difference in root mean squared error ( $\Delta$  RMSE) between the predictability-corrected forecast and the raw survey-implied forecasts. To predict forecast errors, a linear model as specified in Equation 17 is used to predict the error term of Equation A.4. Panel B shows the same difference in  $\Delta$  RMSE between the raw survey-implied forecasts and the predictability-corrected forecast when using just the simple historical average to predict the error term of Equation A.4. The horizontal axes shows the behaviour of  $\Delta$  RMSE for varying moving window sizes of 60 to 120 observations. The vertical axis shows the  $\Delta$  RMSE in percent ( $\frac{RMSE_{corrected} - RMSE_{survey}}{RMSE_{survey}} * 100$ ). Data is available for the sample period 2000.1 to 2020.12.



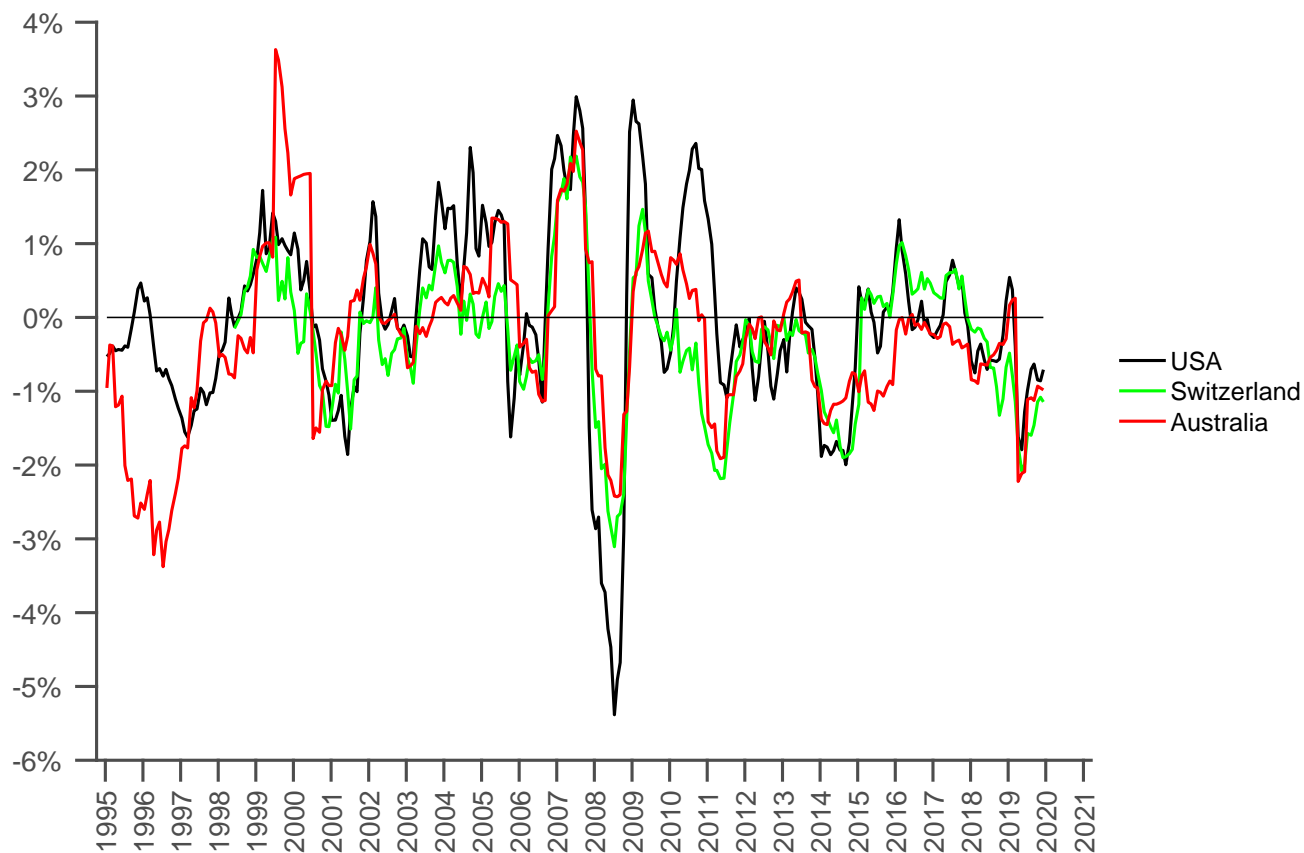
(a)



(b)

**Figure A.12. Correcting Errors:  $\Delta$  RMSE in the Foreign Exchange market**

Figure displays the exploitation of predictable forecast errors using Equation A.4. Panel A shows the difference in root mean squared error ( $\Delta$  RMSE) between the predictability-corrected forecast and the raw survey-implied forecasts. To predict forecast errors, a linear model as specified in Equation 17 is used to predict the error term of Equation A.4. Panel B shows the same difference in  $\Delta$  RMSE between the raw survey-implied forecasts and the predictability-corrected forecast when using just the simple historical average to predict the error term of Equation A.4. The horizontal axes shows the behaviour of  $\Delta$  RMSE for varying moving window sizes of 60 to 120 observations. The vertical axis shows the  $\Delta$  RMSE in percent ( $\frac{RMSE_{corrected} - RMSE_{survey}}{RMSE_{survey}} * 100$ ). Data is available for the sample period 2000.1 to 2020.12.



**Figure A.13. Forecast errors of inflation expectations**

Figure displays the difference between realised inflation and expected inflation as measured by the CPI for the US, Australia, and Switzerland. Expectations are obtained from Consensus Economics for a 12-month horizon while realisations are obtained from FRED, the Australian Bureau of Statistics (ABS), and the Federal Statistical Office of Switzerland, respectively. As the Australian Bureau of Statistics only reports CPI realisations on a quarterly frequency, we compute weighted averages of quarterly observations to obtain monthly estimates. The sample period is 1995.1 to 2020.12.