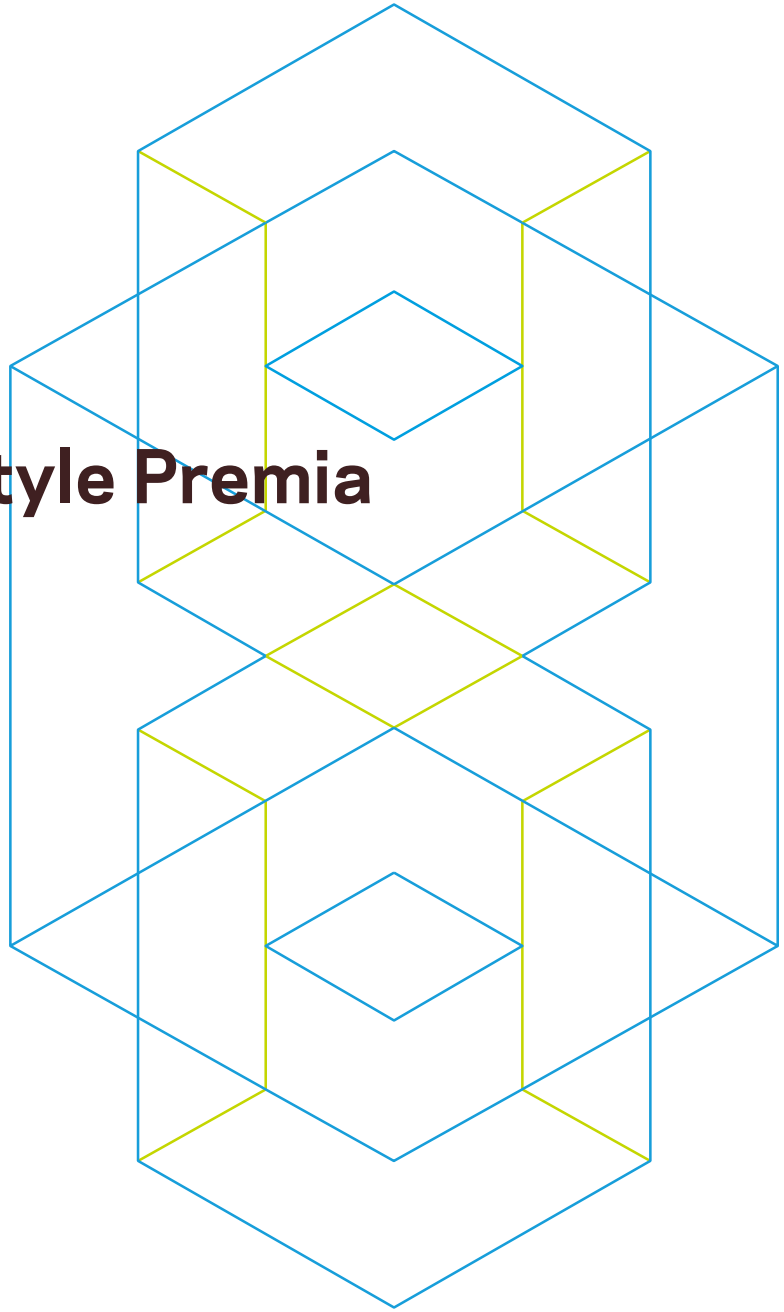
A Century of Evidence on Style Premia

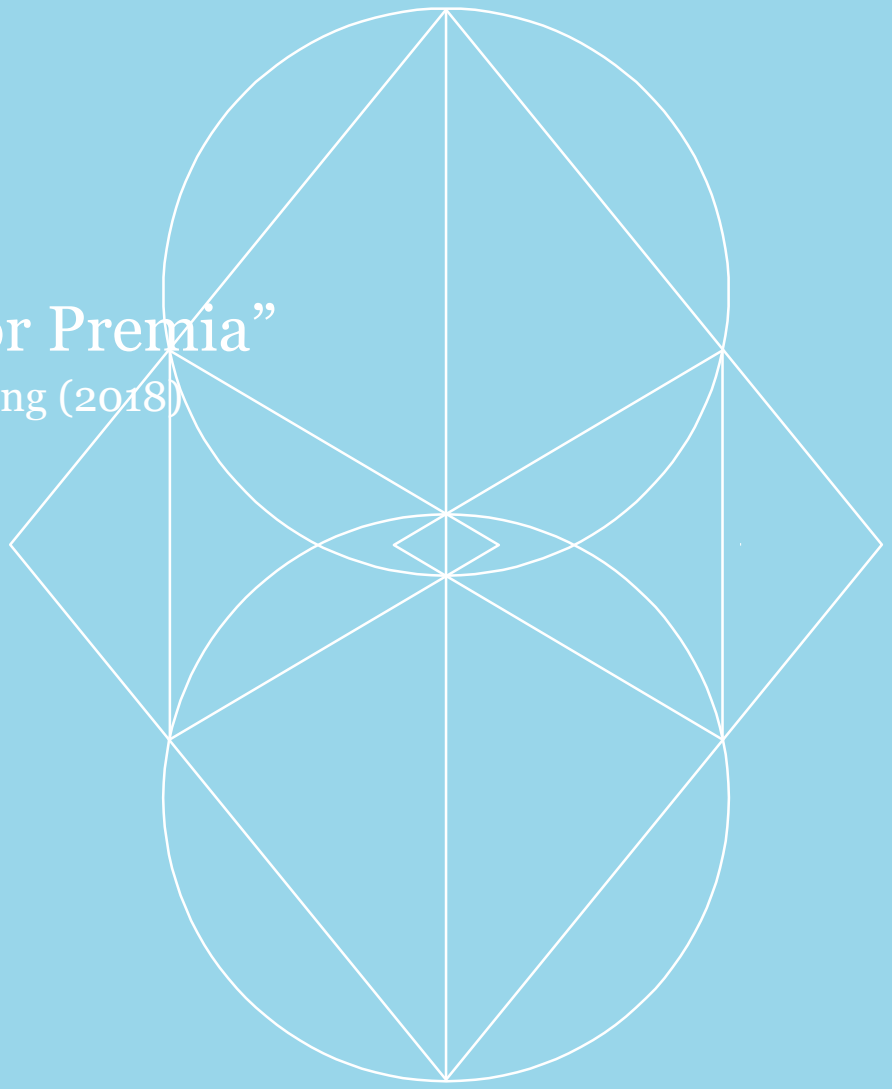
Tobias Moskowitz, Ph.D
Dean Takahashi Professor of Finance and
Economics, Yale University,
Principal, AQR,
Research Associate, NBER

March 2018



Introduction

Based off of “A Century of Factor Premia”
by Ilmanen, Israel, Moskowitz, Thapar, and Wang (2018)

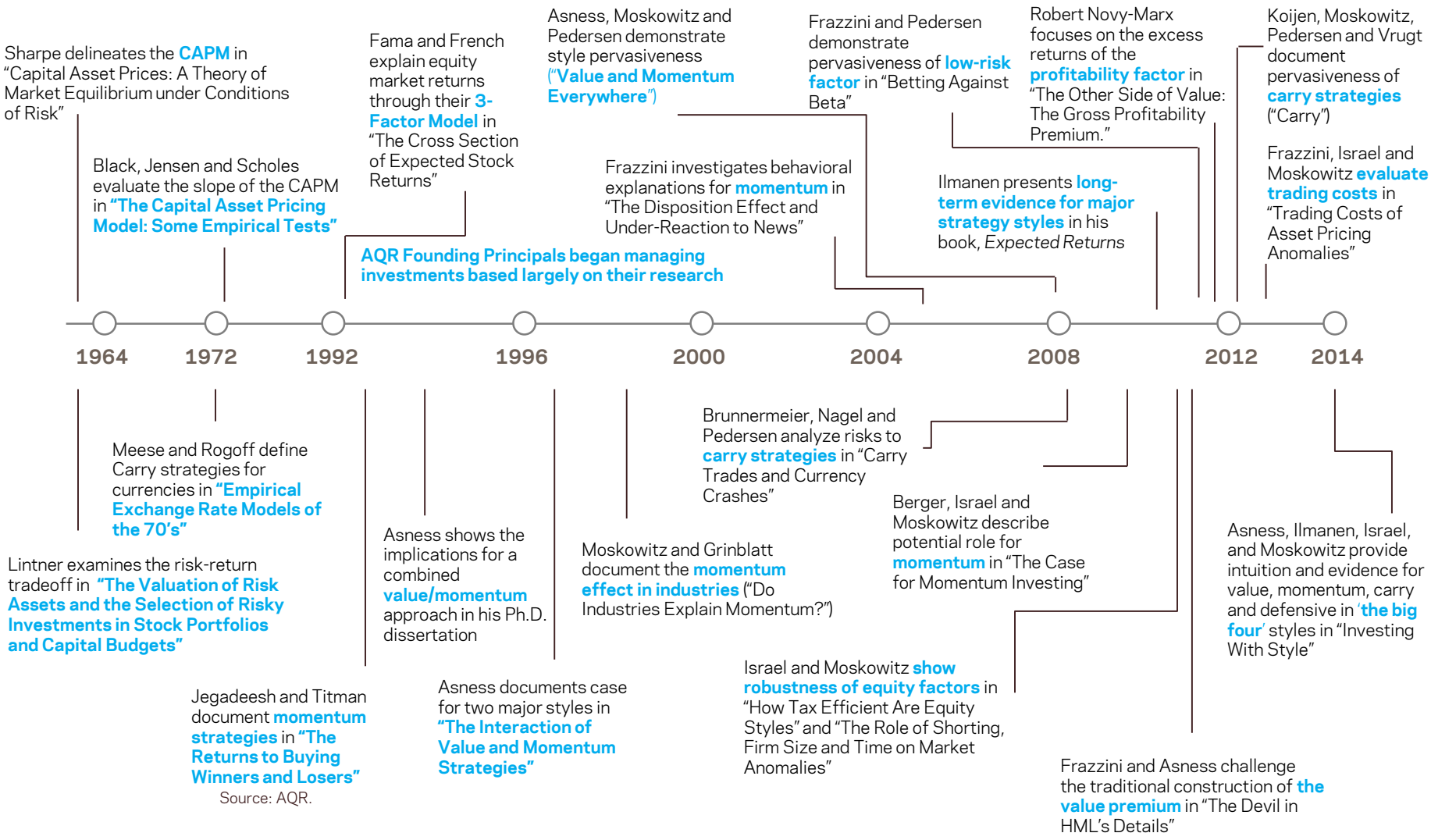


What Are Style Premia?

We Focus On Four Intuitive and Well-Researched Styles

Value	**The tendency for relatively cheap assets to outperform relatively expensive ones**
Momentum	**The tendency for an asset's recent relative performance to continue in the near future**
Carry	**The tendency for higher-yielding assets to provide higher returns than lower-yielding assets**
Defensive	**The tendency for lower-risk and higher-quality assets to generate higher risk-adjusted returns**

Significant History of Research on Style Premia



Source: AQR.

So, What Else Could We Possibly Learn About Styles?

Many questions still remain (some more informed than others).

1. **"Are styles just data mined or over-fitted to a specific sample?"**
2. **"If they do exist, are they behavioral? Are they risk-based?"**
3. **"Do style returns depend on macroeconomic conditions?"**
4. **"Can I time the styles?"**
5. **"Has the alpha of these styles decayed over time?"**

Many of these questions simply aren't answerable without a very long data sample…

This Is Precisely Where 100 Years of Data Comes In Handy

1. **"Are styles just data mined or over-fitted to a specific sample?"**
   - Then would expect to see poor out of sample performance

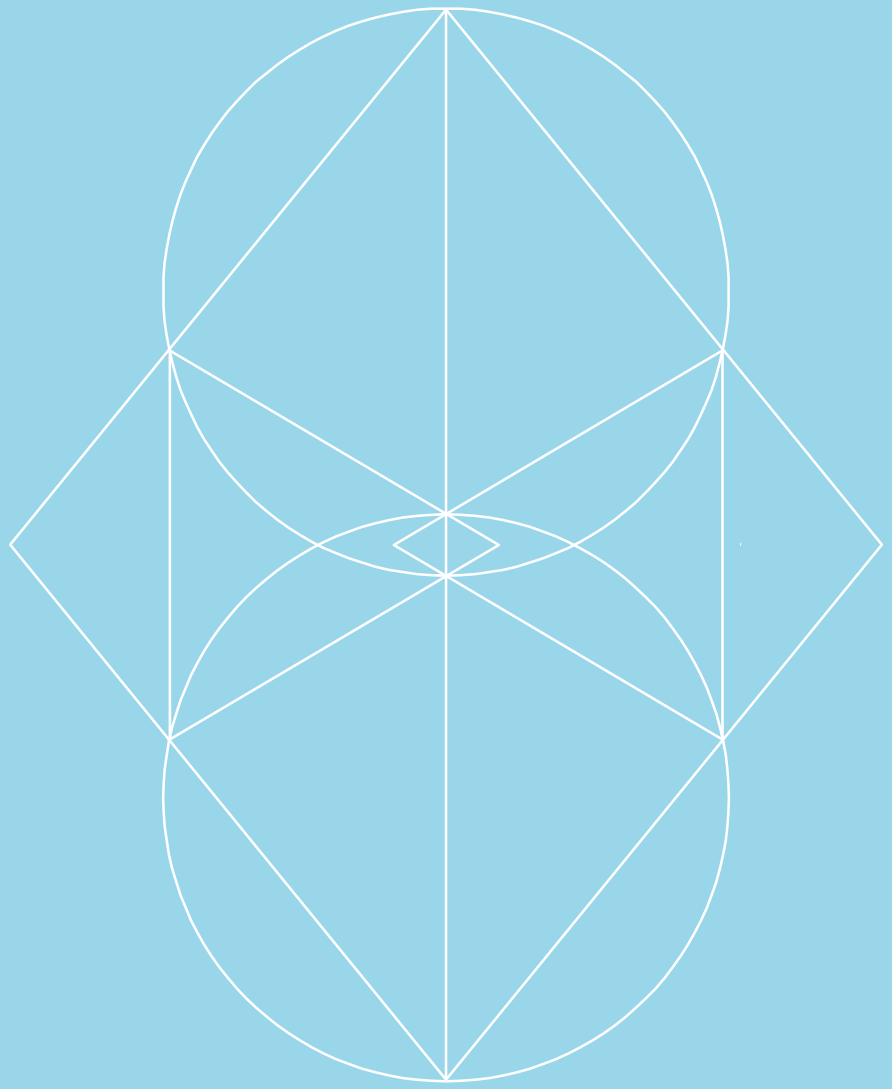
2. **"If they do exist, are they behavioral? Are they risk-based?"**
   - Properties should change during crashes or diminish after discovery

3. **"Do style returns depend on macroeconomic conditions?"**
   - 100 years of macro events should reveal something

4. **"Can I time the styles?"**
   - 100 years to try to time this!

5. **"Has the alpha of these styles decayed over time?"**
   - Some hope of measuring whether alpha has changed over time

A Century of Data



A Century's Worth of Style Data

Using the Following Asset Class Data

Asset Class Definitions

Equity Indices	43 equity markets
Fixed Income	10 year government bonds from 26 countries
U.S. Stocks	All U.S. stocks
Commodities	Futures prices of 40 commodities
Currencies	Forward exchange rates for 20 developed markets

A Century's Worth of Style Data

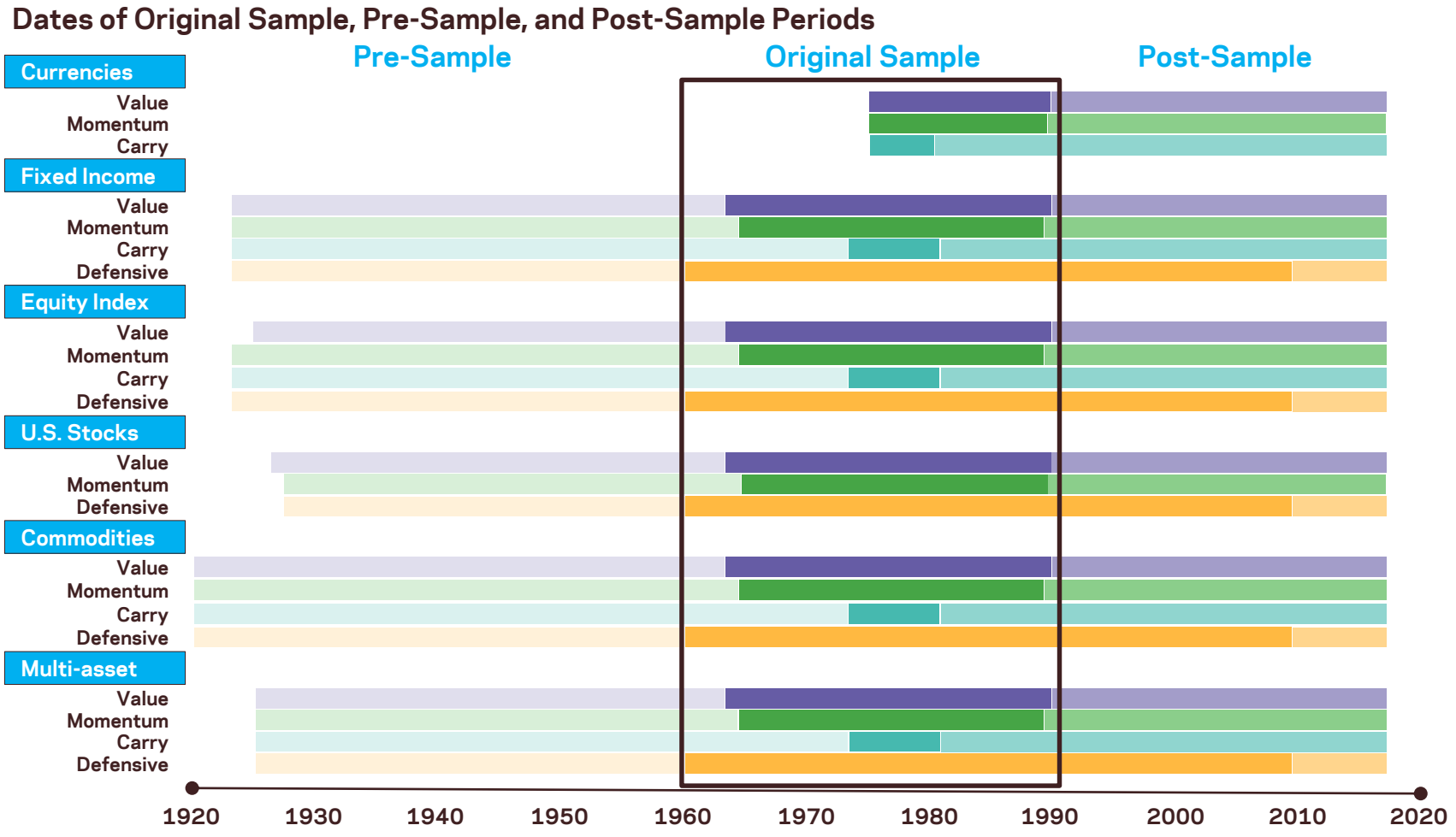
And Four Intuitive Styles

Style Premia Definitions Per Asset Class

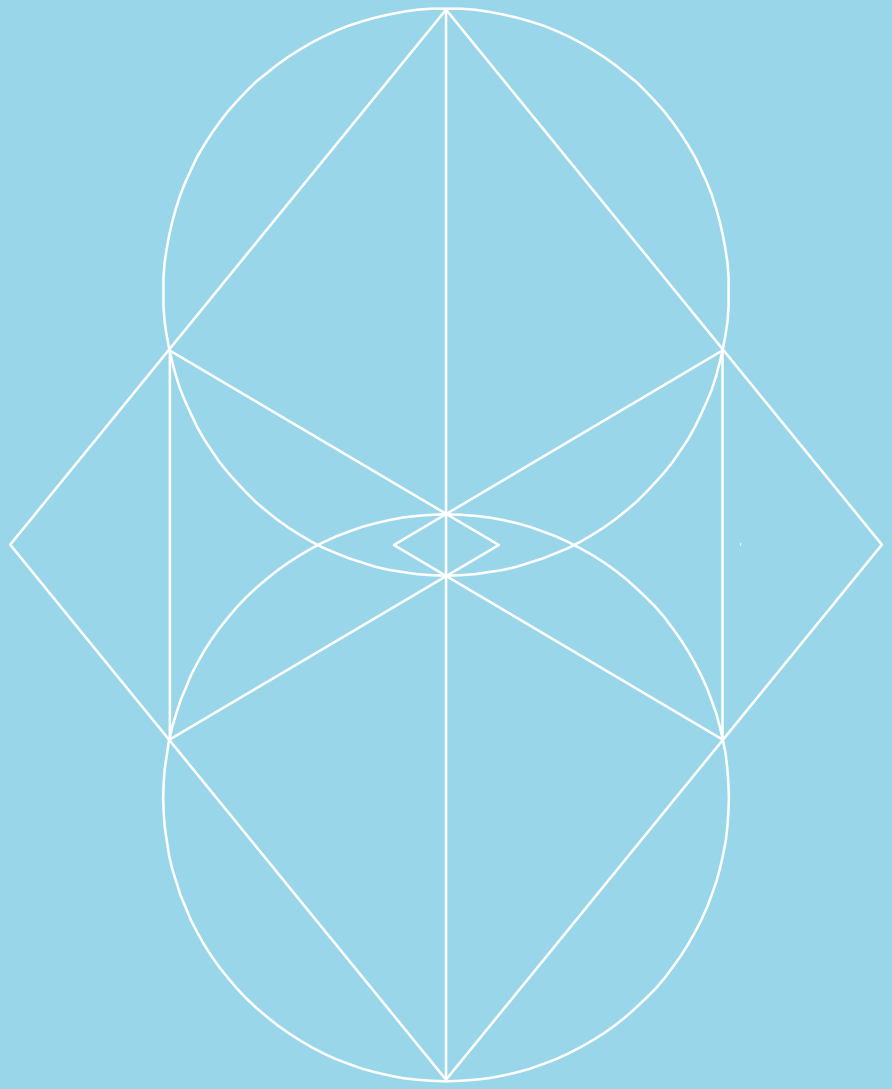
	Equity Indices	Global Bonds	U.S. Stocks	Commodities	Currencies
Value	CAPE	Real Bond Yield	B/P	5 Year Reversal	PPP
Momentum	Past 12 Month Price Return (excluding Most Recent Month)				
Carry	D/P	Term Premium	—	Futures Curve Rolldown	Short Term Interest Rate
Defensive	Beta				—

A Century's Worth of Style Data

Out of Sample Evidence Both Before and After the Original Sample



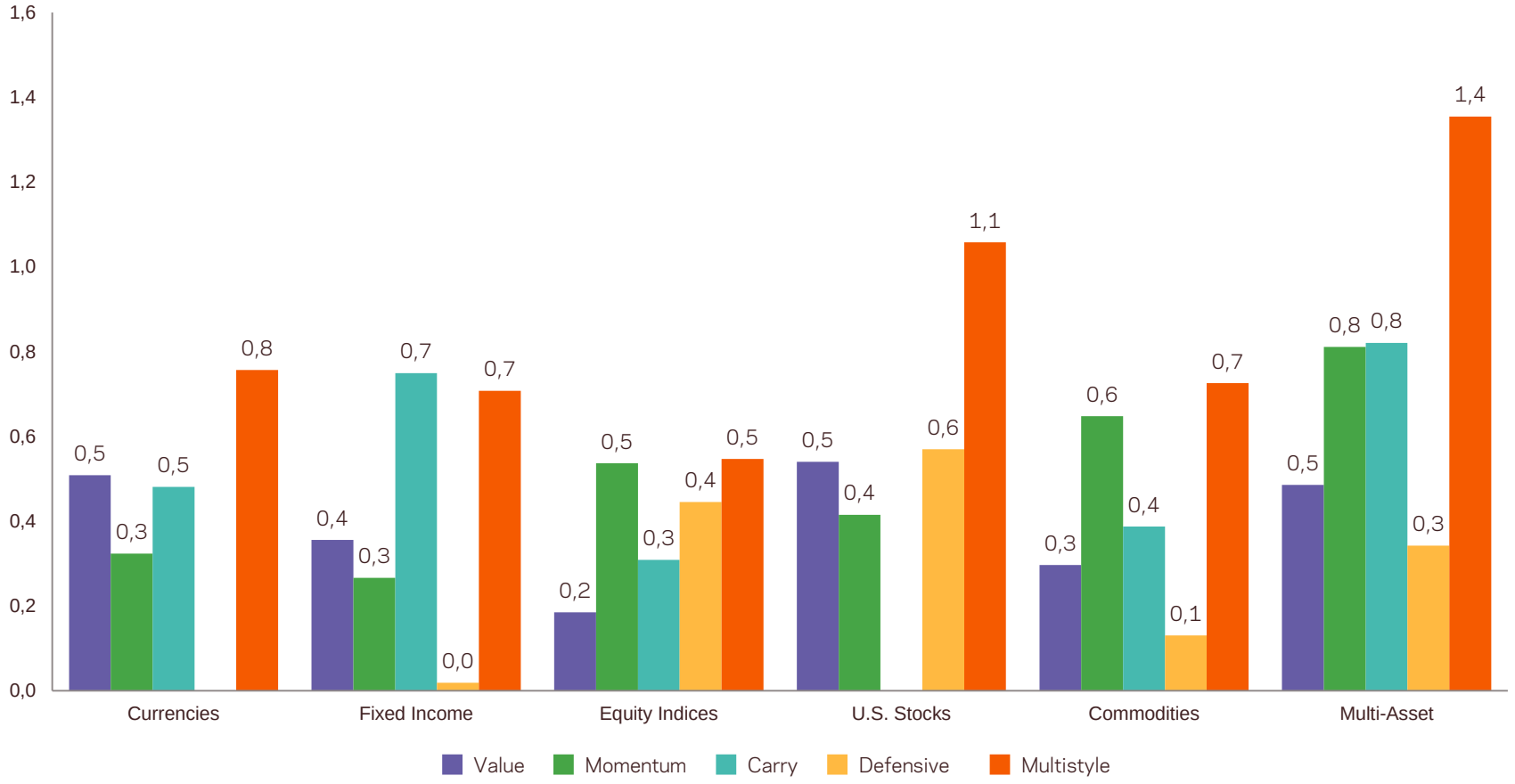
Results



Let's Consider the Full 100 Year Period

All Styles Have Positive and High Sharpe Ratios Over This Period

Full Sample Sharpe Ratios Across Styles and Asset Classes

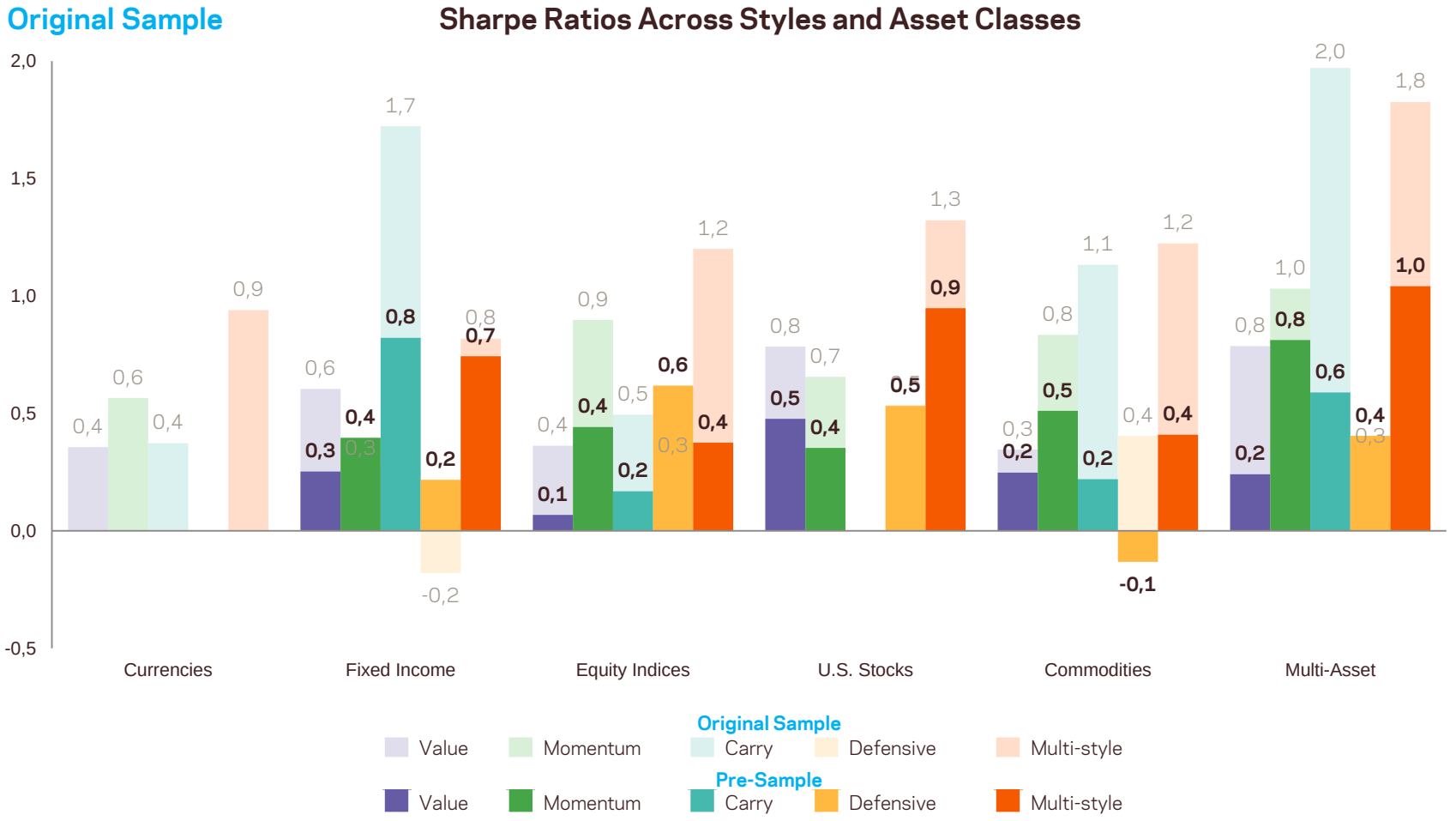


Source: AQR.

How Does the OoS Performance Stack Up to the Original?

Positive and High Sharpe Ratios Pre-Discovery

Original Sample



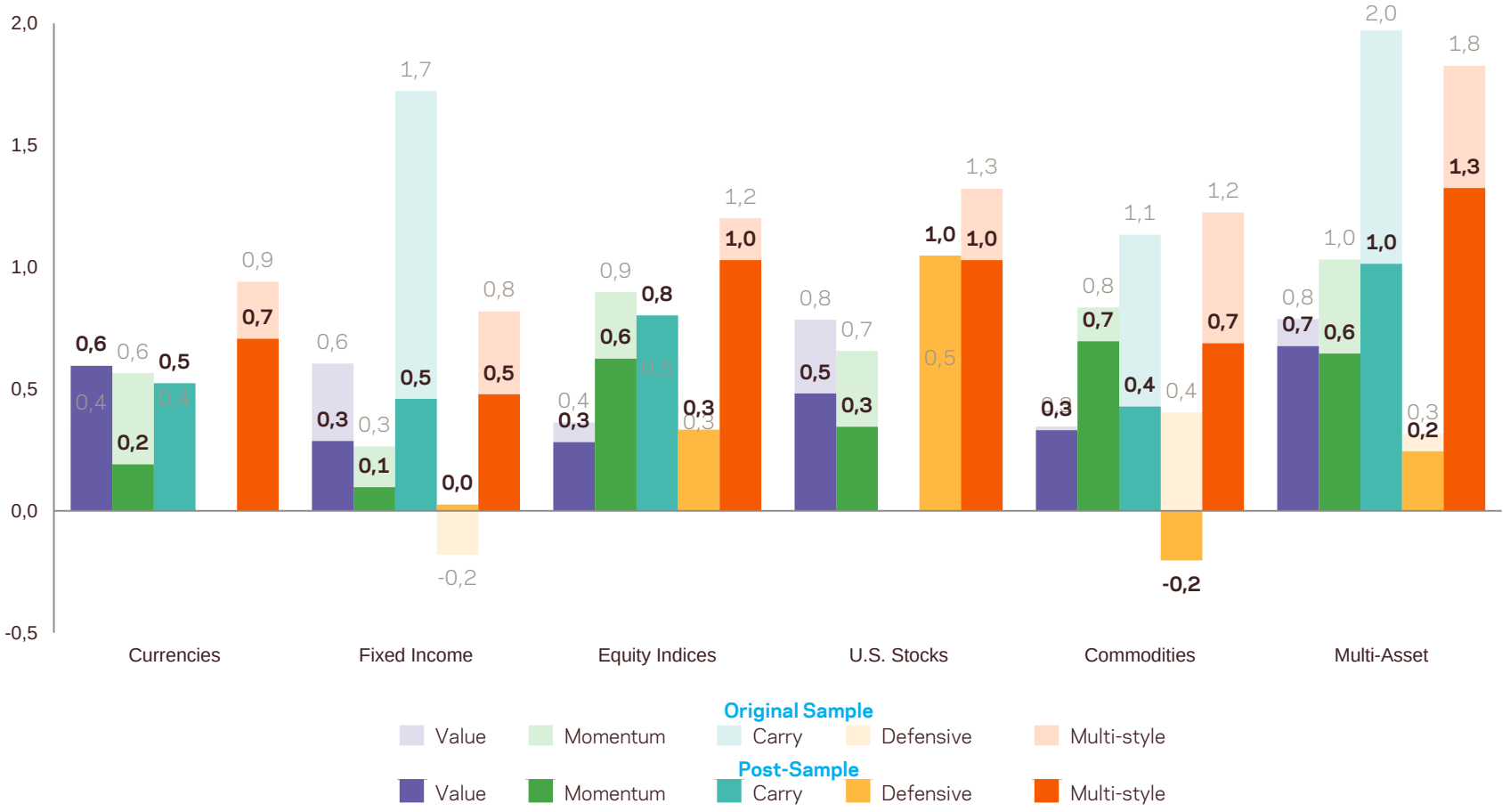
Source: AQR.

How Does the OoS Performance Stack Up to the Original?

Sharpe Ratios Remain Positive and High Post-Discovery

Original Sample

Sharpe Ratios Across Styles and Asset Classes

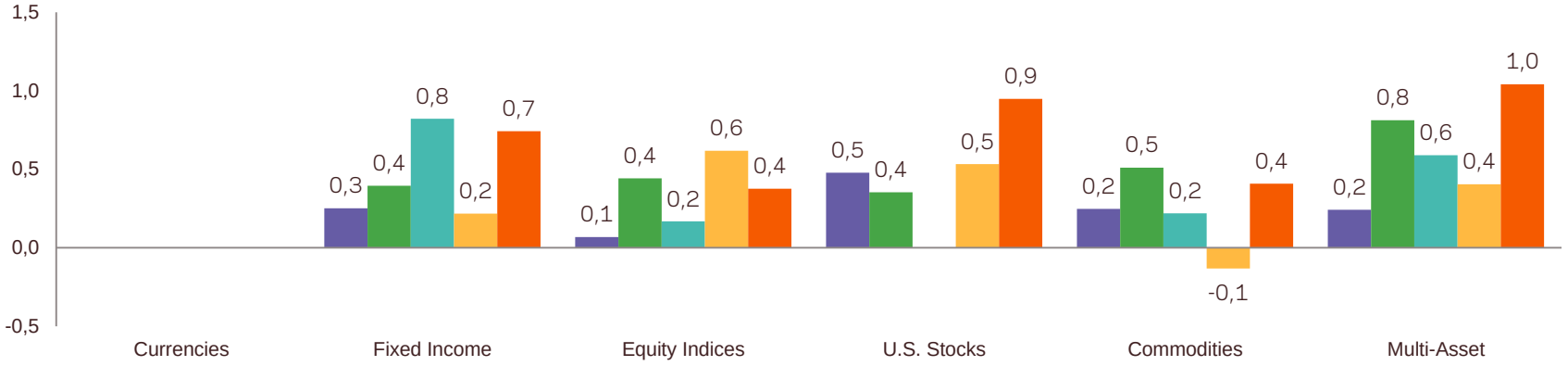


Source: AQR.

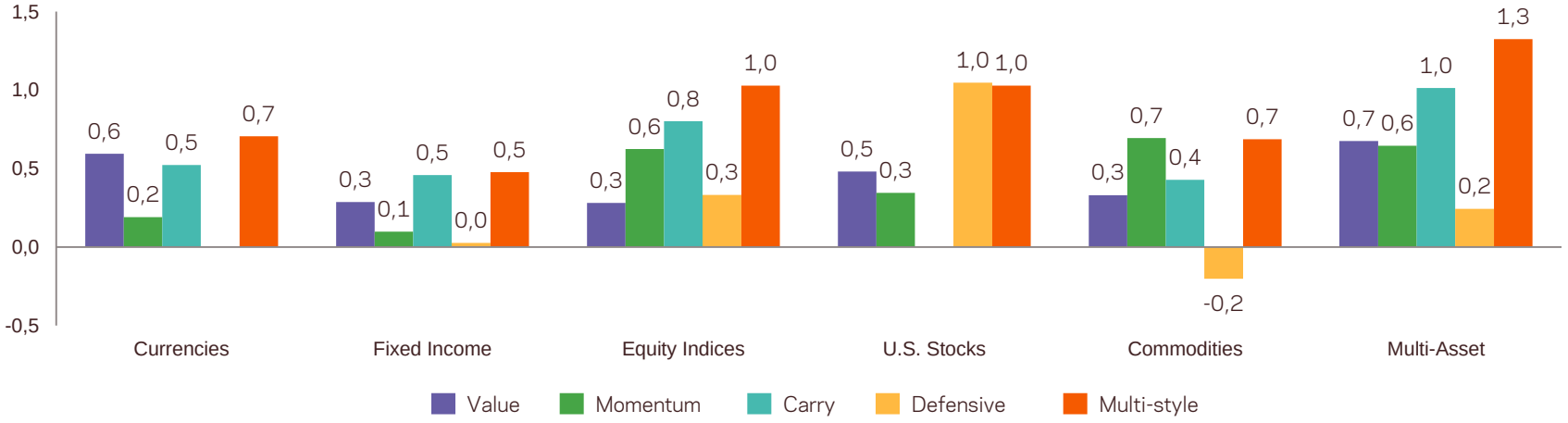
Strong Out of Sample Evidence As Well

Both in the Pre- and Post-Sample periods

Pre-Sample Sharpe Ratios Across Styles and Asset Classes

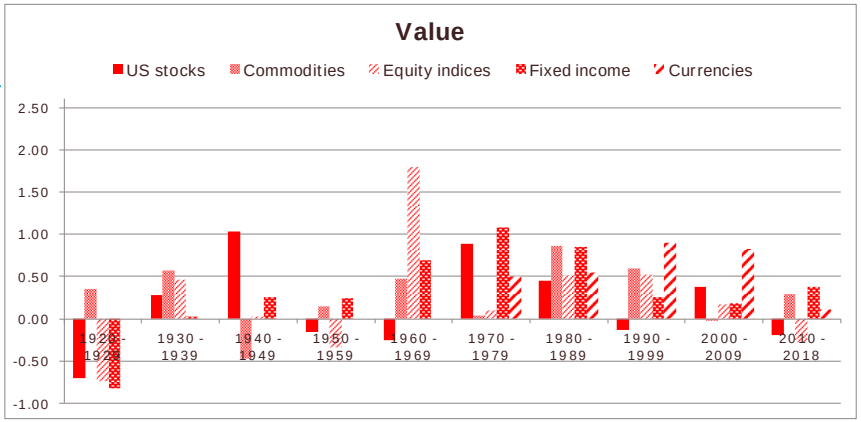


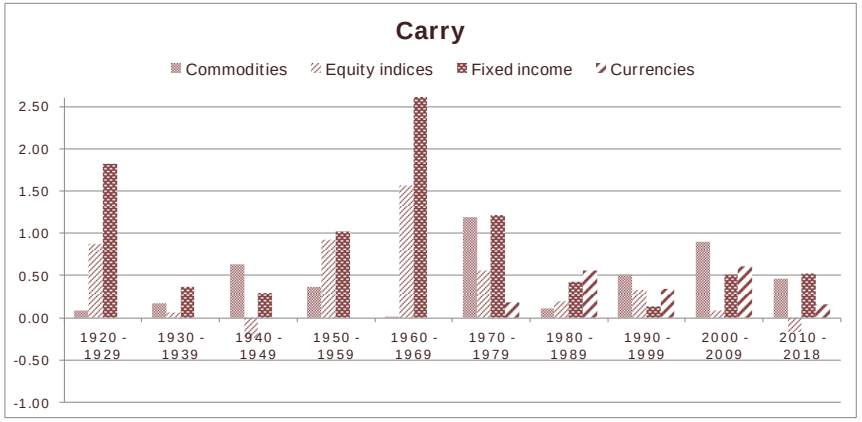
Post-Sample Sharpe Ratios Across Styles and Asset Classes

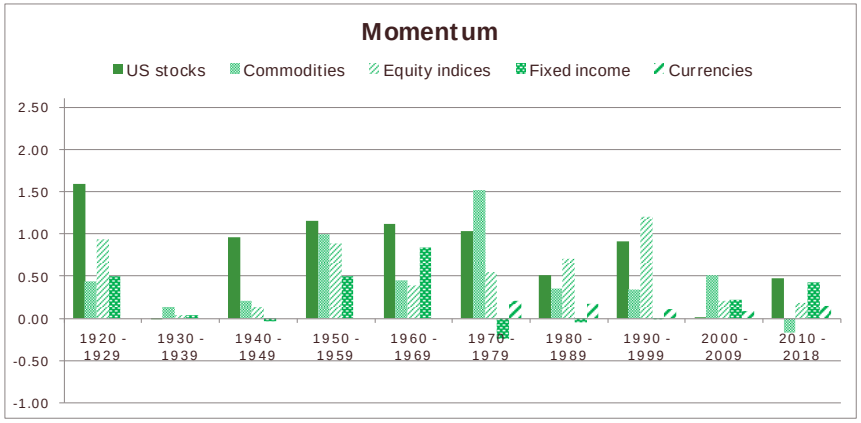


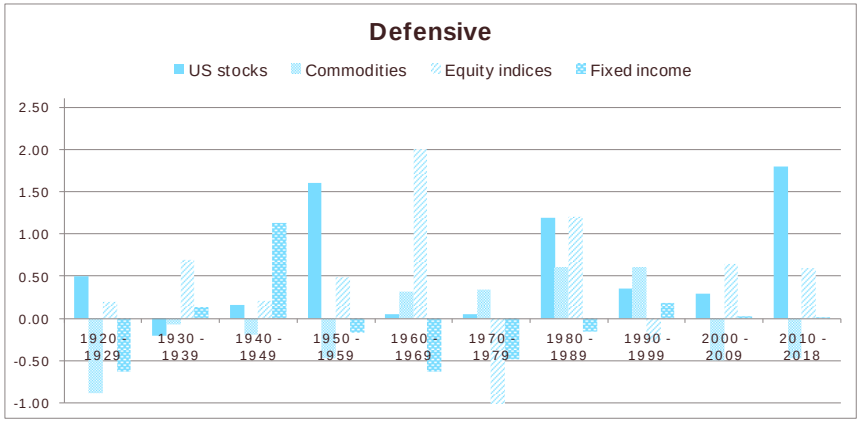
Source: AQR.

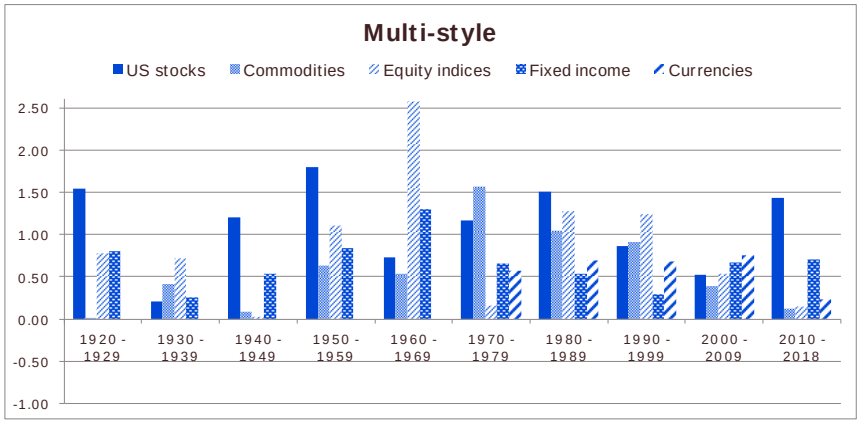
By Decade

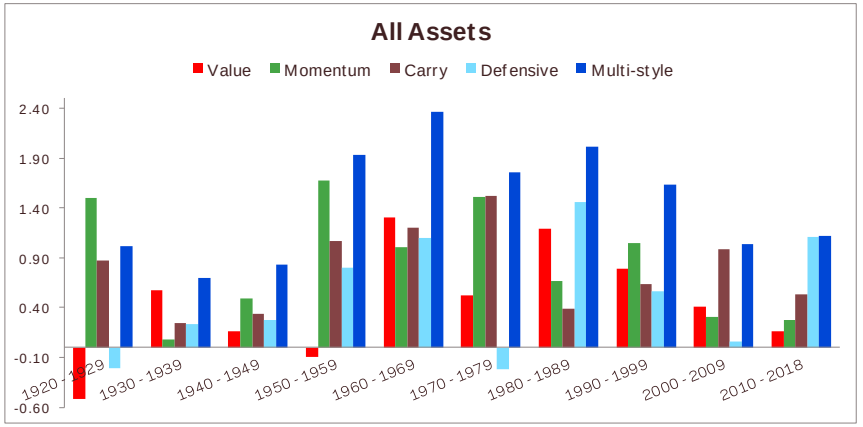

Value
US stocks
Commodities
Equity indices
Fixed income
Currencies
2.50
2.00
1.50
1.00
0.50
0.00
-0.50
-1.00
1920 - 1929
1930 - 1939
1940 - 1949
1950 - 1959
1960 - 1969
1970 - 1979
1980 - 1989
1990 - 1999
2000 - 2009
2010 - 2018


Carry
Commodities
Equity indices
Fixed income
Currencies
2.50
2.00
1.50
1.00
0.50
0.00
-0.50
-1.00
1920 - 1929
1930 - 1939
1940 - 1949
1950 - 1959
1960 - 1969
1970 - 1979
1980 - 1989
1990 - 1999
2000 - 2009
2010 - 2018


Momentum
US stocks
Commodities
Equity indices
Fixed income
Currencies
2.50
2.00
1.50
1.00
0.50
0.00
-0.50
-1.00
1920 - 1929
1930 - 1939
1940 - 1949
1950 - 1959
1960 - 1969
1970 - 1979
1980 - 1989
1990 - 1999
2000 - 2009
2010 - 2018


Defensive
US stocks
Commodities
Equity indices
Fixed income
2.50
2.00
1.50
1.00
0.50
0.00
-0.50
-1.00
1920 - 1929
1930 - 1939
1940 - 1949
1950 - 1959
1960 - 1969
1970 - 1979
1980 - 1989
1990 - 1999
2000 - 2009
2010 - 2018


Multi-style
US stocks
Commodities
Equity indices
Fixed income
Currencies
2.50
2.00
1.50
1.00
0.50
0.00
-0.50
-1.00
1920 - 1929
1930 - 1939
1940 - 1949
1950 - 1959
1960 - 1969
1970 - 1979
1980 - 1989
1990 - 1999
2000 - 2009
2010 - 2018


All Assets
Value
Momentum
Carry
Defensive
Multi-style
2.40
1.90
1.40
0.90
0.40
-0.10
-0.60
1920 - 1929
1930 - 1939
1940 - 1949
1950 - 1959
1960 - 1969
1970 - 1979
1980 - 1989
1990 - 1999
2000 - 2009
2010 - 2018

Correlations Over the Full Sample

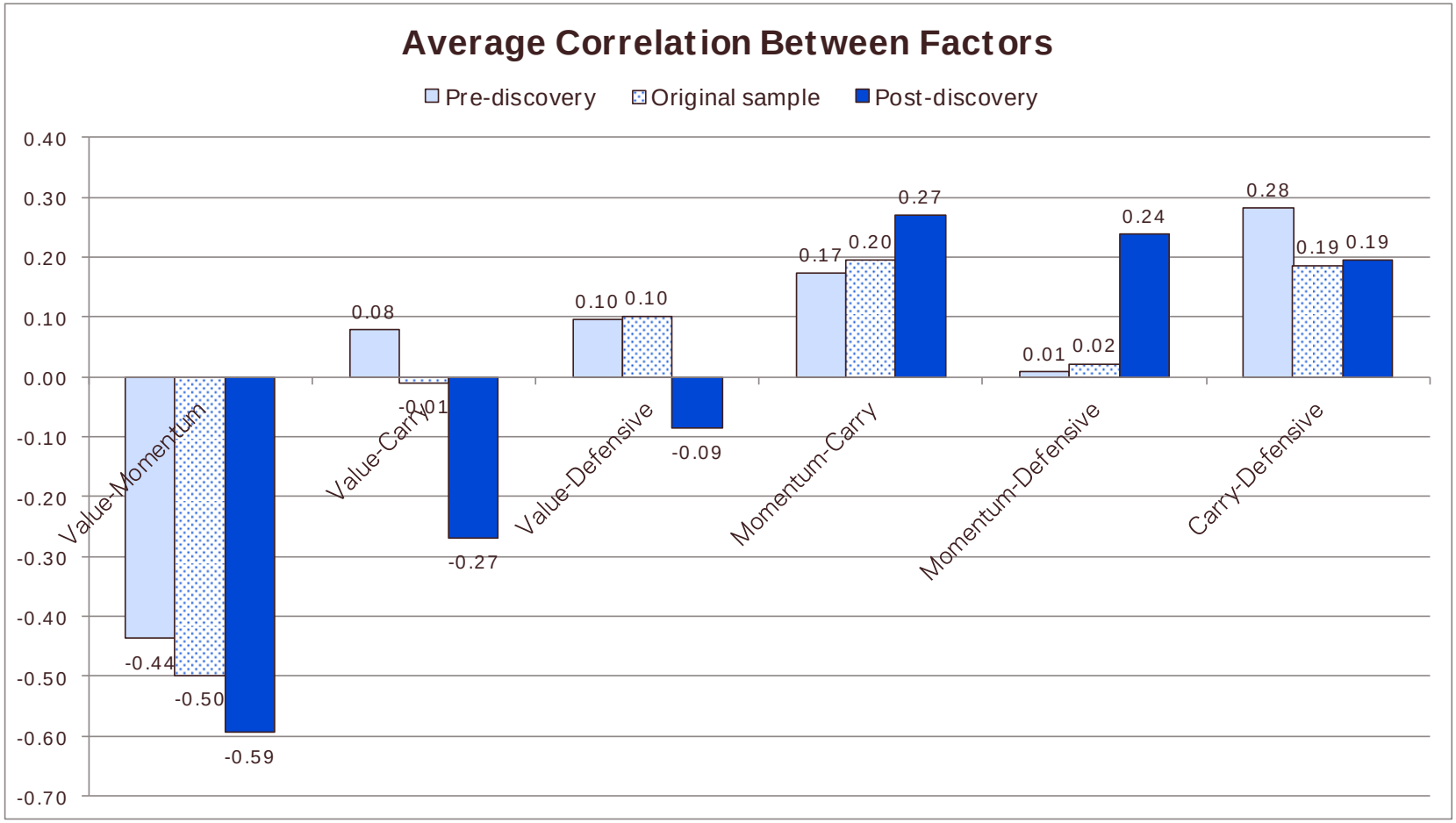
Correlations Across Styles

	Value	Momentum	Carry	Defensive
Panel A: US Stocks				
Value	1	-0.56		0.09
Momentum		1		-0.08
Carry				
Defensive				1
Panel B: Equity Indices				
Value	1	-0.36	0.21	0.05
Momentum		1	0.07	0.20
Carry			1	0.26
Defensive				1
Panel C: Fixed Income				
Value	1	-0.22	0.28	-0.03
Momentum		1	0.11	0.06
Carry			1	0.04
Defensive				1
Panel D: Currencies				
Value	1	-0.24	0.25	
Momentum		1	0.18	
Carry			1	
Defensive				
Panel E: Commodities				
Value	1	-0.45	-0.32	0.13
Momentum		1	0.43	0.00
Carry			1	0.03
Defensive				1
Panel F: All Assets				
Value	1	-0.50	-0.01	0.10
Momentum		1	0.20	0.02
Carry			1	0.19
Defensive				1

Source: AQR.

Do Correlations Change Over Time?

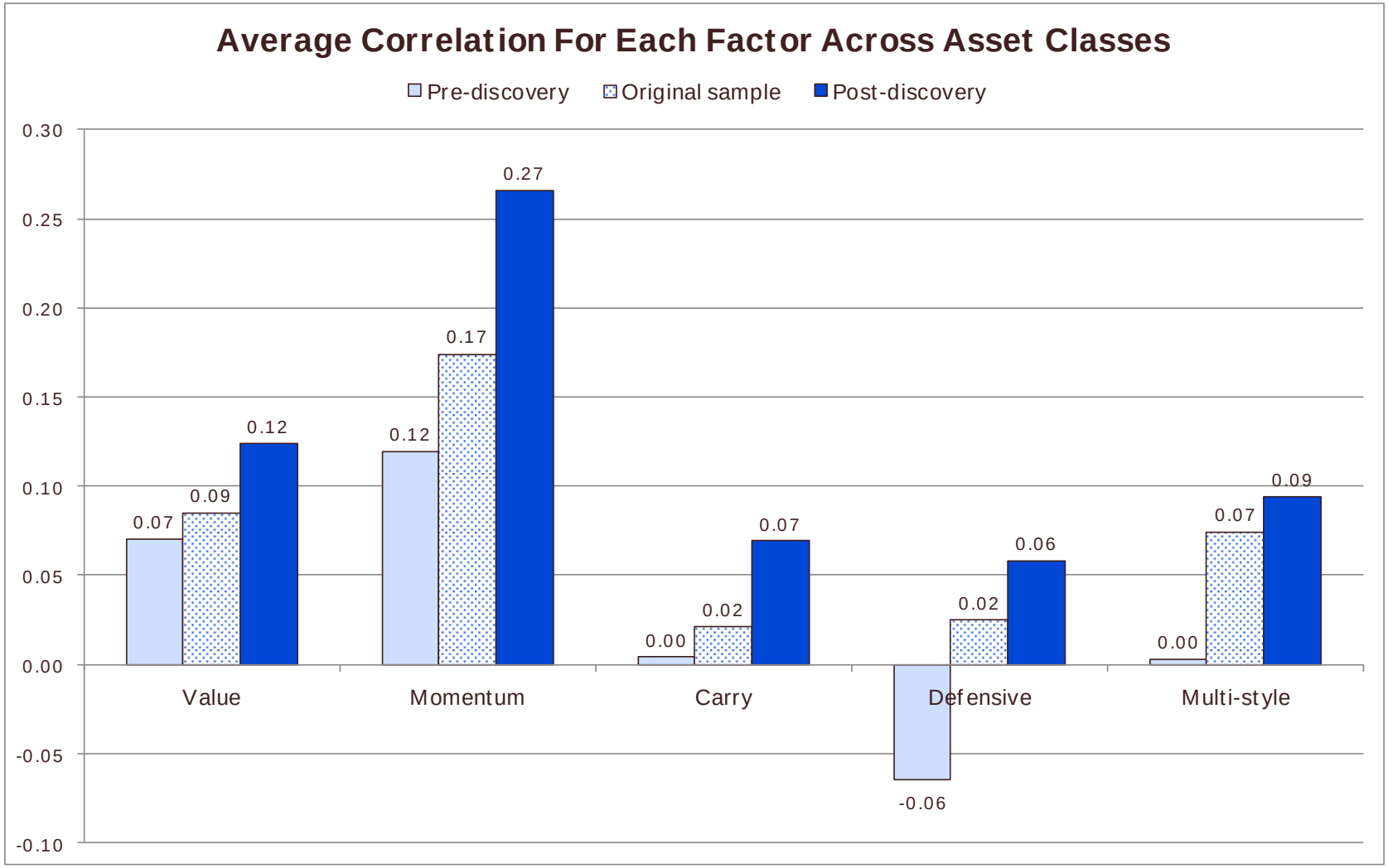
Pre-, Original, and Post-Sample Periods



Source: AQR.

Do Correlations Change Over Time?

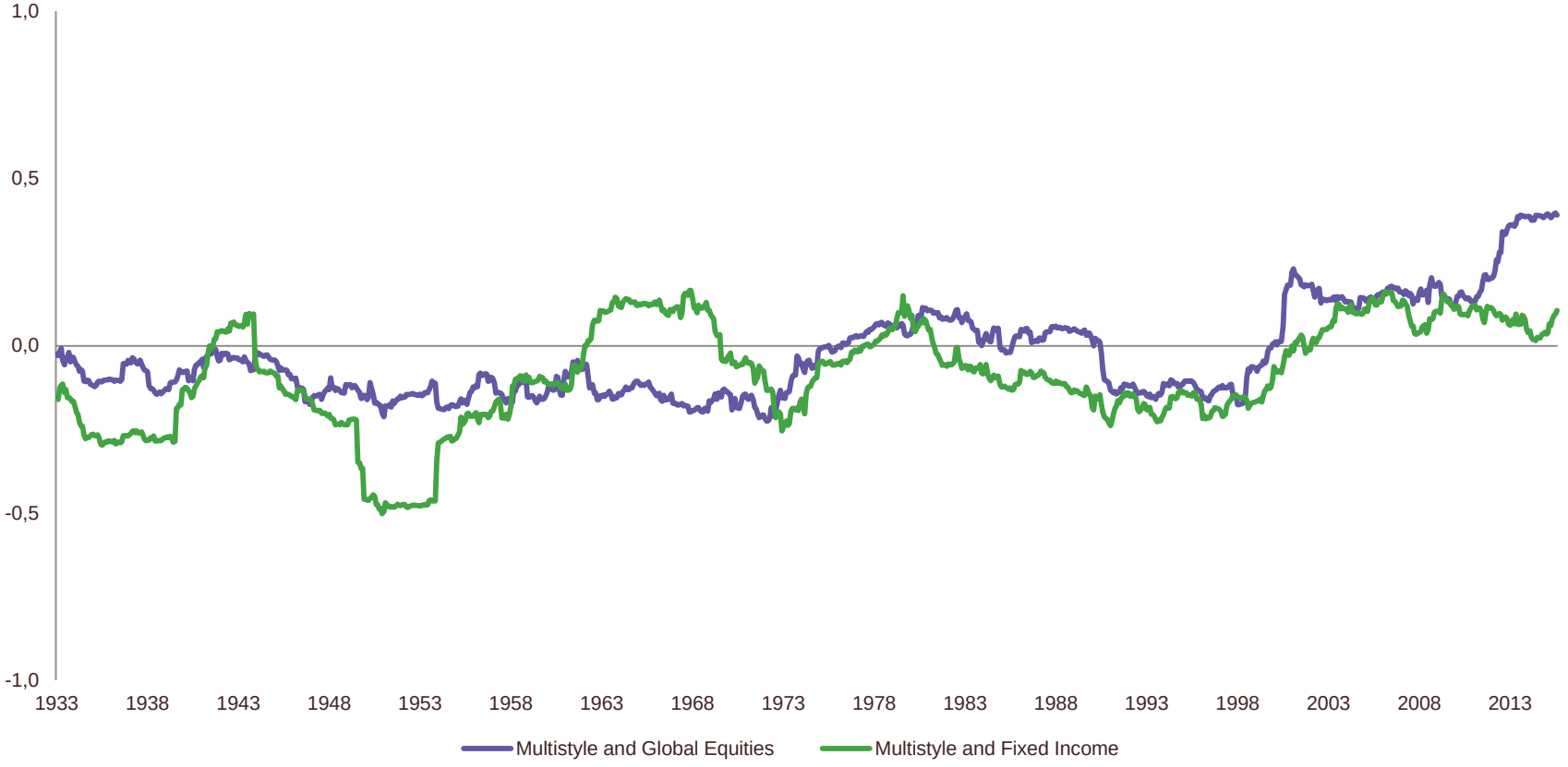
Pre-, Original, and Post-Sample Periods



And Not Just Between Styles

Correlations to Traditional Markets Also Remain Low Through Time

Full Sample Rolling 10 Year Correlation Between Multi-asset Multistyle and Traditional Markets



Source: AQR.

So, We're Pretty Sure Styles Aren't Just Data Mined

But What About All Those Other Questions?

Recall:

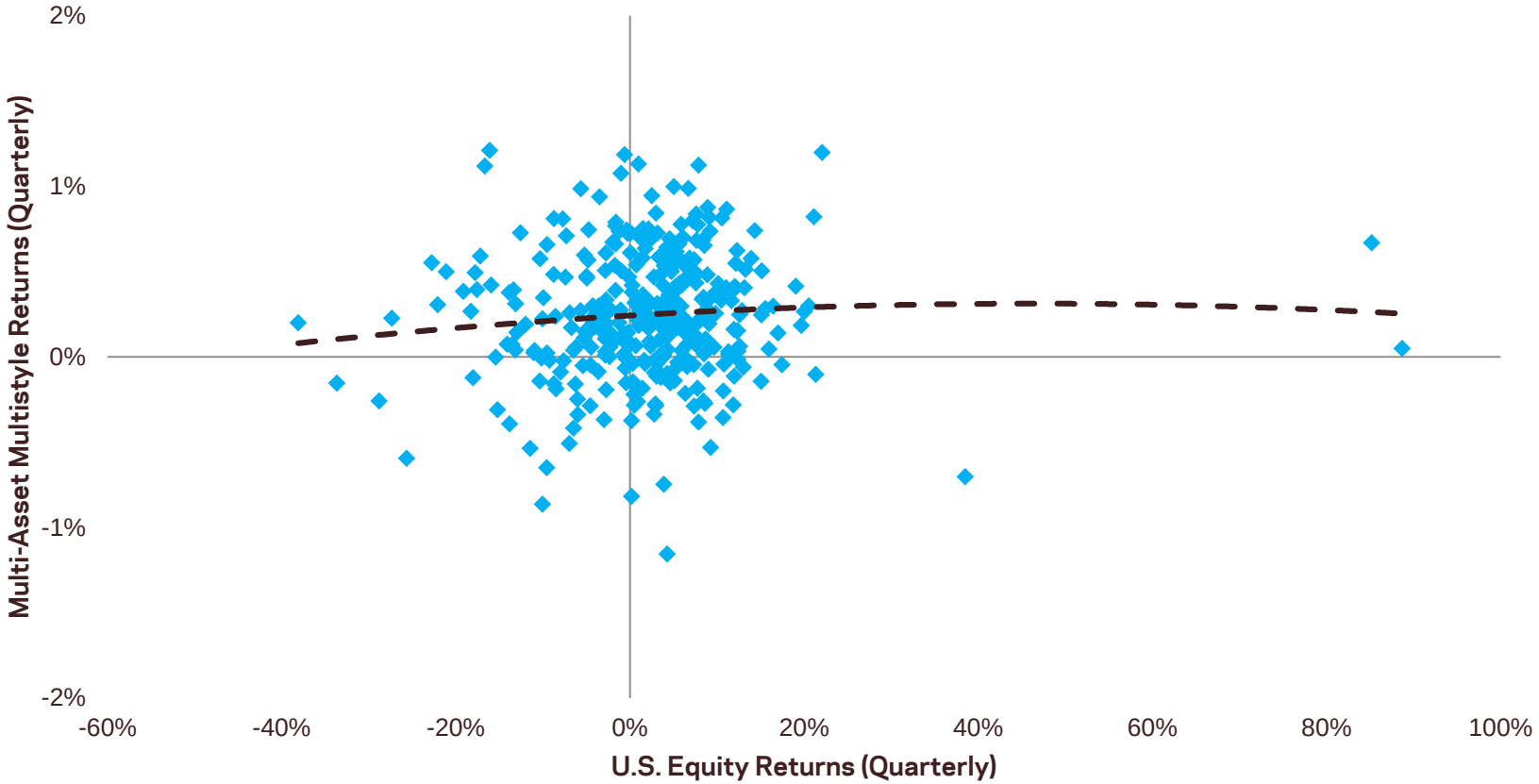
1. ~~"Are styles just data mined or over-fitted to a specific sample?"~~
2. **"If they do exist, are they behavioral? Are they risk-based?"**
3. **"Do style returns depend on macroeconomic conditions?"**
4. **"Can I time the styles?"**
5. **"Has the alpha of these styles decayed over time?"**

Let's dive a bit deeper and explore some of these other questions relating to macroeconomic dependencies, timing, alpha decay, etc.

How Do Styles Behave During Crises?

Styles Perform Equally Well in Bull and Bear Markets

Full Sample U.S. Equity Returns versus Multi-Asset Multistyle Returns



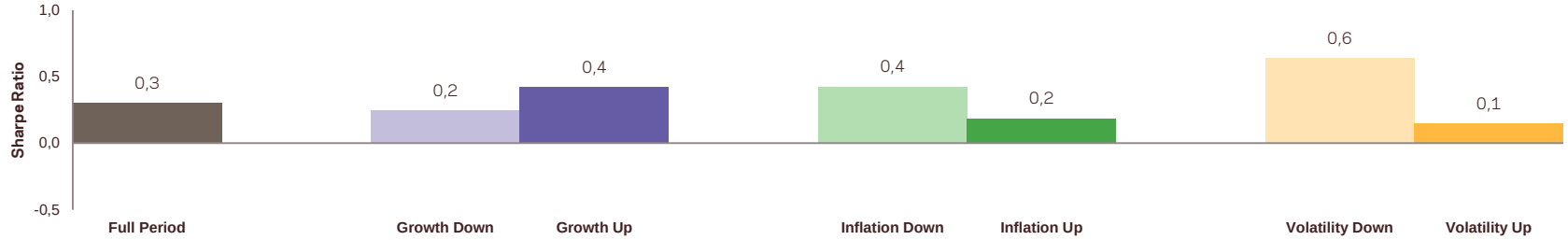
Source: AQR.

Are the Styles Sensitive to Macroeconomic Conditions?

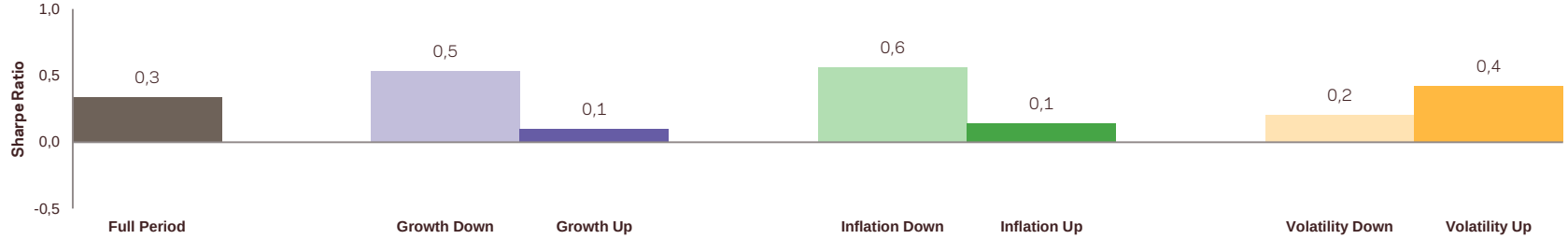
Sharpe Ratios Similar in Both "Up" and "Down" Macro Regimes

Full Sample Sharpe Ratios in Different Macroeconomic Environments

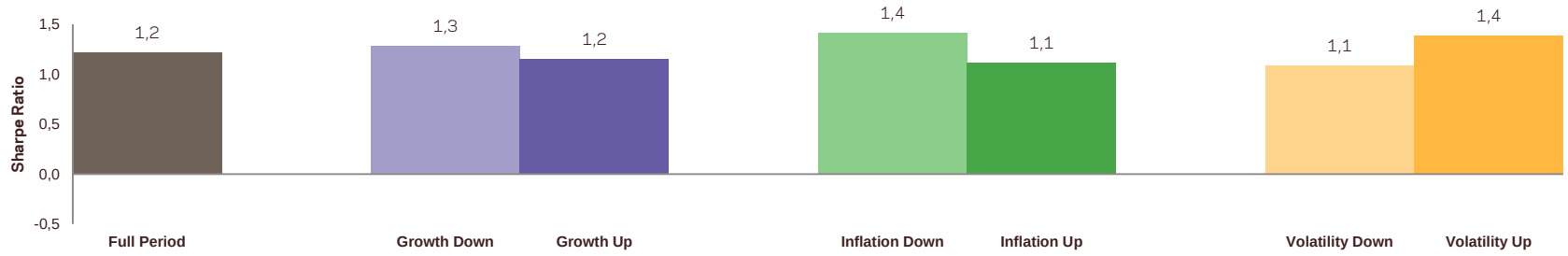
Global Equities



Fixed Income



Multi-asset Multistyle

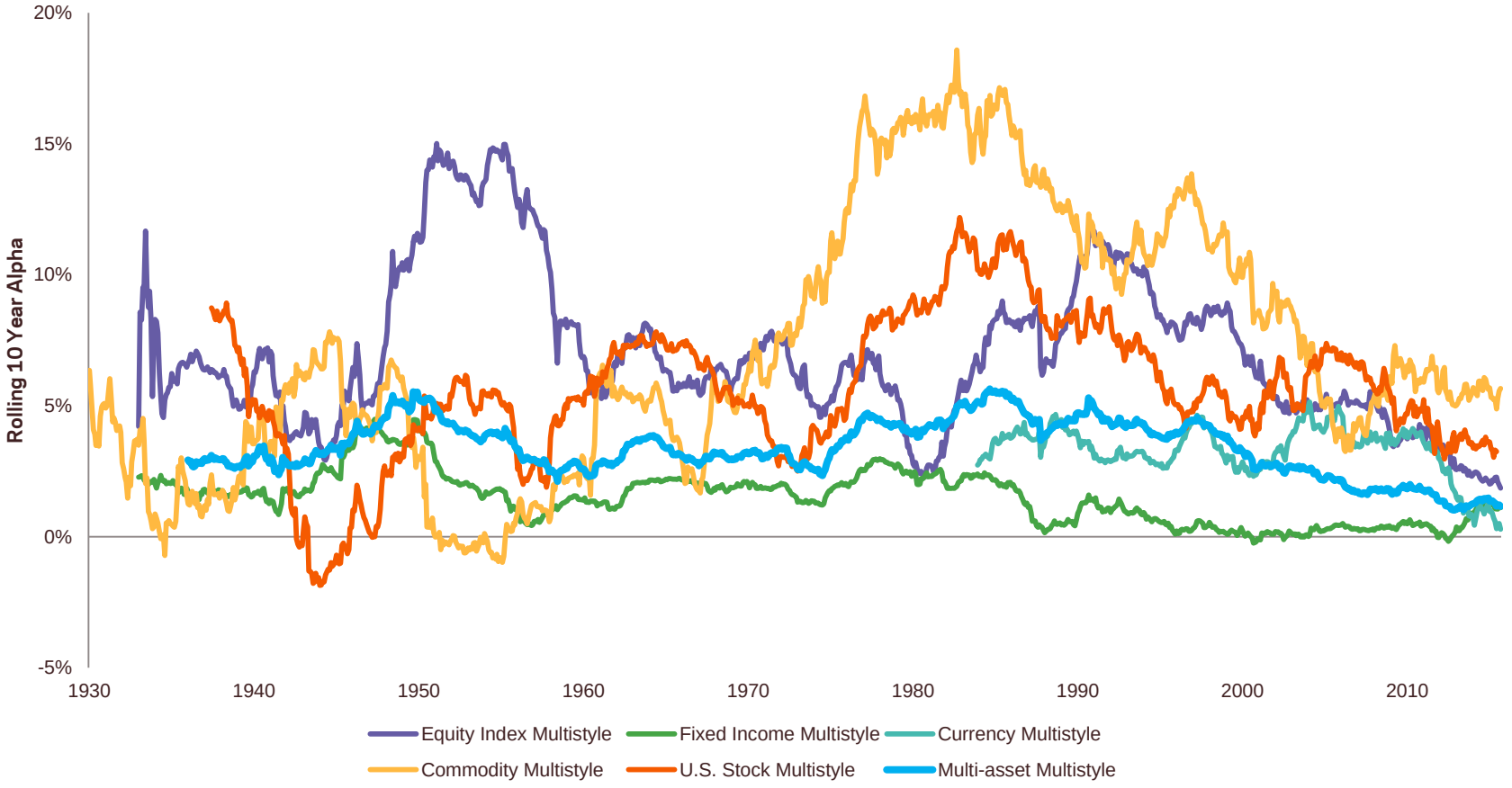


Source: AQR.

What About "Alpha Decay"?

Alpha Has Been Consistently Positive Through Time

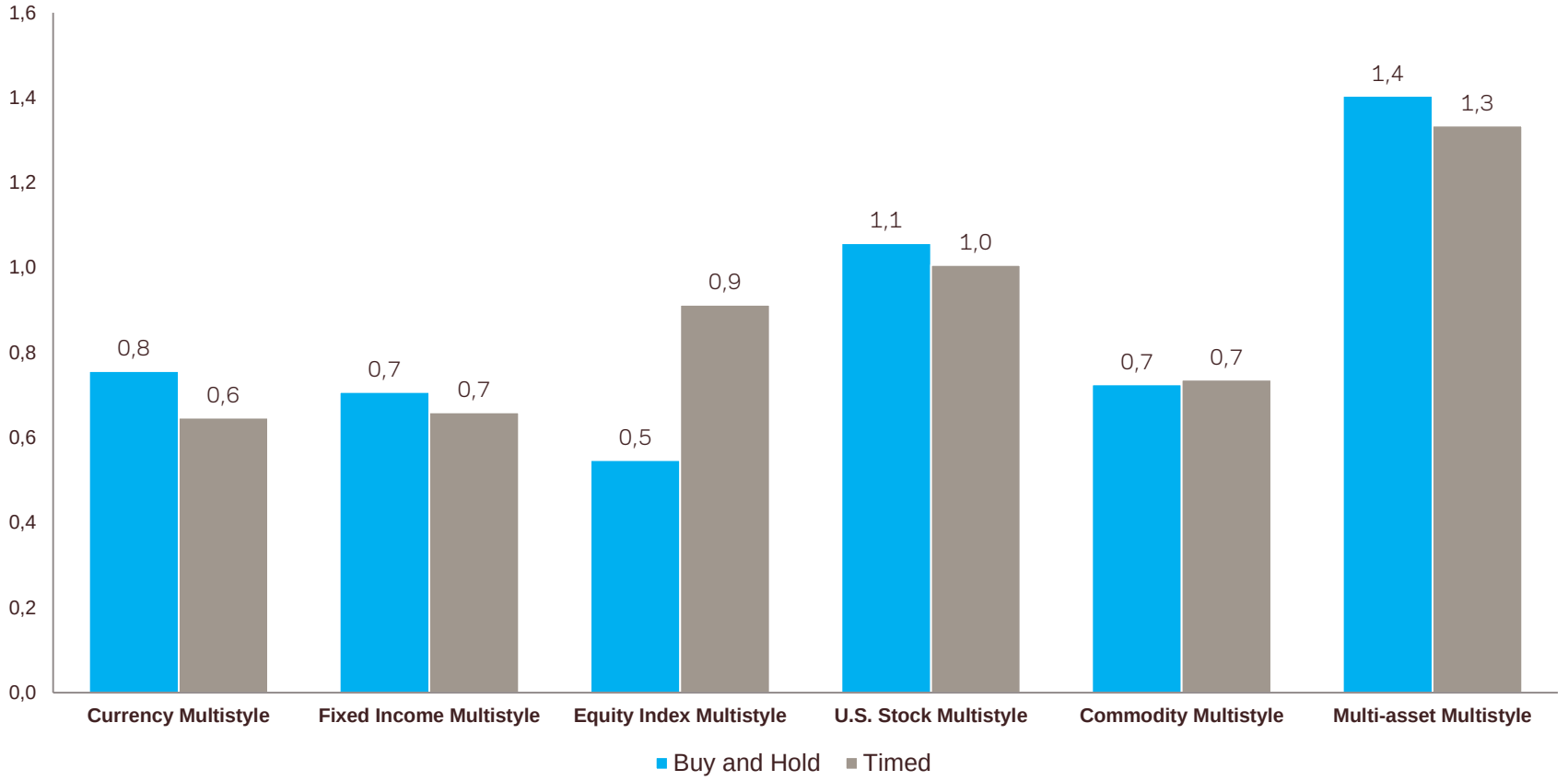
Full Sample Rolling 10 Year Alpha of Multistyle Portfolios to Global Equities and Fixed Income



Source: AQR.

Can I Get Even More Outperformance Through Timing?

Full Sample Sharpe Ratios of Buy and Hold versus Timed Backtest by Asset Class



Source: AQR.

Did We Learn Anything New?
We Think So…

There will always be naysayers, but with over a century of evidence…

1. **"Are styles just data mined or over-fitted to a specific sample?"**
   - **Definitely not data-mined**

2. **"If they do exist, are they behavioral? Are they risk-based?"**
   - **Some combination of risk-based and behavioral explanations**

3. **"Do style returns depend on macroeconomic conditions?"**
   - **A century of diverse macroeconomic conditions suggests no significant relationship.**

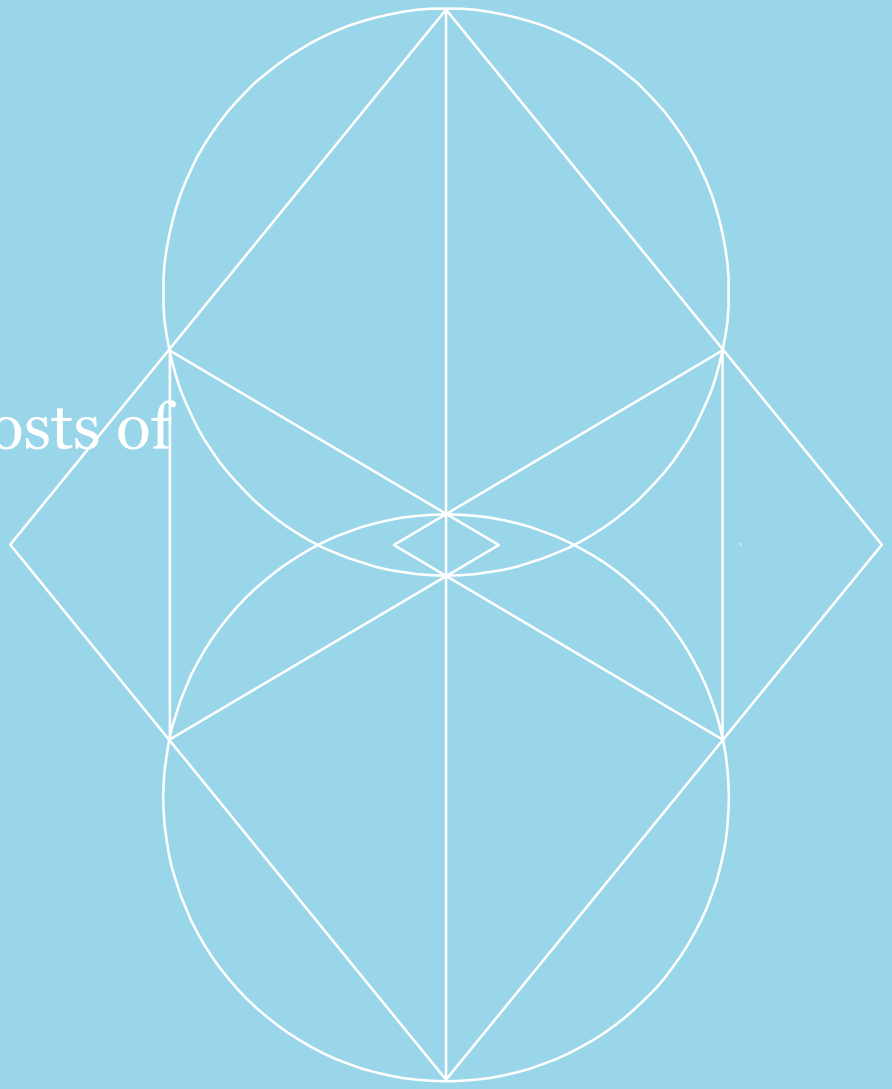
4. **"Can I time the styles?"**
   - **Even with 100 years of hindsight, the results are underwhelming.**

5. **"Has the alpha of these styles decayed over time?"**
   - **Maybe, but multi-asset multistyle's alpha remains consistent and positive**

Source: AQR.

Implementation Costs

Based off of two papers:
"Trading Costs" and "Trading Costs of Asset Pricing Anomalies"
by Frazzini, Israel, and Moskowitz (2015, 2018)



Motivation

Cross-section of expected returns typically analyzed gross of transactions costs

Questions regarding market efficiency should be net of transactions costs

- Are profits within trading costs?

Research Questions:

- How robust are anomalies in the literature after realistic trading costs?
- At what size do trading costs start to constrain arbitrage capital?
- What happens if we take transactions costs into account ex ante?
  - Tradeoff between expected returns and trading costs varies across anomalies

Objectives

Use real-world tcosts of a large trader/arbitrageur

Understand the cross-section of *net* returns on anomalies

Model of trading costs for descriptive and prescriptive purposes

Constructing optimized portfolios

What We Do

Take all (longer-term) equity orders and executions from AQR Capital

- 1998 to 2016, $1.7 trillion worth of trades, traded using automated algorithms
- U.S. (NYSE and NASDAQ) and 20 international markets—
- *Exclude "high frequency" (intra-day) trades

Use *actual* trade sizes and prices to calculate

- Price impact and implementation shortfall (e.g., Perold (1988))

More accurate picture of real-world transactions costs and tradeoffs

- Get vastly different measures than the literature
- Actual costs are 1/10 the size of those estimated in the literature

- Why?
  1) *Average* trading cost ≠ cost facing an arbitrageur
  2) Design portfolios that endogenously respond to expected trading costs

Trading Execution Algorithm

***The portfolio generation process is separate from the trading process - algorithms do not make any explicit aggregate buy or sell decisions**

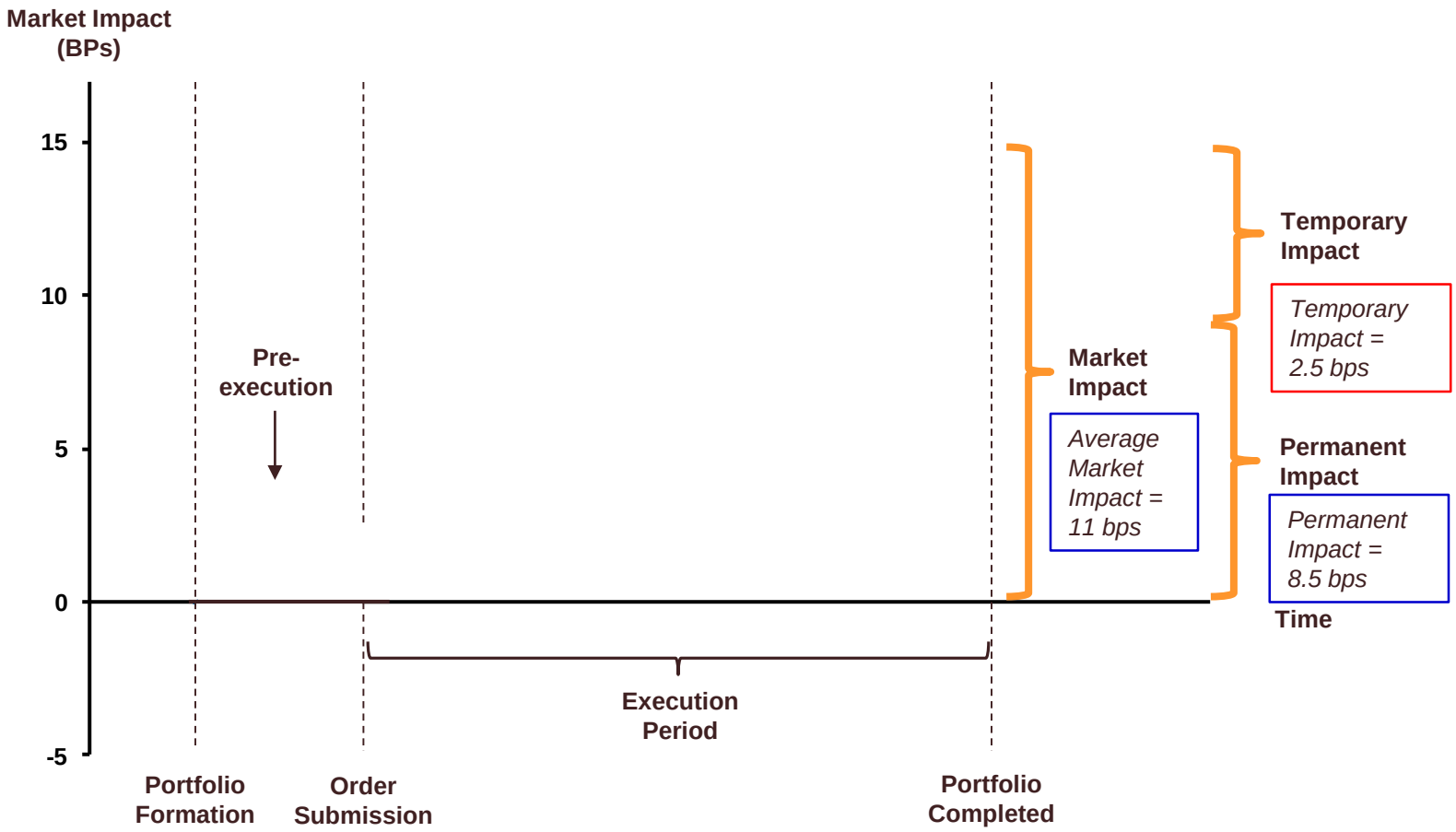
- Merely determine duration of a trade (most within 1 day)

The trades are executed using proprietary, automated trading algorithms designed and built by the "manager" (aka Ronen)

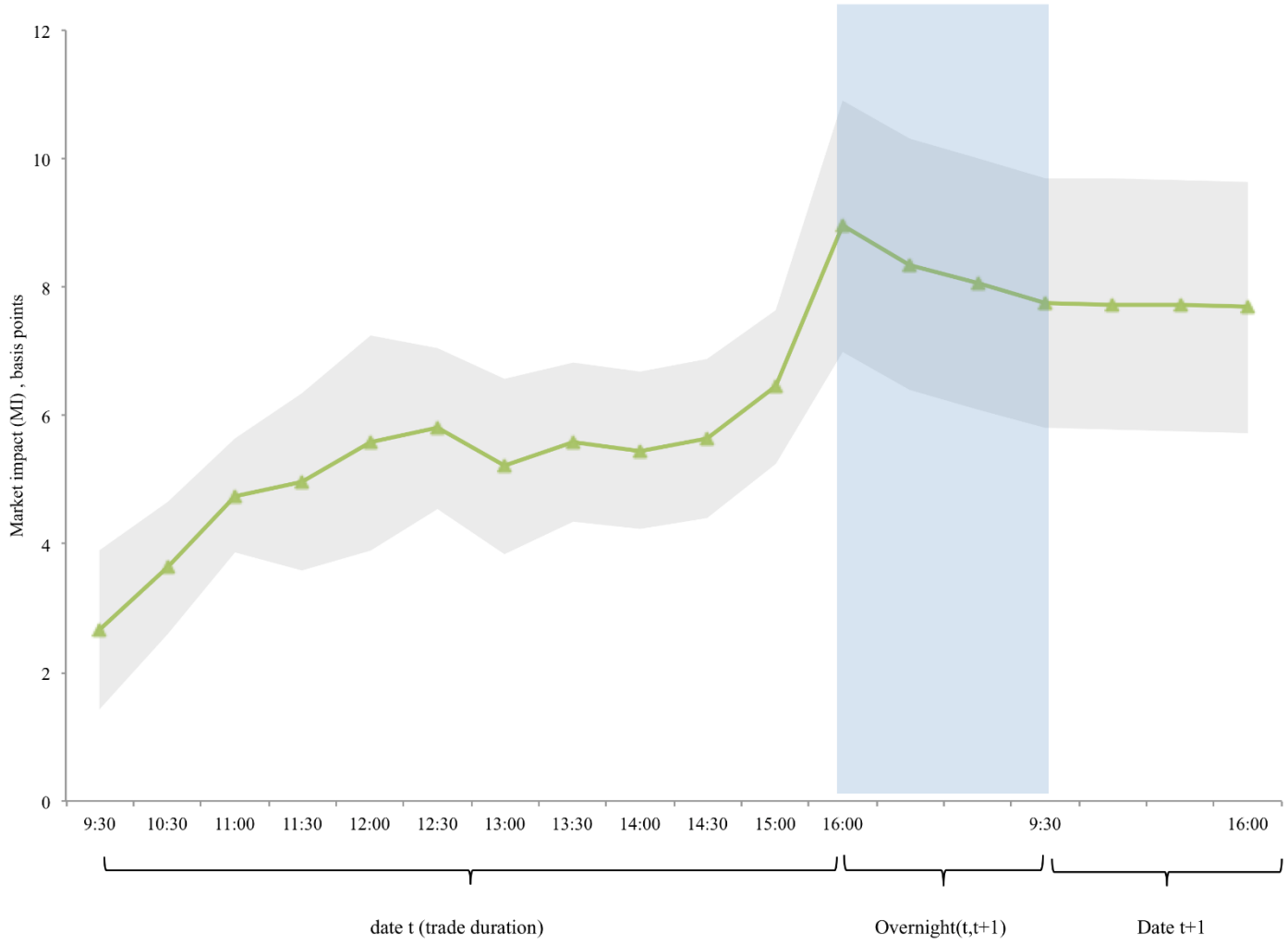
- Direct market access through electronic exchanges
- Provide rather than demand liquidity using a systematic approach that sets opportunistic, liquidity-providing limit orders
- Break up total orders into smaller orders and dynamically manage them
- Randomize size, time, orders, etc. to limit market impact
- Limit prices are set to buy stocks at bid or below and sell stocks at ask or above generally

We consider all of the above as part of the "trading cost" of a large arbitrageur

Measuring Market Impact: A *Theoretical* Example

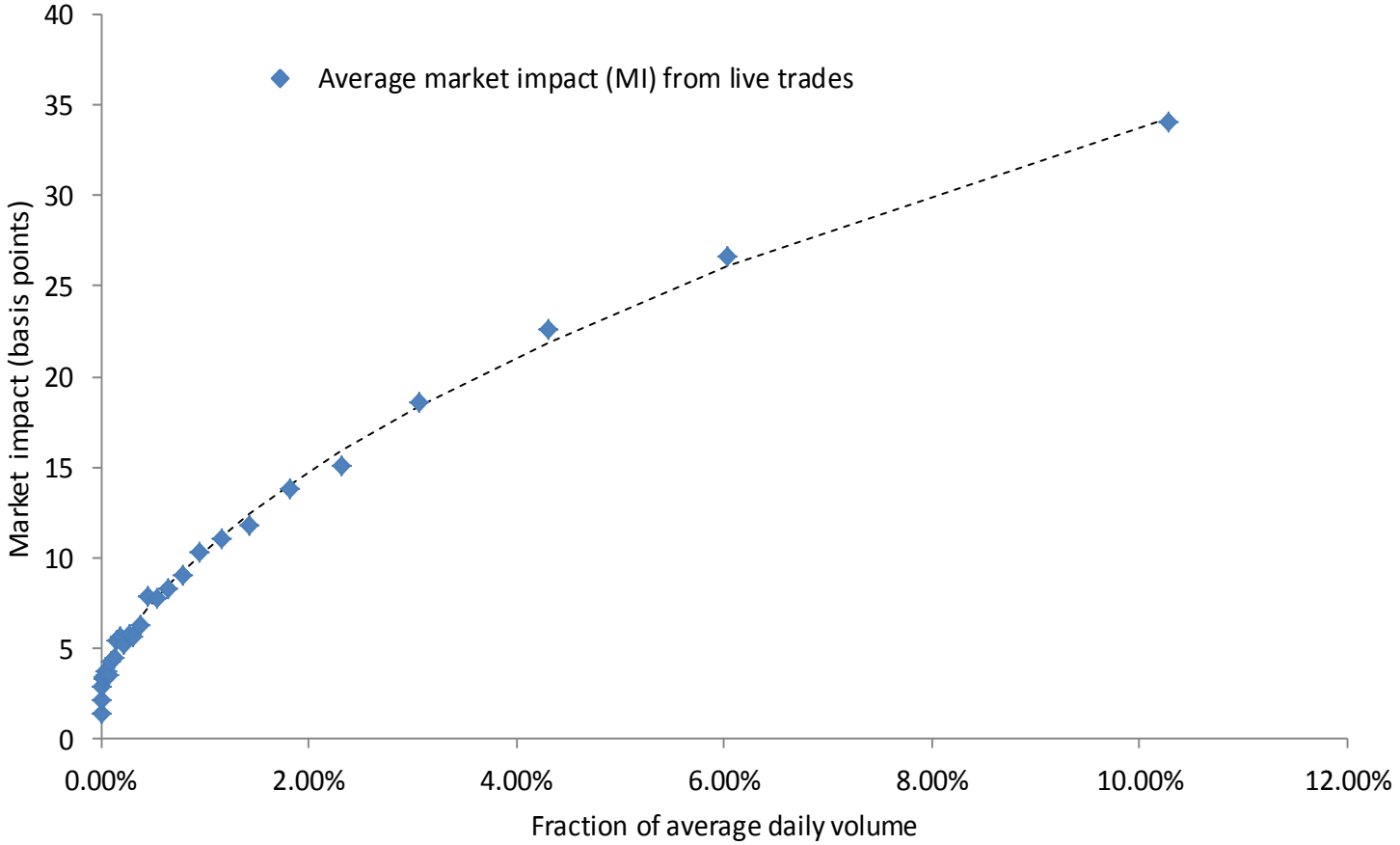


Measuring Market Impact: *Empirical* Average



Market Impact by Fraction of Trading Volume, 1998 – 2013

This figure shows average Market Impact (MI). We sort all trades in our datasets into 30 bins based on their fraction of daily volume and compute average and median market impact for each bucket.



Break-Even Fund Sizes (aka "capacity")

Panel A: U.S. sample	Full Sample premium , 1926 - 2013				Recent sample premium, 1980 - 2013			
	SMB	HML	UMD	Combo	SMB	HML	UMD	Combo
Gross return (annualized %)	2.95	4.95	8.20	9.71	1.76	4.05	5.60	6.90
Turnover (monthly)	0.29	0.44	1.02	0.89	0.29	0.44	1.02	0.89
Break-even NAV (billion)	**275.52**	**214.28**	**56.16**	**98.69**	**102.21**	**153.78**	**26.60**	**54.64**
Average fraction of daily volume traded (%)	36.67	39.95	22.83	38.63	13.60	28.67	10.81	21.39
Average market impact (bps)	87.85	92.54	66.81	90.67	50.93	75.98	45.58	64.47
Total cost (annualized %)	3.03	4.93	8.20	9.71	1.76	4.05	5.60	6.90
Panel B: International sample	Full Sample premium , 1986 - 2013				Recent sample premium, 1993 - 2013			
	SMB	HML	UMD	Combo	SMB	HML	UMD	Combo
Gross return (annualized %)	-0.17	5.78	7.64	7.23	0.24	5.18	6.18	6.86
Turnover (monthly)	0.43	0.51	1.11	0.99	0.43	0.51	1.11	0.99
Break-even NAV (billion)	**0.00**	**95.48**	**18.87**	**23.40**	**0.00**	**79.66**	**12.34**	**21.17**
Average fraction of daily volume traded (%)	0.00	41.57	17.25	19.40	0.00	34.68	11.28	17.55
Average market impact (bps)	11.27	94.83	57.48	61.15	11.27	84.97	46.50	58.00
Total cost (annualized %)	0.59	5.78	7.64	7.23	0.59	5.18	6.18	6.86

Optimized Portfolios

So far, have ignored trading costs when building portfolios

How can portfolios take into account trading costs to reduce total costs substantially?

- Can we change the portfolios to reduce trading costs without altering them significantly?
- Tradeoff between trading costs (market impact) and opportunity cost (tracking error)

Construct portfolios that minimize trading costs while being close to the "benchmark" paper portfolios (SMB, HML, UMD, …)

$$\min_{\boldsymbol{w}} Total\ Trading\ Cost\ (\boldsymbol{w})$$

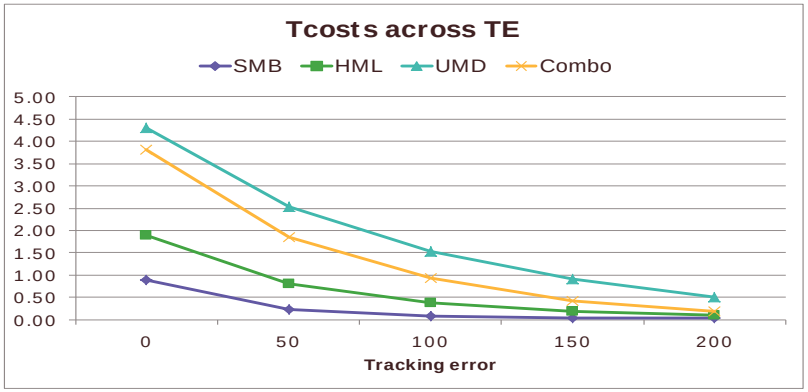
Subject to:

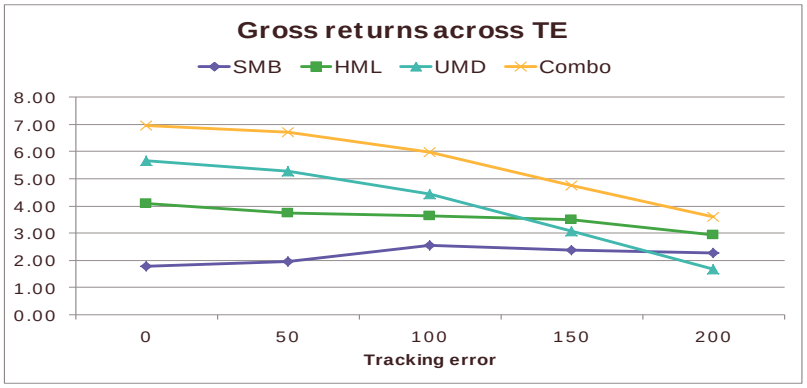
Tracking Error Constraint: $\sqrt{(\boldsymbol{w}-\boldsymbol{B})\boldsymbol{\Omega}(\boldsymbol{w}-\boldsymbol{B})} \leq 1\%$

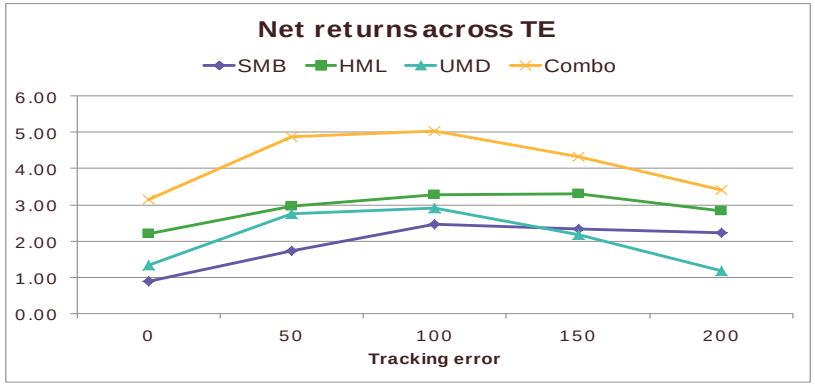
$1 long and $1 short: $\boldsymbol{w}'\boldsymbol{i} = 0$ and $|\boldsymbol{w}|'\boldsymbol{i} = 2$

Trading Constraint: Fraction of daily volume <=5%

Tracking Error Frontiers

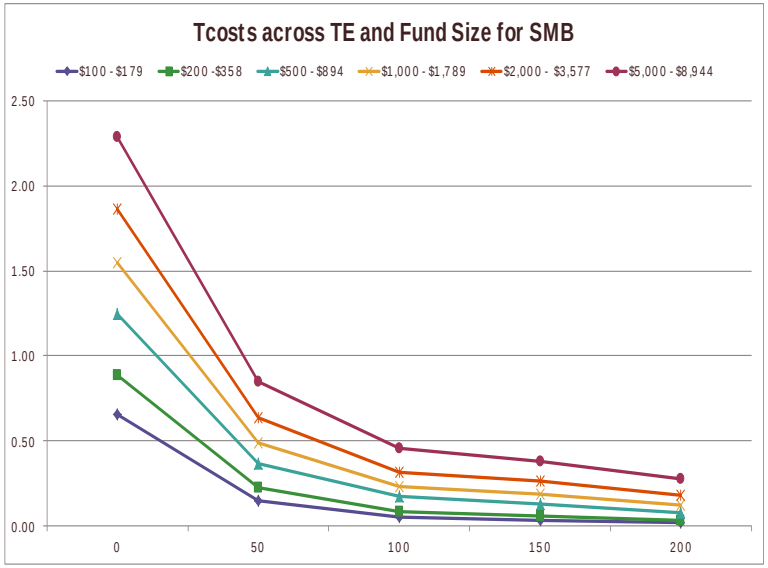


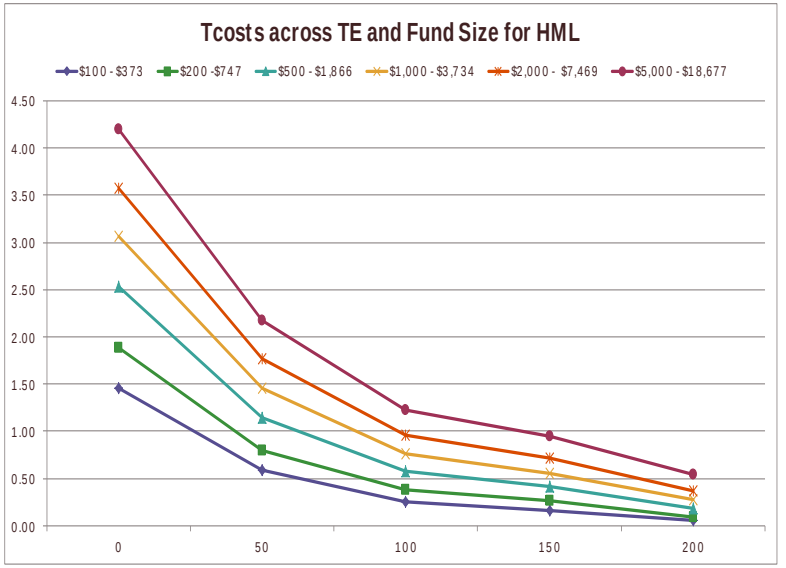


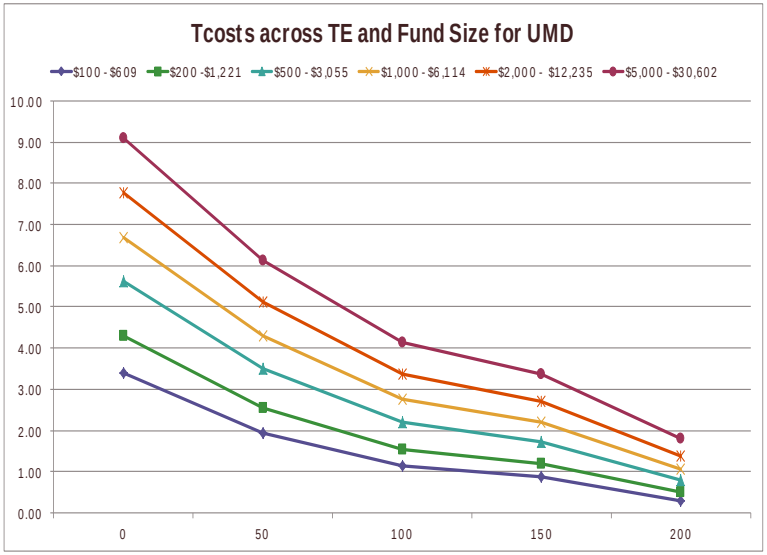


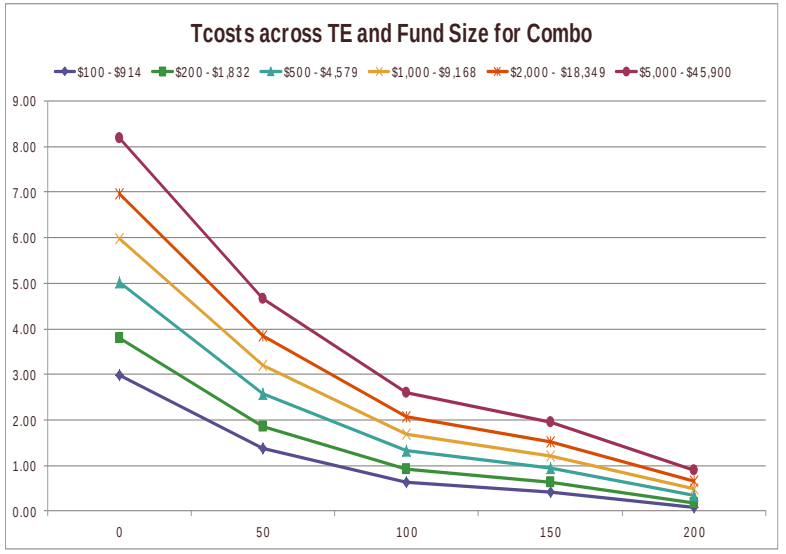
Trading Costs of Asset Pricing Anomalies - Frazzini, Israel, and Moskowitz

Tracking Error vs. Fund Size



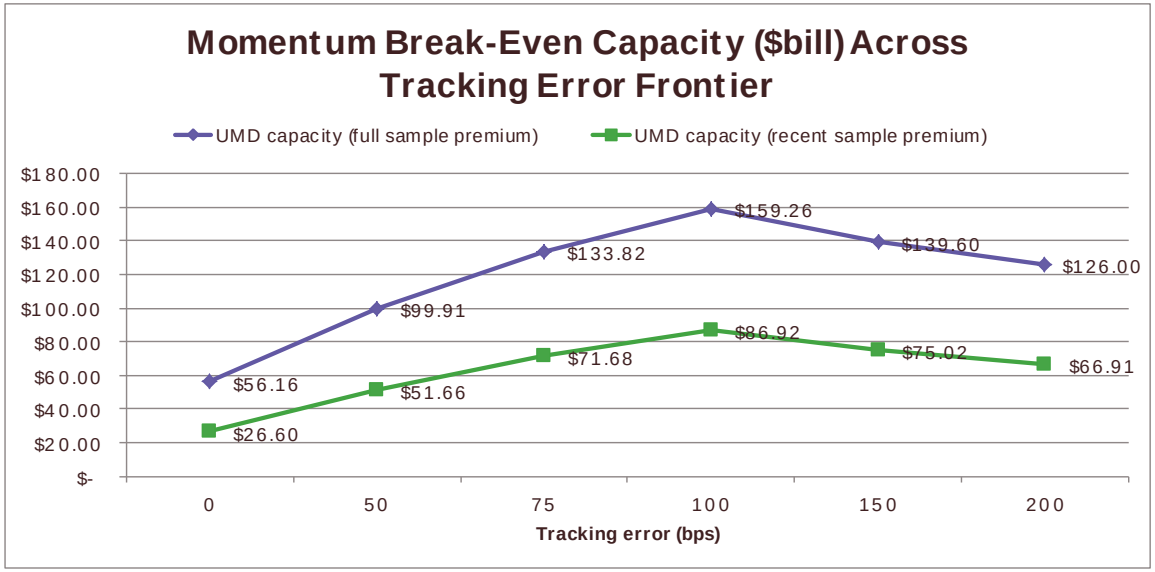


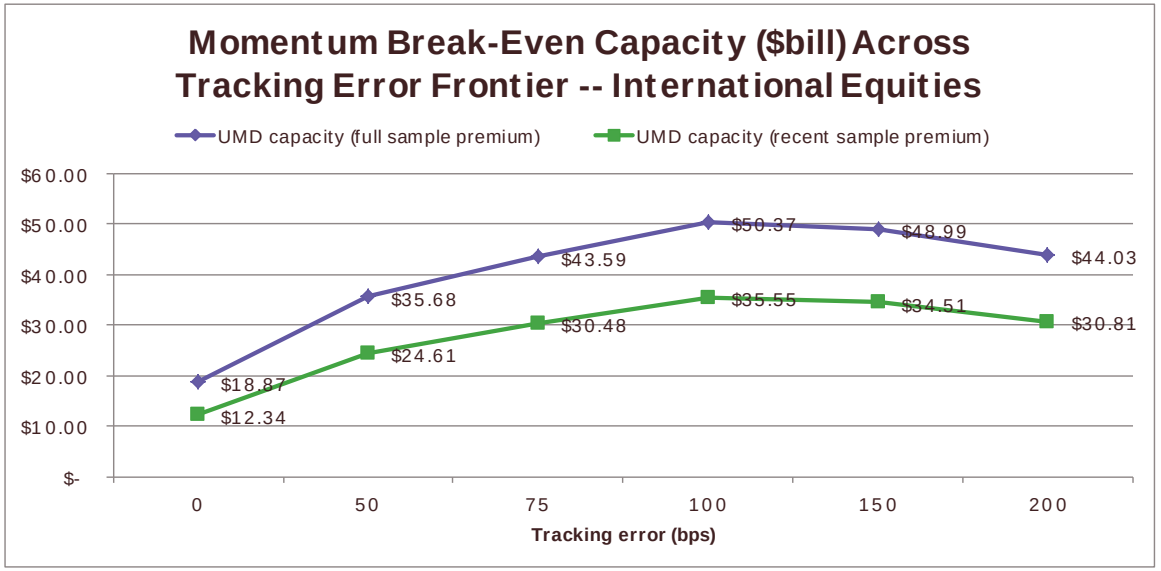




Trading Costs of Asset Pricing Anomalies - Frazzini, Israel, and Moskowitz

Momentum Break-Even Capacity as an Example





Trading Costs of Asset Pricing Anomalies - Frazzini, Israel, and Moskowitz

Conclusions

Unique dataset of live trades to approximate the real trading costs of a large institutional trader/arbitrageur

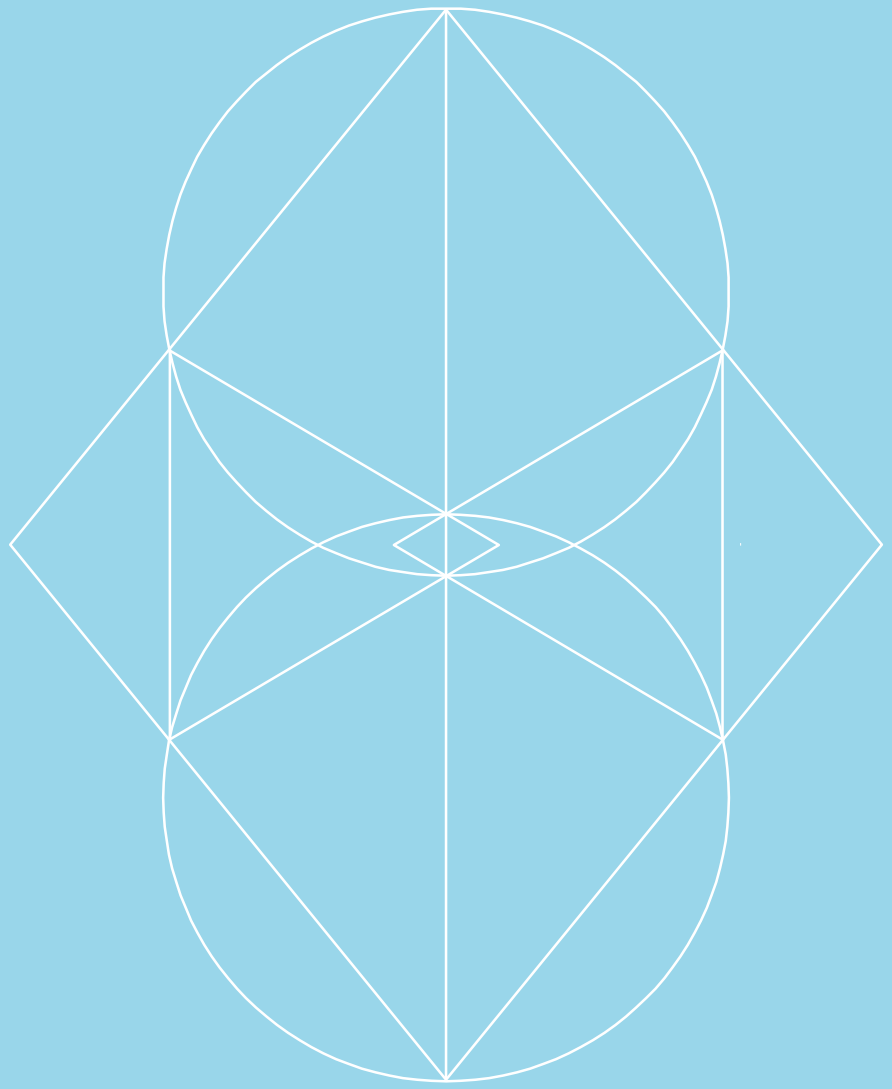
Our trading cost estimates are many times smaller (and break even capacities many times larger) than those previously claimed:

Size, Val, Mom all survive tcosts at high capacity, but STR does not

Fit a model from live traded data to compute expected trading costs based on observable firm and trade characteristics

- We plan to make the coefficients and the price impact breakpoints available to researchers to be used to evaluate trading costs

Appendix



Data Descriptions

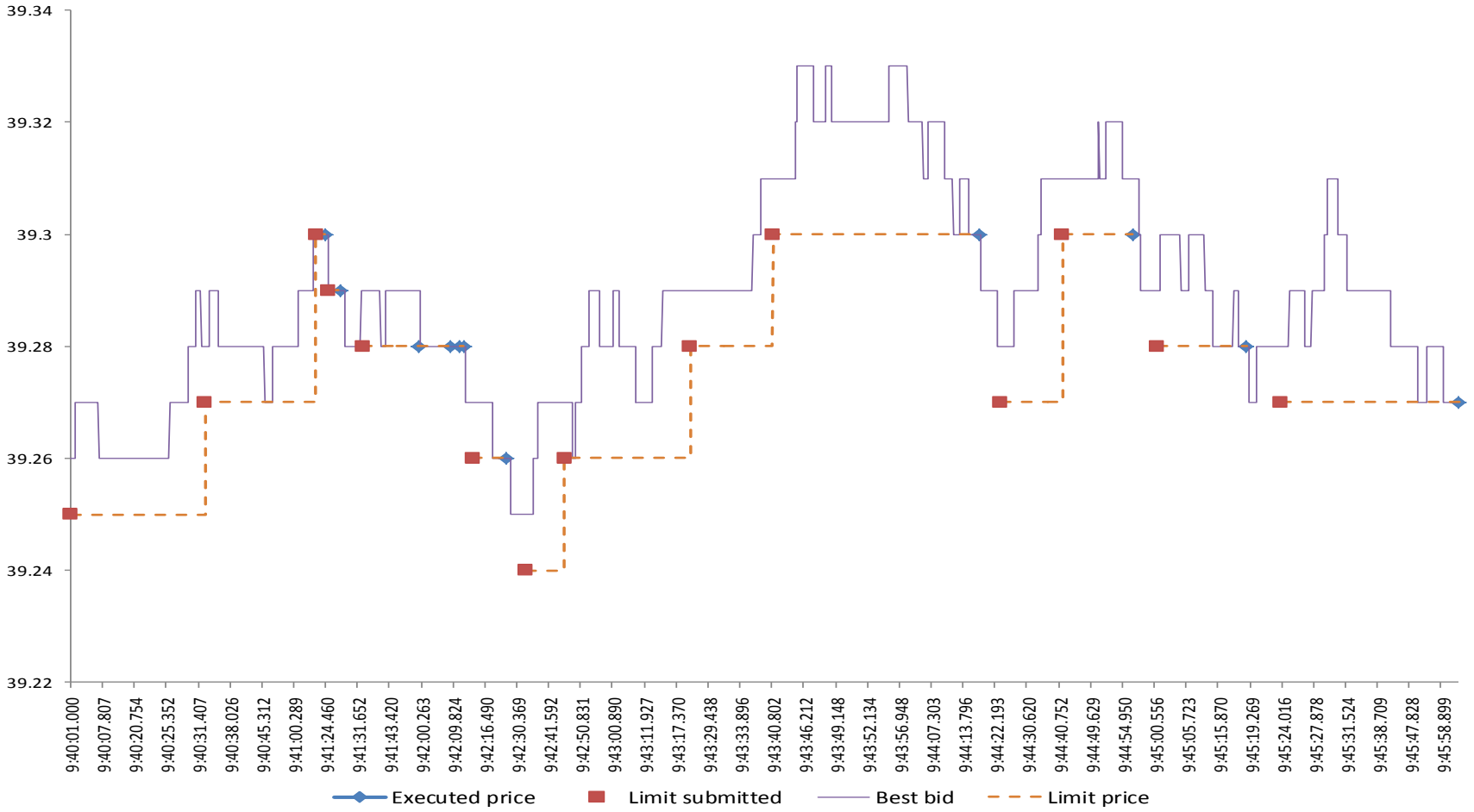
Global Equity Indices
Returns on equity indices from 43 equity markets international which include all countries in the MSCI World Index as of 10/31/2016. Since most countries have multiple equity indices, we use the index that is investable, has the most coverage of the total sock market of that country, and has the longest history. We source monthly total returns from Global Financial Data and futures returns from Bloomberg and Datastream.

Global Fixed Income
Nominal yield and total returns data of 10-year local currency government bonds as well as 3-month interest rates for 26 countries covering North America, Northern Europe, Japan, and Australia/New Zealand, sourced from Global Financial Data, Bloomberg, and Datastream.

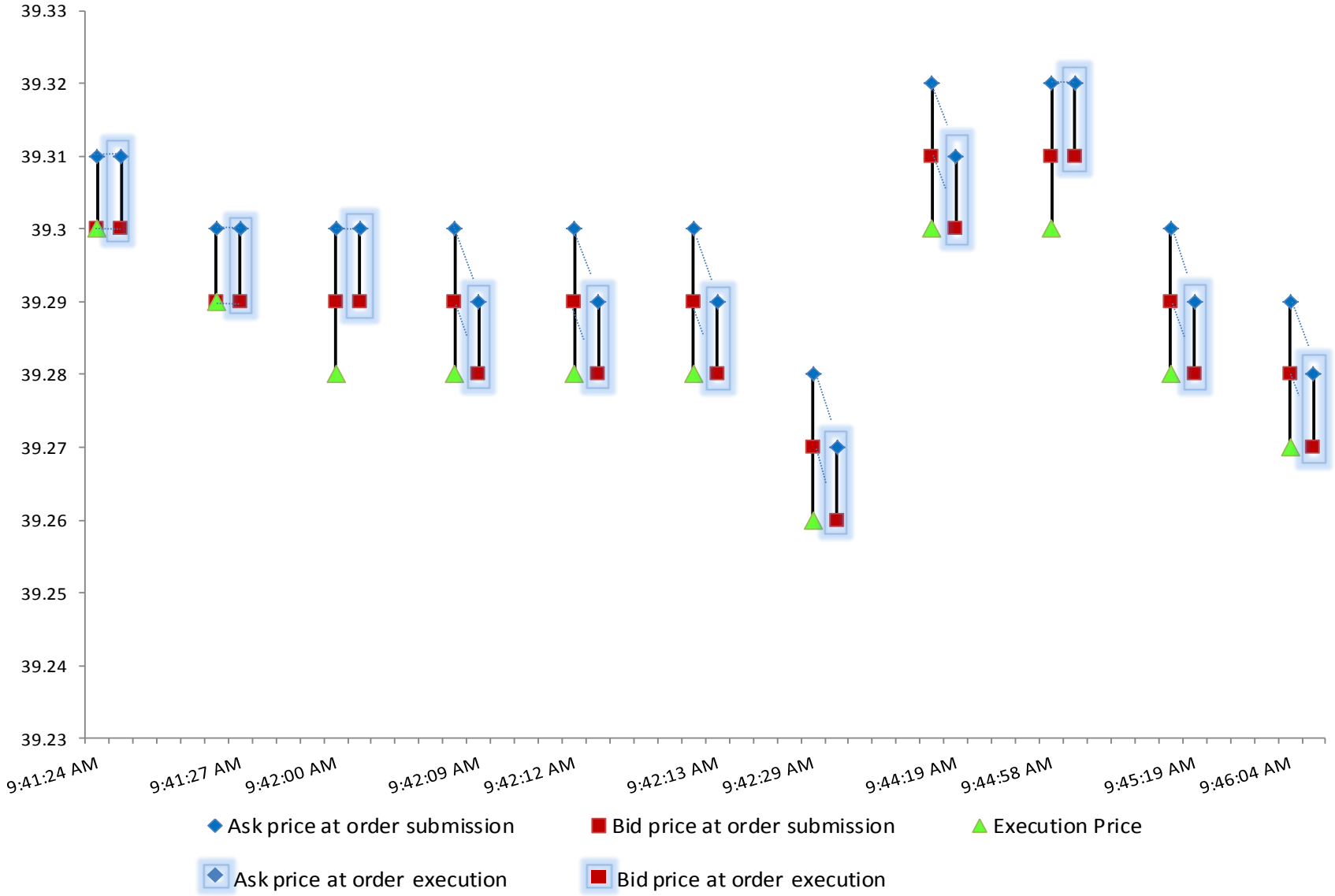
Global Currencies
Spot and 1-, 2-, 3-, and 6-month forward exchange rates from AQR's production data base and interpolate the forward exchange rate for the next quarterly IMM date. This simulates a strategy of buying and holding the forward contract maturing at the near IMM date and rolling to the far contract 5 days before the maturity date. Before 1990, we use changes in spot exchange rates plus the carry of the currency for the total return. This includes data from 20 developed market currencies (Australia, Eurozone, Canada, Japan, Norway, New Zealand, Sweden, Switzerland, United Kingdom, and the U.S., and Belgium, Spain, Finland, France, Germany, Ireland, Italy, Netherlands, Austria, and Portugal).

Commodity Futures
Monthly futures prices of 40 commodities starting in 1877, sourced from the Annual Report of the Trade and Commerce of the Chicago Board of Trade, Commodity Systems Inc., and Bloomberg. For base metals and platinum, rolled return series from the S&P, Goldman Sachs, and Bloomberg are used.

Anatomy of a Trade Execution



Anatomy of a Trade Execution

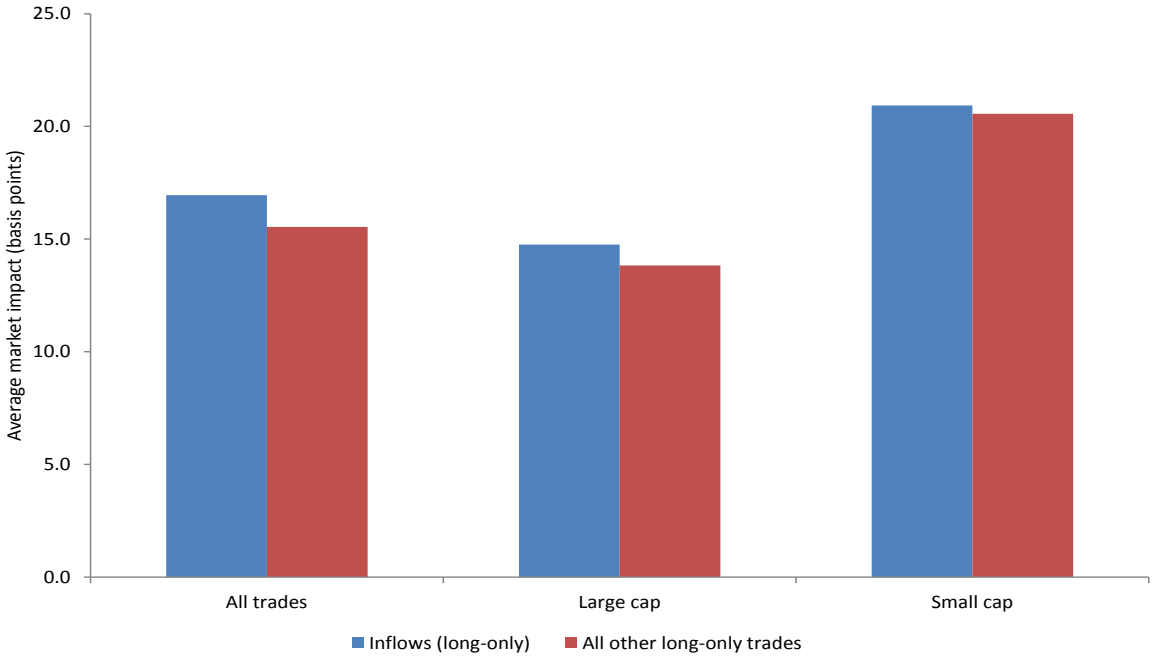


Trade Execution Data, 1998 – 2016. Summary Stats

Panel A: Amount Traded (Billion USD)							
		By region		By size		By portfolio type	
Year	Total	U.S.	International	Large Cap	Small Cap	Long short	Long only
1998*	2.96	1.29	1.67	2.96		2.96	
1999	5.29	1.99	3.30	5.29		5.29	
2000	1.99	0.76	1.23	1.99		1.86	0.13
2001	1.08	0.55	0.53	1.08		1.00	0.08
2002	4.21	0.71	3.50	4.21	0.00	1.40	2.81
2003	5.43	2.69	2.75	5.43	0.00	4.17	1.26
2004	10.00	2.95	7.05	9.99	0.01	6.38	3.62
2005	16.16	8.06	8.10	15.75	0.41	11.45	4.71
2006	67.01	34.79	32.22	64.23	2.78	44.69	22.31
2007	129.46	50.70	78.76	125.21	4.25	96.65	32.81
2008	108.29	25.06	83.24	104.27	4.02	69.30	38.99
2009	111.12	18.58	92.54	108.12	2.99	85.50	25.62
2010	117.17	29.15	88.02	113.78	3.38	91.94	25.23
2011	146.50	56.62	89.88	141.93	4.58	115.69	30.81
2012	179.09	121.39	57.70	173.41	5.68	141.97	37.13
2013	173.94	112.75	61.18	167.11	6.82	117.25	56.69
2014	223.34	153.72	69.62	217.41	5.93	169.99	53.35
2015	263.26	167.39	95.87	256.04	7.22	185.30	77.96
2016*	135.10	82.85	52.25	130.87	4.23	93.33	41.77
Total	1,701.39	871.99	829.40	1,649.07	52.32	1,246.11	455.28

*Data begins September 1998 and ends in June of 2016, so only a partial year of trading for 1998 and 2016.

Exogenous Trades—Initial Trades from Inflows



Panel A: Market impact of trades from new flows					
Long-only trades, 199808 - 201606	Trade type	Inflows only	All other trades	Difference	*t*-statistic
MI mean	All trades	14.99	13.57	1.42	0.36
MI median	All trades	11.77	8.92	2.85	0.77
MI vw mean	All trades	11.40	15.24	-3.84	-1.08
MI mean	Large cap	14.16	11.24	2.92	0.62
MI median	Large cap	11.29	7.43	3.86	0.88
MI vw mean	Large cap	11.30	14.63	-3.34	-0.84
MI mean	Small cap	17.62	18.90	-1.27	-0.28
MI median	Small cap	13.37	13.45	-0.08	-0.02
MI vw mean	Small cap	24.08	22.78	1.30	0.22

Regression Results: Tcost Model

This table shows results from pooled regressions. The left-hand side is a trade's Market Impact (MI), in basis points. The explanatory variables include the contemporaneous market returns, firm size, volatility and trade size (all measured at order submission).

	All sample					United States					International				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Beta*IndexRet*buysell	**0.25**	**0.25**	**0.25**	**0.25**	**0.23**	**0.30**	**0.30**	**0.30**	**0.30**	**0.28**	**0.22**	**0.22**	**0.22**	**0.22**	**0.14**
	(25.76)	(25.78)	(25.79)	(25.81)	(11.77)	(13.96)	(13.96)	(13.96)	(13.95)	(11.07)	(21.22)	(21.21)	(21.19)	(21.31)	(15.02)
Time trend (Jun 1926 = 1)	**-0.04**	-0.03	**-0.03**	0.00	-0.01	-0.02	0.00	-0.01	0.02	0.01	**-0.06**	**-0.05**	**-0.06**	**-0.03**	**-0.03**
	(-2.72)	(-1.96)	(-2.29)	(-0.31)	(-0.82)	(-0.82)	(-0.13)	(-0.46)	(1.00)	(0.54)	(-4.55)	(-3.67)	(-3.96)	(-2.14)	(-3.50)
Log of ME (Billion USD)	**-3.66**	**-2.61**	**-1.90**	**-0.62**	**-0.62**	**-3.28**	**-2.23**	**-1.56**	-0.20	-0.14	**-4.39**	**-3.17**	**-2.47**	**-1.18**	**-1.40**
	(-18.04)	(-13.90)	(-10.00)	(-5.14)	(-4.60)	(-14.17)	(-10.83)	(-6.91)	(-1.10)	(-0.77)	(-17.18)	(-12.70)	(-10.00)	(-8.09)	(-9.45)
* Fraction of daily volume		**1.97**	**0.36**	0.22	-0.13		**2.56**	0.58	0.35	-0.53		**1.69**	**0.34**	0.25	**0.29**
		(15.29)	(2.30)	(1.55)	(-0.72)		(10.34)	(1.67)	(1.06)	(-1.37)		(12.43)	(2.12)	(1.72)	(2.05)
* Sqrt(Fraction of daily volume)			**7.33**	**8.27**	**8.89**			**7.88**	**9.32**	**11.21**			**6.57**	**7.22**	**5.97**
			(11.26)	(13.23)	(10.39)			(7.11)	(8.56)	(8.54)			(11.00)	(13.18)	(12.72)
Idiosyncratic Volatility				**0.30**	**0.28**				**0.32**	**0.31**				**0.29**	**0.25**
				(10.67)	(9.50)				(7.87)	(7.49)				(9.76)	(8.94)
Vix				**0.17**	**0.15**				**0.13**	0.12				**0.21**	**0.20**
				(2.74)	(2.91)				(2.06)	(1.95)				(2.61)	(2.83)
DGTW-adjusted return*buysell				.	0.04					0.03					**0.13**
					(1.54)					(1.33)					(14.51)
Observations (1,000s)	3,470	3,470	3,470	3,470	3,470	1,722	1,722	1,722	1,722	1,722	1,748	1,748	1,748	1,748	1,748
Adjusted R^2	0.103	0.105	0.105	0.106	0.149	0.117	0.118	0.119	0.119	0.152	0.094	0.095	0.096	0.096	0.212
Country Fixed Effects	Yes	Yes	Yes	Yes	Yes	No	No	No	No	No	Yes	Yes	Yes	Yes	Yes

- Use regression coefficients to compute predicted trading costs for all stocks

Regression Results: Other Tcost Measures

	Panel A: United States											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Modified Roll	**0.03**	0.01									0.00	0.01
	(3.01)	(1.40)									(0.00)	(1.24)
Amihud			0.04	-0.03							-0.07	-0.05
			(1.60)	(-0.87)							(-1.62)	(-1.39)
PropZero					102.31	-54.00					22.57	-34.85
					(1.32)	(-0.75)					(0.53)	(-0.84)
TAQ Effective Spread							**0.30**	-0.04			0.10	0.01
							(2.21)	(-0.33)			(1.39)	(0.16)
TAQ Lambda									**98.45**	-10.59	**141.03**	32.42
									(3.14)	(-0.30)	(3.42)	(1.27)
Beta*IndexRet*buysell	**0.30**	0.01	**0.30**	0.01	**0.30**	0.01	**0.30**	0.01	**0.30**	0.01	**0.30**	0.01
	(13.96)	(0.53)	(13.96)	(0.54)	(13.96)	(0.53)	(13.95)	(0.55)	(13.95)	(0.55)	(13.95)	(0.55)
Time trend	-0.04	0.02	**-0.04**	0.01	**-0.04**	0.02	**-0.04**	0.02	-0.02	0.02	-0.02	0.02
	(-1.81)	(1.04)	(-2.31)	(0.93)	(-2.31)	(0.95)	(-1.98)	(0.99)	(-1.21)	(0.98)	(-0.96)	(1.03)
Log of ME (Billion USD)		-0.13		-0.42		-0.25		-0.22		-0.29		-0.19
		(-0.79)		(-1.33)		(-1.31)		(-1.01)		(-0.70)		(-0.50)
Fraction of daily volume		0.35		0.41		0.37		0.43		0.43		0.49
		(1.00)		(1.27)		(1.07)		(1.24)		(1.24)		(1.52)
Sqrt(Fraction of daily volume)		**9.03**		**9.07**		**9.03**		**8.86**		**8.87**		**8.93**
		(8.40)		(8.38)		(8.32)		(8.01)		(7.96)		(8.01)
Idiosyncratic Volatility		**0.32**		**0.33**		**0.32**		**0.33**		**0.33**		**0.30**
		(7.81)		(8.39)		(8.05)		(7.83)		(7.72)		(7.37)
Vix		0.12		**0.12**		**0.12**		**0.13**		**0.13**		0.11
		(1.95)		(2.03)		(2.00)		(2.05)		(2.01)		(1.83)
DGTW Ret*buysell		**0.27**		**0.27**		**0.27**		**0.27**		**0.27**		**0.27**
		(22.19)		(22.27)		(22.23)		(22.11)		(22.13)		(22.15)
Adj. R^2	0.1154	0.1561	0.1155	0.1561	0.1154	0.1561	0.1155	0.1559	0.1159	0.1559	0.1161	0.1560
Adj. R^2 after beta and trend	0.0001	0.0408	0.0002	0.0408	0.0001	0.0408	0.0002	0.0406	0.0006	0.0406	0.0008	0.0407

Regression Results: Other Tcost Measures

	Panel B: International							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Modified Roll	**0.03**	0.01					**0.02**	0.01
	(3.10)	(1.66)					(2.17)	(1.38)
Amihud			**0.21**	**0.06**			**0.20**	**0.06**
			(11.17)	(4.82)			(10.89)	(4.78)
PropZero					**38.46**	15.22	3.47	11.44
					(2.71)	(1.29)	(0.25)	(0.98)
Beta*IndexRet*buysell	**0.22**	**-0.05**	**0.22**	**-0.05**	**0.22**	**-0.05**	**0.22**	**-0.05**
	(21.20)	(-6.63)	(21.20)	(-6.63)	(21.20)	(-6.63)	(21.21)	(-6.63)
Time trend	**-0.07**	**-0.03**	**-0.07**	**-0.03**	**-0.08**	**-0.03**	**-0.06**	**-0.03**
	(-4.93)	(-2.36)	(-4.44)	(-2.35)	(-5.64)	(-2.64)	(-4.81)	(-2.59)
Log of ME (Billion USD)		**-1.42**		**-0.90**		**-1.35**		**-0.89**
		(-10.08)		(-5.02)		(-8.99)		(-4.88)
Fraction of daily volume		0.19		0.18		0.19		0.18
		(1.38)		(1.30)		(1.38)		(1.30)
Sqrt(Fraction of daily volume)		**6.81**		**6.68**		**6.83**		**6.67**
		(13.81)		(13.58)		(13.87)		(13.55)
Idiosyncratic Volatility		**0.27**		**0.26**		**0.28**		**0.26**
		(10.06)		(9.29)		(10.07)		(9.33)
Vix		**0.18**		**0.18**		**0.18**		**0.18**
		(2.98)		(2.98)		(3.00)		(3.03)
DGTW Ret*buysell		**0.27**		**0.27**		**0.27**		**0.27**
		(47.63)		(47.61)		(47.63)		(47.62)
Adj. R^2	0.0921	0.1532	0.0933	0.1533	0.0920	0.1532	0.0933	0.1533
Adj. R^2 after beta and trend	0.0000	0.0612	0.0012	0.0613	0.0000	0.0612	0.0013	0.0613

Returns Results – Trade Execution Sample – U.S.

Actual dollar traded in each portfolio (past 6 month) to estimate trading costs at each rebalance

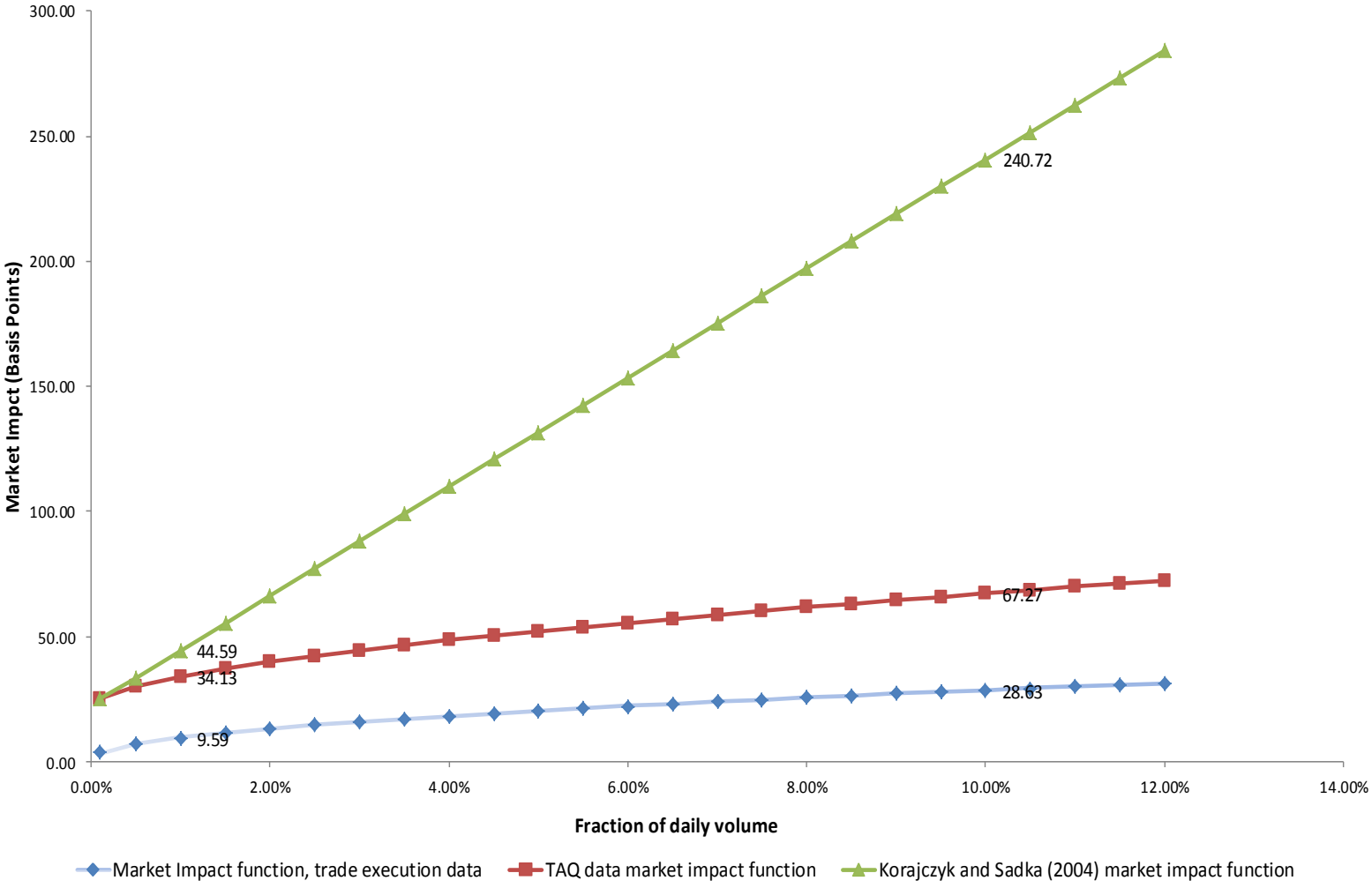
Trading costs and implied fund size are based on *actual traded sizes and actual trading costs*

- No estimation here!

	Panel A: U.S. trade execution sample, 1998 - 2013				Panel B: International trade execution sample, 1998 - 2013			
	SMB	HML	UMD	Combo	SMB	HML	UMD	Combo
Dollar traded per month (billion USD)	9.69	5.97	6.18	9.69	11.80	7.15	8.38	12.77
Implied fund size (billion USD)	18.18	9.42	5.21	16.91	17.88	10.09	6.85	19.74
Correlation to portfolio over full universe	0.78	0.96	0.97	0.88	0.53	0.91	0.94	0.79
Realized cost	1.47	1.35	3.03	1.46	1.70	1.54	2.24	1.24
Break-even cost	2.95	4.95	8.20	5.39	-0.17	5.78	7.65	4.68
Realized minus breakeven	**-1.48**	**-3.61**	**-5.17**	**-3.93**	**1.87**	**-4.24**	**-5.40**	**-3.44**
Full sample historical mean:								
Return (Gross)	**2.95**	**4.95**	**8.20**	**5.39**	-0.17	**5.78**	**7.65**	**4.68**
	(2.72)	(3.10)	(4.79)	(9.13)	(-0.12)	(3.01)	(2.98)	(5.22)
Return (Net)	1.48	**3.61**	**5.17**	**3.93**	-1.87	**4.24**	**5.40**	**3.44**
	(1.40)	(2.25)	(3.02)	(6.66)	(-1.30)	(2.20)	(2.10)	(3.75)
Live trading sample mean:								
Return (Gross)	**7.98**	4.86	2.26	**5.04**	1.17	5.59	4.02	**3.59**
	(3.01)	(1.12)	(0.40)	(3.17)	(0.75)	(1.83)	(0.92)	(2.88)
Return (Net)	**6.52**	3.51	-0.77	**3.58**	-0.53	4.05	1.78	2.35
	(2.48)	(0.80)	-(0.14)	(2.23)	-(0.33)	(1.32)	(0.41)	(1.86)
Turnover (monthly)	0.53	0.63	1.19	0.57	0.66	0.71	1.22	0.65
MI (bps)	22.94	17.71	21.30	21.22	21.42	18.12	15.27	16.02
Sharpe ratio (gross)	0.78	0.29	0.10	0.82	0.20	0.47	0.24	0.75
Sharpe ratio (net)	0.65	0.21	-0.04	0.58	-0.09	0.34	0.11	0.48
Number of months	178	178	178	178	178	178	178	178

Comparing Market Impact Functions

This figure shows average Market Impact (MI) from the Korajczyk and Sadka (2004) model and data (TAQ)



Other Comparisons to Literature

Fund size =	Actual total market cap of S&P 500	Actual total market cap of S&P 500	Amount benchmarked to S&P 500	Amount benchmarked to S&P 500	Actual total market cap of Russell 2000	Actual total market cap of Russell 2000	Amount benchmarked to Russell 2000	Amount benchmarked to Russell 2000
tcost estimate =	FIM, trade data	linear, TAQ	FIM, trade data	linear, TAQ	FIM, trade data	linear, TAQ	FIM, trade data	linear, TAQ
	Panel A: S&P 500 index*				**Panel B: Russell 2000 index****			
Gross return (annualized %)	5.28	5.28	5.28	5.28	6.67	6.67	6.67	6.67
Turnover (monthly)	0.4%	0.4%	0.4%	0.4%	1.6%	1.6%	1.6%	1.6%
NAV ($billion)	**21,299.97**	**21,299.97**	**7,800.00**	**7,800.00**	**4,434.40**	**4,434.40**	**1,186.67**	**1,186.67**
Average fraction of daily volume traded (%)	136.54	136.54	50.00	50.00	34.29	34.29	9.18	9.18
Average market impact (bps)	110.26	2,999.56	71.04	1,113.72	60.81	771.33	37.16	224.08
Estimated total cost (annualized bps)	**5.5**	**150.0**	**3.6**	**55.7**	**11.7**	**148.9**	**7.2**	**43.2**

*Vanguard S&P 500 Index Fund annual tcosts = 4 bps per year; iShares S&P 500 ETF annual tcosts = 7 bps per year.

**Vanguard Russell 2000 Index Fund annual tcosts = 15 bps per year; iShares Russell 2000 ETF annual tcosts = 19 bps per year.

Comparison to Costs from Brokers

Panel A: Comparison of Trading Costs Across Trade Size (%DTV)

Average costs from 2008 -2011	Actual trading costs					Estimated trading costs		
%DTV	ITG	DB	JPM	Average	AQR	Trade data, FIM model	TAQ data, square root	TAQ data, linear
0.25-0.50%	4.00	4.50	8.00	5.50	5.27	5.08	28.87	30.97
0.50-1.0%	8.00	10.00	14.00	10.67	7.75	7.21	32.22	39.14
1.0-1.5%	10.00	13.00	16.00	13.00	10.57	9.86	36.49	52.76
1.5-2.0%	10.00	13.00	16.00	13.00	13.08	11.53	39.25	63.66
2.0-5.0%	17.00	17.50		17.25	18.66	16.01	46.91	101.80
5.0-10.0%	22.00			22.00	23.52	23.49	60.41	188.96

Panel B: Comparison of Trading Costs Over Time

	AQR average costs		ANcerno average costs	
	MI	Commissions	MI	Commissions
Avg. trade size = 2.4% DTV				
1999-2008	15.4	0.5	24.1	8.8
2007-2008	18.2	0.3	24.5	7.5
1999-2006	28.7	0.6	24.0	9.2
Avg. trade size = 0.5% DTV				
1999-2014	7.10	0.6	10.52	12.0