Dynamic Principal Agent Models: A Continuous Time Approach

Lecture IV

Extensions and Applications
(He 2009, DeMarzo et al. 2011, Hoffmann and Pfeil 2010, 2012, Piskorski and Westerfield 2011)

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Extensions and Applications I

He (2009): Optimal Executive Compensation when Firm Size follows a GBM

Basic Setting

Similar to DeMarzo and Sannikov (2006):

- Time is continuous with $t \in [0, \infty)$,
- all players are risk-neutral,
- agent has limited liability and limited wealth, so principal has to cover operating losses and initial set up costs K.

BUT:

- Agent controls firm size instead of instantaneous cash flows,
- agent is only weakly more impatient than the principal $\rho \geq r$.

Firm Size Follows a GBM

- Firm size $\delta \geq 0$ follows a geometric Brownian motion

$$d\delta_t = A_t \delta_t dt + \sigma \delta_t dZ_t,$$

  where $A_t \in \{0, \mu\}$ denotes the agent's effort.
- Firm produces cash flows at rate δ (i.e. 1:1 proportional to size).
- Principal discounts at rate $r > \mu$, so first best firm value as of time t is

$$E_t \left[\int_t^\infty e^{-r(s-t)} \delta_s ds \right] = \frac{\delta_t}{r - \mu}.$$

- When setting $A_t = 0$, the agent enjoys shirking benefits $\phi \delta_t dt$.

Contracting Problem

- Upon liquidation, the principal receives scrap value $L\delta_t$.
- The principal offers the agent a contract specifying cash payments $\{C_t, t \geq \tau\}$ and a stopping time $\tau \geq 0$ to maximize

$$F_0 = E^{A^*=\mu}\left[\int_0^{\tau} e^{-rt}\left(\delta_t dt - dC_t\right) + e^{-r\tau}L\delta_\tau\right].$$

  Note: we implicitly assume that $A_t = \mu$, $t \geq 0$ is optimal (it has to be checked later whether this is true, as revelation principle does not apply here).

- Where A^* maximizes the agent's expected utility

$$W_0 = E^{A}\left[\int_0^{\tau} e^{-\rho t}\left(dC_t + \phi\left(1 - \frac{A_t}{\mu}\right)\delta_t dt\right) + e^{-\rho\tau}R\delta_\tau\right].$$

- Observe that the problem is homogenous with respect to firm size, which will allow us to get rid of the additional state variable δ.

Agent's Continuation Value and Incentive Compatibility

- By analogous arguments as in DeMarzo and Sannikov, the agent's continuation value evolves according to

$$dW_t = \rho W_t - dC_t + \Gamma_t \underbrace{(d\delta_t - \mu\delta_t dt)}_{=\delta_t \sigma dZ_t \text{ if } A_t = 0}.$$

- *High effort ($A_t = \mu, t \geq 0$) is incentive compatible iff*

$$\Gamma_t \geq \underbrace{\phi/\mu}_{:=\lambda}.$$

- Intuition: If the agent shirks,
  - he enjoys a private benefit of $\phi\delta_t$,
  - his continuation value is reduced by $\Gamma_t \mu \delta_t$.

Derivation of HJB for Principal's Value Function

- Denote the highest profit that the principal can obtain, given the agent's expected payoff is W and the current firm size is δ, by

$$F(\delta, W).$$

- $F(\delta, W)$ is concave in W (because inefficient termination occurs when $W = 0$, the principal becomes "risk-averse" wrt W)
  - No cash payments as long as

$$F_W(\delta, W) := \partial F / \partial W > -1.$$

  - Cash payments dC cause W to reflect at the compensation boundary $\overline{W}(\delta)$ defined by

$$F_W\left(\delta, \overline{W}(\delta)\right) = -1.$$

Derivation of HJB for Principal's Value Function

- Over the interval $\left[R\delta, \overline{W}(\delta)\right]$, the principal's value function has to satisfy the HJB equation

$$\underbrace{rF(\delta, W)\,dt}_{\text{required return}} = E\Big[\underbrace{\delta dt}_{\text{cash flow}} + \underbrace{dF(\delta, W)}_{\text{change in value}}\Big].$$

- This is now a PDE, as $dF(\delta, W)$ involves derivatives with respect to both state variables δ and W!

Size Adjusted Value Function

- Using Itô's Lemma, the HJB becomes, more explicitly,

$$rF = \delta + F_\delta \mu\delta + \rho W F_W + \frac{1}{2}\left(\sigma^2\delta^2 F_{\delta\delta} + 2\lambda\sigma^2\delta^2 F_{\delta W} + \lambda^2\sigma^2\delta^2 F_{WW}\right).$$

- Use that F is homogenous in δ to define principal's scaled value function

$$\delta f(w) = \delta F\left(1, \frac{W}{\delta}\right).$$

- From this we immediately get the derivatives

$$\begin{aligned} F_\delta &= f(w) - \delta f'(w), \\ F_W &= f'(w), \\ \delta F_{\delta\delta} &= -\delta w F_{\delta W} = \delta w^2 F_{WW} = w^2 f''(w), \end{aligned}$$

which gives us the size adjusted version of the HJB.

Size Adjusted Value Function

- Over the interval $[R, \overline{w}]$, the principal's scaled value function $f(w)$ satifies

$$(r-\mu) f(w) = 1 + (\rho - \mu) w f'(w) + \frac{1}{2} (\lambda - w)^2 \sigma^2 f''(w)$$

with the usual boundary conditions

$$\begin{aligned} f(R) &= 0 && \textit{value matching}, \\ f'(\overline{w}) &= -1 && \textit{smooth pasting}, \\ f''(\overline{w}) &= 0 && \textit{super contact}. \end{aligned}$$

- And the agent's scaled continuation value evolves according to

$$dw = (\rho - \mu) w dt + (\lambda - w) \sigma dZ - dc,$$

where cash payments dc cause w to reflect at $\overline{w}$.

Comparison to Arithmetic Brownian setting

ABM Setting	**GBM Setting**
Agent controls	
instantaneous cash flows dY_t	change in cash flow *rate* $d\delta_t$
Cash flows	
unbounded from below dY_t	always *positive* $\delta_t dt$

"Free" Incentives in the GBM Setting

- Shirking benefits are equal to

$$\lambda,$$

but instantaneous volatility of w is only

$$(\lambda - w)\,\sigma.$$

- The agent's scaled continuation value w itself provides some incentives.
- Intuition:
  - w represents the agent's "stake in the firm"
  - If size changes by $d\delta$, agent's continuation value $W = w\delta$ changes by

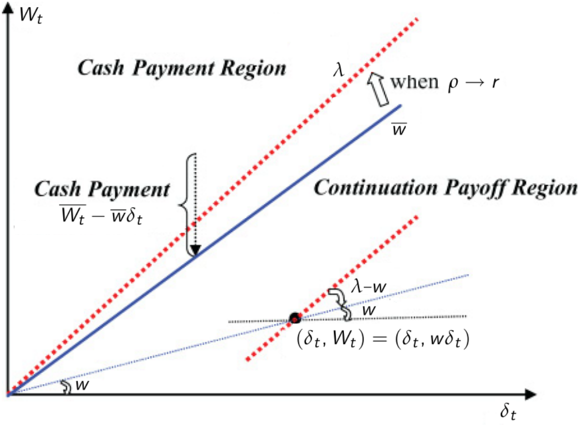
$$wd\delta.$$

- If the agent's share in the firm is sufficiently high, ($w = \lambda$), the volatility in w becomes zero (absorbing state).

$\Rightarrow$ Agent's inside stake is sufficient to provide incentives for working.

Incentive Provision in the GBM Setting

- IC requires that $\partial W / \partial \delta = \lambda$,
- "free" incentives: w,
- remaining portion: $(\lambda - w)$.



No Absorbing State with a More Impatient Agent

- *If agent is more impatient than the principal ($\rho > r$), then*

$$\overline{w} < \lambda,$$

  i.e. cash payments keep w from reaching the absorbing state λ

- Intuition: Consider a marginal reduction of $\overline{w}$

  1. **benefit**: the agent is paid earlier and $\rho - r$
     (strictly positive, independent of the level of $\overline{w}$)
  2. **cost**: the probability of termination increases
     (vanishes for $\overline{w} = \lambda$ where no future termination threat)

- Equating marginal benefits (1.) and marginal costs (2.) implies that $\overline{w} = \lambda$ cannot be optimal if $\rho > r$

No Absorbing State with a More Impatient Agent

- Show this a bit more formally: Assume $\overline{w} = \lambda$ and evaluate HJB in $\lambda - \varepsilon$

$$(r-\mu) f(\lambda-\varepsilon) = 1 + (\rho-\mu)(\lambda-\varepsilon) f'(\lambda-\varepsilon) + \frac{\varepsilon^2\sigma^2}{2} f''(\lambda-\varepsilon)$$

- A Taylor expansion of f and f' yields

$$\begin{aligned} f(\lambda-\varepsilon) &= f(\lambda) - f'(\lambda)\varepsilon + \frac{1}{2} f''(\theta_1)\varepsilon^2 \\ f'(\lambda-\varepsilon) &= f'(\lambda) + \frac{1}{2} f''(\theta_2)\varepsilon^2, \end{aligned}$$

where $\theta_i \in (\lambda-\varepsilon, \lambda)$. From the boundary conditions for $\overline{w}$, we get

$$r-\rho = -\frac{r-\mu}{2} f''(\theta_1)\varepsilon - f''(\theta_2)(\rho-\mu)(\lambda-\varepsilon) + \frac{\varepsilon\sigma^2}{2} f''(\lambda-\varepsilon).$$

Letting $\varepsilon \to 0$, the RHS$\to 0$, while the LHS< 0 whenever $\rho > r$.

Absorbing State with an Equally Patient Agent

- *When principal and agent are equally patient ($\rho = r$), then*

$$\overline{w} = \lambda$$

- Intuition:
    - There is no cost from delaying payments to the agent
    - $\overline{w}$ will be raised until marginal benefits from lower probability of termination are zero

Absorbing State with an Equally Patient Agent

- If w_t reaches λ, from then on the agent receives cash payments

$$dc_s = (r - \mu)\,\lambda ds$$

- His **scaled** continuation value, which evolves according to

$$dw_s = (r - \mu)\,w_s dt - dc_s + (\lambda - w_s)\,\sigma dZ = 0,$$

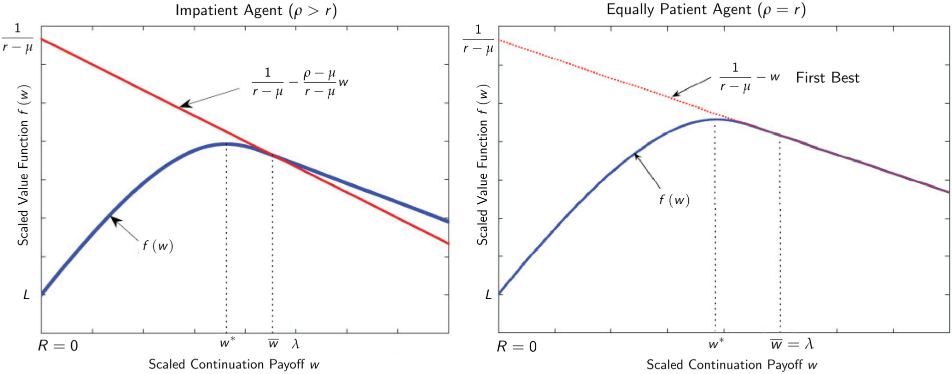
therefore remains constant at $w_s = \lambda$ and, as there is no termination,

$$f(\lambda) + \lambda = \frac{1}{r - \mu} \qquad \text{(first best)}$$

- Equivalently, consider granting the agent $(r - \mu)\,\lambda$ shares of the firm, so his **unscaled** continuation value evolves according to

$$dW_s = (r - \mu)\,\lambda \delta_s ds$$

More Impatient Agent and Equally Patient Agent



Extensions and Applications II

DeMarzo et al. (2011): Dynamic Agency and the q Theory of Investment

Motivation

- Add dynamic agency to the standard neoclassical model of investment.
- Classic Modigliani-Miller: Optimal Investment separable from financing.
- However: External Financing often subject to frictions and financing costs matter (Fazzari et al. 1988, Kaplan and Zingales 1997).
- Here: Frictions arising from agency problem endogenizing costs of external financing (optimal contracts): Hayashi (1982) + DeMarzo and Sannikov (2006).

Basic Setting

Similar to DeMarzo and Sannikov (2006):

- Time is continuous with $t \in [0, \infty)$.
- All players are risk-neutral, agent more impatient ($\rho > r$).
- Agent has limited liability and limited wealth, so principal has to cover operating losses and initial set up costs K.

BUT additionally:

- Capital accumulation: Principal has access to an investment technology increasing firm size.

Basic Setting - Technology

- Capital accumulation:
$$dK_t = (I_t - \delta K_t)\, dt$$
with depreciation $\delta \geq 0$ and investment I.
- Define growth rate (per unit of capital) before depreciation as $i := I/K$, so
$$dK_t = K_t\,(i_t - \delta)\, dt.$$
- Adjustment costs $G(I, K)$ homogeneous of degree one, i.e., total costs of growth at rate i equal
$$c(i)K := I + G(I, K),$$
where c is convex, satisfying $c(0) = 0$. Often choose:
$$c(i) = i + \frac{1}{2}\theta i^2.$$

Basic Setting - Technology

- Constant returns to scale: Output is proportional to capital stock:

$$dY_t = K_t \left(dA_t - c(i_t)dt \right),$$

where dA_t denotes instantaneous productivity.

- Output is subject to stochastic productivity shocks:
  - The instantaneous productivity process satisfies

$$dA_t = \mu dt + \sigma dZ_t.$$

First-best Investment

- Abstracting from agency problem (Hayashi 1982):
  - First best investment satisfies

$$
\begin{aligned}
c'(i^{FB}) &= q^{FB} = Q^{FB}, \\
q^{FB} &= Q^{FB} = \frac{\mu - c(i^{FB})}{r + \delta - i^{FB}}.
\end{aligned}
$$

  - Without agency problem, average Q equals marginal q.
  - Assume growth condition:

$$\mu < c(r + \delta),$$

"Firm cannot profitably grow faster than the discount rate."

Basic Setting - Agency Problem

- Agency problem as in DeMarzo and Sannikov (2006):
  - Agent risk neutral with limited funds/liability and more impatient than firm owners ($\rho > r$).
  - Can take hidden action $a_t \in [0, 1]$ affecting productivity

$$dA_t = a_t \mu dt + \sigma dZ_t,$$

  - Private benefits of $\lambda (1 - a_t) \mu dt$ per unit of capital, with $\lambda \in [0, 1]$.
  - Firm owners observe K and Y and, hence, also I and A.
- In case of (inefficient) liquidation:
  - Firm owners receive lK_t, with $0 \leq l < Q^{FB}$,
  - Agent gets outside option of zero.

Main Findings

- Underinvestment relative to first best.
- History dependent wedge between marginal q and average Q.
- Investment positively correlated with (note: time-invariant investment opportunities!):
  - profits,
  - past investment,
  - financial slack ("maximal cash flow shock that can be sustained without termination").
- Investment decreases with firm-specific risk (note: risk neutral investors and manager!).
- Controlling for average Q, *financial slack* predicts investment.

Contracting Problem

- Principal offers a contract Φ specifying, based on past performance (A):
  - stopping time $\tau \geq 0$,
  - cumulative payments $\{C_t, t \leq \tau\}$,
  - investment policy $\{I_t, t \leq \tau\}$.
- Given Φ, the agent chooses $\{a_t, t \leq \tau\}$ to solve

$$W(\Phi) = \max_{a} E^a \left[\int_0^\tau e^{-\rho t} \left(dC_t + \lambda \left(1 - a_t\right) \mu K_t dt \right) \right].$$

- Firm owners choose (Φ, a) to solve

$$\begin{aligned} F(K_0, W_0) &= \max_{\Phi, a} E^a \left[\int_0^\tau e^{-rt} \left(dY_t - dC_t \right) + e^{-r\tau} l K_\tau \right], \\ s.t. W(\Phi) &= W_0, \ (\Phi, a) \ \textit{is incentive compatible}. \end{aligned}$$

Agent's Continuation Value and Incentive Compatibility

- Focus on incentive compatible contract that induces $a_t = 1 \; \forall t$.
- As in DeMarzo and Sannikov (2006), the agent's continuation value in any incentive compatible contract evolves according to

$$dW_t = \rho W_t dt - dC_t + \Gamma_t K_t \underbrace{(dA_t - \mu dt)}_{=\sigma dZ_t}.$$

- High effort ($a_t = 1 \; \forall t$) is incentive compatible iff

$$\Gamma_t \geq \lambda.$$

- Intuition: If the agent shirks,
    - he enjoys a private benefit of $\lambda (1 - a_t) \mu K_t dt$,
    - and his continuation value is reduced by $\Gamma_t (1 - a_t) \mu K_t dt$.
- IC will bind in optimal contract, i.e., $\Gamma_t = \lambda \; \forall t$.

Derivation of Principal's Value Function

- Denote the highest profit that the principal can obtain given current firm size K and promised wealth to the agent W, by $F(K, W)$.
- $F(K, W)$ is homogeneous in K due to scale invariance of technology:

$$F(K, W) = KF(1, \frac{W}{K}) = Kf(w).$$

- Some properties:
  - Scaled value function $f(w)$ is concave in w,
  - Possibility to compensate cash, hence, $f'(w) \geq -1$,
  - Payment threshold $\overline{w}$: Defer payments as long as $w \leq \overline{w}$, pay cash for $w > \overline{w}$.

Size Adjusted Value Function

- From dynamics of W_t and K_t, evolution of w_t on $[0, \overline{w}]$ is given by

$$dw_t = (\rho - (i_t - \delta)) w_t dt + \lambda \sigma dZ_t.$$

- The scaled value function has to satisfy the HJB equation

$$rf(w) = \sup_i \left\{ \begin{array}{c} \underbrace{(\mu - c(i))}_{\textit{instantaneous cf}} + \underbrace{(i - \delta) f(w)}_{\textit{growth}} \\ + \underbrace{(\rho - (i - \delta)) wf'(w) + \frac{1}{2}\lambda^2 \sigma^2 f''(w)}_{\textit{change in value } E[df]} \end{array} \right\}.$$

with the usual boundary conditions

$$f(0) = l,\ f'(\overline{w}) = -1,\ f''(\overline{w}) = 0.$$

- Liquidation is inefficient as $l < Q^{FB}$:

$$f(w) < f^{FB}(w) = Q^{FB} - w.$$

Optimal Investment

- FOC for investment in HJB shows history dependence

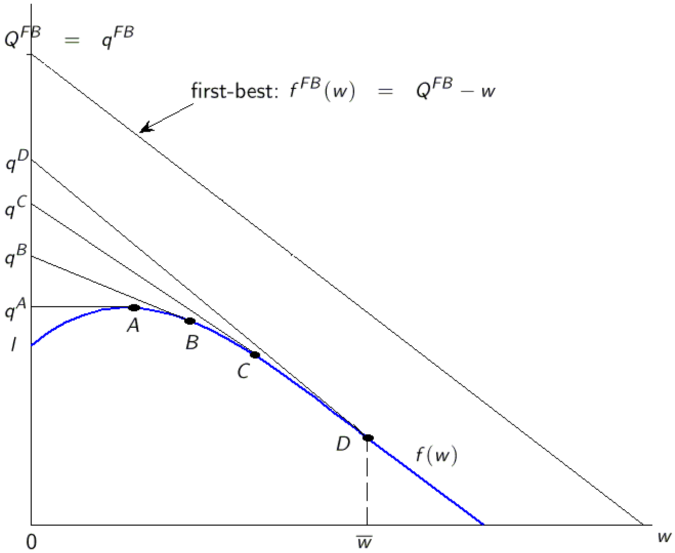
$$\begin{aligned} c'(i) &= f(w) - wf'(w) \\ &= F_K(K, W) = \frac{\partial}{\partial K}\left(KF(1, \frac{W}{K})\right) = q. \end{aligned}$$

- Intuition: "Marginal costs of investment $c'(i)$ equals current per unit value of investment to firm owners $f(w)$ plus the marginal effect of decreasing the agent's per unit payoff w as the firm grows."
- Investment dynamics:

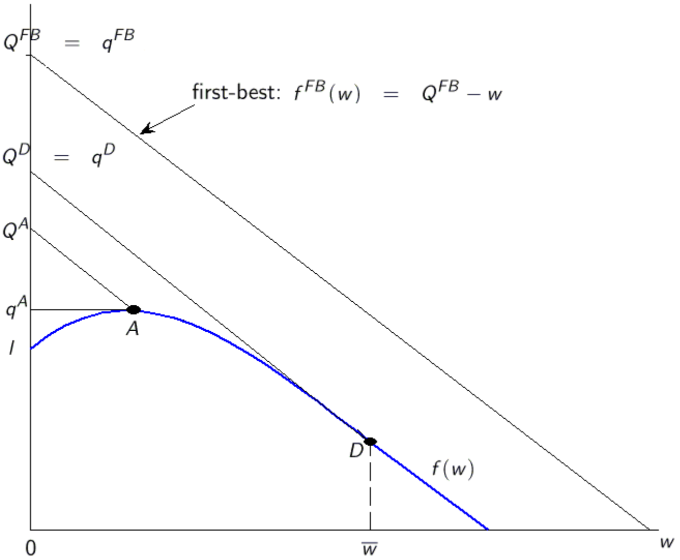
$$i'(w) = -\frac{wf''(w)}{c''(i(w))} \geq 0.$$

- Intuition: In case of high performance:
    - agent gets rewarded
    - his stake in the firm (w) increases
    - this relaxes IC constraint
    - raises the value of investing in more capital.

Optimal Investment



Marginal q and Average Q



Main Findings

- Underinvestment relative to first best.
- History dependent wedge between marginal q and average Q.
- Investment history dependent and positively correlated with w ("financial slack"), past profits and past investment (w persistent).
- Investment decreases with firm-specific risk (comparative statics wrt $\lambda\sigma$) as provision of incentives becomes more costly.
- Controlling for Q, financial slack predicts investment.

Structural Estimation

- Nikolov and Schmid, 2012, "Testing Dynamic Agency Theory via Structural Estimation."
- Use implementation of optimal contract for quadratic adjustment costs with cash reserves, equity and long term debt for structural estimation.
- Dataset with almost 2000 firms (non-financial and non regulated) over period 1992 to 2010.
- SMM approach, matching simulated and actual moments of distribution of cash, investment, leverage and Tobin's Q.
- Parameters estimated using SMM: λ, ρ, μ, σ, δ and θ ($c(i) = i + \frac{1}{2}\theta i^2$).
- Remaining parameters:
  - Interest rate r estimated as average of one-year T-bill rate over sample period.
  - Liquidation value: $l = (Tangibility + Cash) / Total\ Assets$.

Structural Estimation - Results

- Reasonable matches for first moments and serial correlation, volatility usually too low.
- Good matches for:
  - level of cash,
  - investment,
- Bad results for:
  - leverage (model-implied leverage too high as in many dynamic capital structure models based on tax advantage of debt),
  - average Q (model-implied Q too low).
  - Authors suggest that including macroeconomic conditions may provide better results.

Structural Estimation - Results

Panel A: Moments		
	Actual moments	Simulated moments
Average cash	0.1308	0.1313
Variance of cash	0.0047	0.0004
Serial correlation of cash	0.9177	0.7675
Average investment	0.1192	0.1005
Variance of investment	0.0056	0.0002
Serial correlation of investment	0.5662	0.6827
Average Tobin's q	1.9324	1.2301
Serial correlation of Tobin's q	0.7562	0.4841
Average Leverage	0.3463	0.6521
Serial correlation of Leverage	0.8947	0.7792
Covariance of cash and investment	0.0005	0.0002

Structural Estimation - Results

- All parameter estimates are significant.
- Agency parameter $\lambda = 0.423$, i.e., quite substantial.
- Estimated idiosyncratic volatility $\sigma = 0.089$ rather low.
- Estimates for $\theta = 2.219$ and $\delta = 0.152$ are in the range of more direct empirical approaches.
- Managers' discount rate $\rho = 0.045$ rather high compared to investors' rate $r = 0.035$.
- Expected productivity estimated at $\mu = 0.22$.

Extensions and Applications III

Hoffmann and Pfeil (2010): Reward for Luck in a Dynamic Agency Model

The Reward for Luck Puzzle

- Real world compensation contracts fail to filter out exogenous shocks to firm value.
  - Bertrand and Mullainathan 2001, QJE:
    Oil price shocks affect income of CEOs in the oil industry.
  - Jenter and Kanaan 2008:
    CEO replacement caused by negative exogenous shocks.
- Why? – potentially costly to impose more risk on agent, but no incentive effects.
  - Holmström 1979 Bell Journal of Economics:
    "sufficient statistics result".
  - Johnson and Tian 2000, JFE:
    Indexed executive stock options more efficient.
  - "Traditional" explanation: Managerial power approach.
- In Dynamic Context: Optimal to reward agent for "lucky" shocks that are informative about future profitability.

Hoffmann and Pfeil (2010) – "Reward for Luck"

Basic setting is similar to DeMarzo and Sannikov (2006):

- Time is continuous with $t \in [0, \infty)$,
- risk-neutral principal with discount rate r,
- risk-neutral agent with discount rate $\rho > r$,
- agent has limited liability and limited wealth, so principal has to cover operating losses and initial set up costs K.

BUT:

- Drift rate of cash flows is subject to persistent, **exogenous** "lucky" shocks.

Cash Flow Process

- Agent's hidden action A_t affects instantaneous cash flows

$$d\hat{Y}_t = (\mu_t - A_t)\,dt + \sigma dZ_t,$$

- the **observable** drift rate μ_t is subject to Poisson shocks with intensity ν:

$$d\mu_t = dN_t.$$

- For simplicity, we stop the μ-process after the first shock has occurred:

$$\text{in any instant } [t, t+dt]: \qquad \mu^l \begin{array}{c} \nearrow \\ \rightarrow \end{array} \begin{array}{l} \mu^h \text{ with probability } \nu dt \\ \\ \mu^l \text{ with probability } 1 - \nu dt \end{array}$$

The Principal's Problem

- Find the profit-maximizing full commitment contract at $t = 0$
- A contract specifies cash payments to the agent $C = \{C_t, t \geq 0\}$ and a stopping time $\tau \geq 0$ when the firm is liquidated and the receives scrap value L, to maximize principal's profit

$$E\left[\int_0^\tau e^{-rt}\left(\mu_t dt - dC_t\right) + e^{-r\tau} L\right],$$

- subject to delivering the agent an initial value of W_0

$$W_0 = E^{A=0}\left[\int_0^\tau e^{-\rho t} dC_t + e^{-\rho\tau} R\right],$$

- and incentive compatibility

$$W_0 \geq E^{\tilde{A}}\left[\int_0^\tau e^{-\rho t}\left(dC_t + \lambda \tilde{A}_t dt\right) + e^{-\rho\tau} R\right], \text{ given } \tilde{A} \geq 0.$$

Model Solution After a Shock Has Occurred

- Since there are no further shocks, the after-jump scenario is identical to DeMarzo and Sannikov (2006).
- The agent's continuation value W_t follows

$$dW_t = \rho W_t dt - dC_t + \lambda\sigma dZ_t,$$

- for $W_t \in [R, \overline{W}^h]$ the principal's value function $F^h(W) := F\left(\mu^h, W\right)$ satisfies

$$rF^h = \mu^h + \rho W_t F^h_W + \frac{1}{2}\lambda^2\sigma^2 F^h_{WW},$$

  with boundary conditions $F^h(R) = L$ and cash payments reflecting W_t at $\overline{W}^h$, where

$$\begin{aligned} F^h_W(\overline{W}^h) &= -1 \\ F^h_{WW}(\overline{W}^h) &= 0. \end{aligned}$$

Model Solution Prior to the Shock – Timing

- With Poisson shocks, the timing in any instant $[t, t + dt]$ matters.
- This differs from the pure diffusion setting, where all processes had continuous paths.
- Sequence of events:

1. The agent takes his action A_t
   (A is predictable with respect to the filtration generated by (Z, μ)).
2. There is a one-off shock to drift rate μ_t with probability νdt.
3. The agent receives cash payment $dC_t \geq 0$
   (C is adapted to to the filtration generated by (Y, μ)).
4. The principal decides whether to terminate the project
   (τ is a (Y, μ)-measurable stopping time).

Evolution of Agent's Continuation Value W

- Again we define the t-expectation of the agent's lifetime utility under $A = 0$:
$$V_t = \int_0^t e^{-\rho s} dC_s + e^{-\rho t} W_t,$$
which, by MRT, can be written as
$$V_t = V_0 + \int_0^t e^{-\rho s} \Gamma_s \left(d\hat{Y}_s - \mu_s ds \right) + \int_0^t e^{-\rho s} \Psi_s \big(dN_s - \nu ds \big).$$
- Recall that $P\left(dN_t = 1\right) = \nu dt$, so that $E\left[dN_t - \nu dt\right] = 0$.
- Differentiating the two expressions for V yields the evolution of W:
$$dW_t = \rho W_t dt - dC_t + \Gamma_t \sigma \left(d\hat{Y}_t - \mu_t dt \right) + \Psi_t \big(dN_t - \nu dt \big).$$

Incentive Compatibility Constraint

- The agent's continuation value evolves acording to:

$$dW_t = \rho W_t dt - dC_t + \Gamma_t \sigma \left(d\hat{Y}_t - \mu_t dt \right) + \Psi_t \left(dN_t - \nu dt \right).$$

- *Truth-telling ($A_t = 0$ for $t \geq 0$) is incentive compatible iff*

$$\Gamma_t \geq \lambda, \text{ for } t \geq 0.$$

- Note in particular, that Ψ does not matter for incentive compatibility.

Left Limit of the Agent's Continuation Value

- To apply MRT when we have Poisson shocks (jumps), the sensitivities Γ and Ψ have to be predictable.
- Intuitively: Ψ_t denotes the agent's reward in case there is a shock in t, BUT the size of Ψ_t must not depend on whether there is a shock in t.
- For the recursive representation of the model we want to express them as deterministic functions of the state variable.
  - But W_t is not predictable wrt the filtration generated by N (W_t jumps up by Ψ_t if $dN_t = 1$).

  $\longrightarrow$ Use the left hand limit of the agent's continuation value (which is predictable) as state variable:

$$W_{t^-} := \lim_{s \uparrow t} W_s.$$

- Intuitively, it is reflected in W_t whether a drift rate shock occurred in t, while W_{t^-} denotes the agent's continuation value before this uncertainty is resolved.

Derivation of the HJB for the Principal's Value Function

- As before, the agent will receive cash payments at $\overline{W}^l$, where

$$\begin{aligned} F_W^l(\overline{W}^l) &= -1, \\ F_{WW}^l(\overline{W}^l) &= 0. \end{aligned}$$

- The principal's value function has to satisfy the HJB equation

$$\underbrace{rF^l(W)\,dt}_{\text{required return}} = E\Big[\underbrace{\mu^l dt}_{\text{cash flow}} + \underbrace{dF^l(W)}_{\text{change in value}}\Big].$$

- What is the change in value $dF^l(W)$ when there are jumps in W?

Method: Change in Variables Formula for Jump Processes

- *Assume that the process X follows*

$$dX_t = \alpha_t dt + \beta_t dZ_t + \pi_t dN_t,$$

with α, β, and π predictable processes and let $f(X_{t^-})$ be a twice continuously differentiable function. Then it holds that

$$\begin{aligned} df(X_{t^-}) &= \left[\alpha_t \frac{\partial f}{\partial X} + \frac{1}{2}\beta_t^2 \frac{\partial^2 f}{\partial X^2}\right] dt + \beta_t \frac{\partial f}{\partial X} dZ_t \\ &\quad + \Big[f\left(X_{t^-} + \pi_t\right) - f\left(X_{t^-}\right) \Big] dN_t \end{aligned}$$

Exercise: Apply change in variables formula to derive the differential of F^I

The HJB for the Principal's Value Function Before a Shock

- Substituting dF^l, we find that for $W \in [R, \overline{W}^l]$, the principal's value function prior to a drift rate shock has to satisfy the HJB

$$\begin{aligned} rF^l(W) &= \mu^l + (\rho W - \nu\psi) F^l_W(W) + \frac{1}{2}\lambda^2\sigma^2 F^l_{WW}(W) \\ &\quad + \nu\left[F^h(W+\psi) - F^l(W)\right], \end{aligned}$$

  with the usual boundary conditions

$$\begin{aligned} F^l(R) &= L, \\ F^l_W(\overline{W}^l) &= -1, \\ F^l_{WW}(\overline{W}^l) &= 0. \end{aligned}$$

- After a jump in μ, the contract is replaced by optimal after-shock contract with starting value $W + \psi$.

The Optimal Response to "Luck" Shocks

- Recall that the sensitivities ψ does not have any incentive effects.
- Still it is optimal because of efficiency reasons to set $\psi > 0$.
- Why is that the case? Recall the fundamental trade off:
  - Because of limited liability, the project has to be shut down when $W = 0$ and the principal foregoes all future cash flows of the project.
  - Postponing the agent's pay is costly, as the agent is more impatient than the principal.
- An increase in μ means that the principal looses higher cash flows if the project is shut down

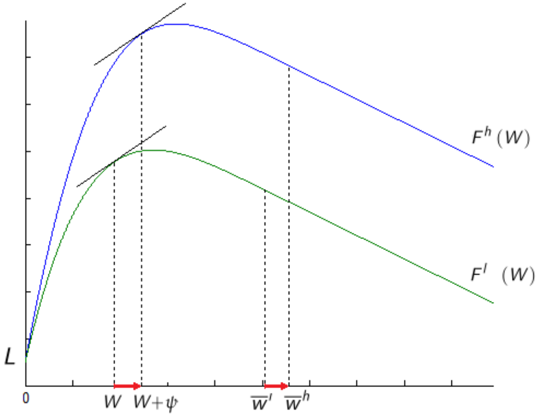
$\Longrightarrow$ Termination becomes "more costly" when μ jumps up.

$\Longrightarrow$ Optimal to raise W in response to a shock, making termination less likely when it is more costly.

The Optimal Response to "Luck" Shocks

- Differentiating HJB on the last slide w.r.t.ψ yields first order condition

$$F_W^h\left(W+\psi\right)=F_W^l\left(W\right).$$



The Optimal Response to "Luck" Shocks

- *For* $W \in [R, \overline{W}^l]$ *it holds that* $\psi(W) > 0$.
- Idea of the proof:

1. From $F^h(R) = F^l(R) = L$ and $F^h(W) > F^l(W)$ for $W > R$ it follows that $\psi(x) > 0$ for $x \in [R, R + \varepsilon]$.
2. Show that ψ has an interior minimum (i.e. $\psi'(y) = 0$ and $\psi''(y) > 0$), then $\psi(y) \geq 0$.
3. Show that $\overline{W}^h > \overline{W}^l$, implying that $\psi(\overline{W}^l) > 0$.

- Therefore, ψ can never turn negative over the whole range $[R, \overline{W}^l]$.

Extensions and Applications IV

Hoffmann and Pfeil (2012): Delegated Investment in a Dynamic Agency Model

Hoffmann and Pfeil (2012)

- Managers have to take care of day-to-day business:
  - Managerial effort: Sannikov (2007),
  - cash flow diversion: Biais et al. (2007), DeMarzo & Sannikov (2006).
- But also have to take strategic actions to increase future profitability.
- This paper: Optimal dynamic contract when manager can take two hidden actions:
  a) Diversion of funds for own consumption (transitory, short-term action).
  b) Allocation of funds inside the firm: investment in future profitability (persistent, long-term action).

Investment Technology

- Investment as the choice of absorptive capacity:

  "A firm's capability of assimilating new, external information and apply it to commercial ends."

  (Cohen & Levinthal 1990, Board & Meyer ter Vehn 2010)

- Unpredictable technology shocks: availability of a new technology:
  - If firm is able to adopt new technology "investment success" $\rightarrow$ high future profitability.
  - If firm can not adopt new technology "investment failure" $\rightarrow$ low future profitability.

- Probability that firm is able to adopt new technology increases with investment (absorptive capacity).

Interaction Between the two Problems

- Cash flow diversion problem (à la DeMarzo & Sannikov 2006).
- Contract ties agent's compensation to cash flow reports to induce truthtelling.
- Aggravates investment problem: Incentives to (mis)use funds and inflate cash flow reports instead of investing.
- Contract ties agent's compensation also to investment outcome which creates "agency costs of investment".

Cash Flow Process

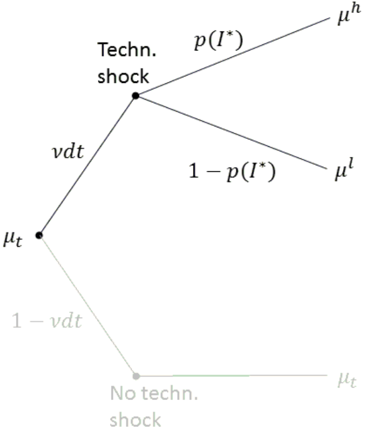
- Firm's cash flows net of investment I_t are given by

$$dY_t = (\mu_t - I_t)dt + \sigma dZ_t.$$

- The principal cannot observe cash flows dY_t, but has to rely on the agent's report $d\hat{Y}_t$.
- Agent controls investment process I_t, which is also not observed by principal.
- The evolution $d\mu_t$ depends stochastically on the agent's investment choice I_t.

Evolution of Profitability

- In any instant $[t, t+dt]$ there is a technology shock w.p. νdt
- If there is no shock, profitability remains unchanged at μ_t
- If there is a shock,
- $\mu_{t^+} = \mu^h$ w.p. $p(I^*)$ (Investment success).
- $\mu_{t^+} = \mu^l$ w.p. $1 - p(I^*)$ (Investment failure).



Evolution of Profitability

- Note symmetry: In any instant, independent of current state μ_t,
  - an investment **success** occurs with probability
    $$\nu p\left(I_t\right) dt,$$
  - and an investment **failure** with probability
    $$\nu\left(1-p\left(I_t\right)\right) dt.$$
- First best investment is given by
  $$\nu p'\left(I^{FB}\right) \underbrace{\frac{1}{r+\nu}\left(\mu^h-\mu^l\right)}_{:=\Delta}=1,$$
- as μ stays constant between shocks
  $\rightarrow$ persistent effect of investment (as shocks are infrequent).

Agent's Problem

- Agent can consume only fraction $\lambda \in (0, 1]$ of diverted cash flows.
- Risk-neutral agent is protected by limited liability and discounts at rate ρ.
- Given a long-term contract $\{U, \tau\}$, agent chooses strategy $S = \{\hat{Y}, I\}$ to maximize his future income

$$W_0 = E\left[\int_0^\tau e^{-\rho t}\left(dC_t + \lambda\left(dY_t - d\hat{Y}_t\right)\right)\right].$$

Principal's Problem

- Risk-neutral principal discounts at rate $r < \rho$.
- Principal offers a long-term contract $\{C, \tau\}$ with $dC \geq 0$.
- And specifies a recommended strategy $S^* = \{\hat{Y}^*, I^*\}$ to maximize his expected profits until replacement in τ

$$F_0 = E\left[\int_0^\tau e^{-\rho t}\left(d\hat{Y}_t^* - dC_t\right) + e^{-r\tau} L_\tau\right].$$

- Recommended strategy S^* is incentive compatible if it maximizes W_0.
- Revelation principle $\Longrightarrow$ Optimal to implement truth-telling: $\hat{Y}^* = Y$.

Agent's Continuation Value and Incentives

- If the agent follows S^*, his continuation value follows

$$\begin{aligned} dW_t &= \rho W_t dt - dC_t + \Gamma_t \Big[d\hat{Y}_t - (\mu_t - I_t^*)\, dt \Big] \\ &\quad + \Psi_t^g \Big[dN_t^g - \nu p\,(I_t^*)\, dt \Big] \\ &\quad + \Psi_t^b \Big[dN_t^b - \nu\,(1 - p\,(I_t^*))\, dt \Big]. \end{aligned}$$

1. If agent would divert cash flows
   - Immediate consumption: $\lambda(dY_t - d\hat{Y}_t)$,
   - reduction of future income: $\Gamma_t(dY_t - d\hat{Y}_t)$.

- No incentives to divert cash flows if

$$\Gamma_t \geq \lambda.$$

Agent's Continuation Value and Incentives

- If the agent follows S^*, his continuation value follows

$$\begin{aligned} dW_t &= \rho W_t dt - dC_t + \Gamma_t \Big[d\hat{Y}_t - (\mu_t - I_t^*)\, dt \Big] \\ &\quad + \Psi_t^g \Big[dN_t^g - \nu p\,(I_t^*)\, dt \Big] \\ &\quad + \Psi_t^b \Big[dN_t^b - \nu\,(1 - p\,(I_t^*))\, dt \Big]. \end{aligned}$$

2. Given $\Gamma_t \geq \lambda$, if agent would reduce I_t marginally below I_t^* would lead to
    - an increases in cash flows $d\hat{Y}_t \to W$ grows by Γ_t,
    - a reduction of success Prob: $\nu p'\,(I_t^*)$,
    - an increase of failure Prob: $\nu p'\,(I_t^*)$.

- No incentives to **decrease** I_t below I_t^* if

$$\Gamma_t \leq \nu p'\,(I_t^*) \left(\Psi_t^g - \Psi_t^b \right).$$

Agent's Continuation Value and Incentives

- If the agent follows S^*, his continuation value follows

$$\begin{aligned} dW_t &= \rho W_t dt - dC_t + \Gamma_t \left[d\hat{Y}_t - (\mu_t - I_t^*)\, dt \right] \\ &\quad + \Psi_t^g \left[dN_t^g - \nu p\,(I_t^*)\, dt \right] \\ &\quad + \Psi_t^b \left[dN_t^b - \nu\,(1 - p\,(I_t^*))\, dt \right]. \end{aligned}$$

3. Increasing I_t above I_t^*: analogous but with opposite signs.

- No incentives to **increase** I_t above I_t^* if

$$\Gamma_t \geq \nu p'\,(I_t^*) \left(\Psi_t^g - \Psi_t^b \right).$$

Local Incentive Compatibility

- To induce truth-telling: Tie compensation sufficiently strong to cash flow reports

$$\Gamma_t \geq \lambda.$$

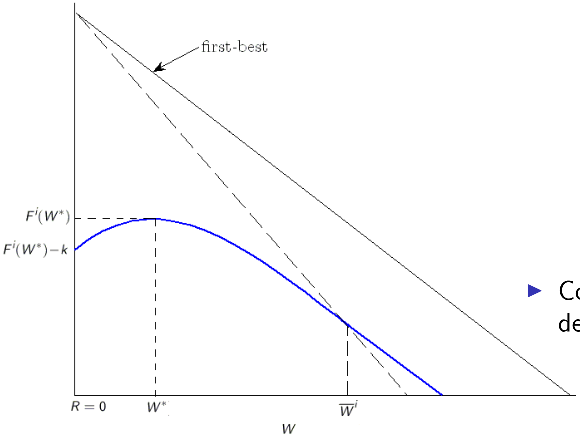
- To induce investment according to I^*: Balance incentives based on investment outcome with incentives based on cash flow reports

$$\Psi_t^g - \Psi_t^b = \frac{\Gamma_t}{\nu p'\left(I_t^*\right)}.$$

- Limited liability requires that for all t,

$$\Psi_t^i \geq -W_t.$$

Principal's Value Function



- If agent is fired, principal has to find a new agent:
  - Search costs k,
  - contract with new agent starts at $F(W^*)$.
  - Lower boundary condition becomes

$$F^i(R) = F^i(W^*) - k.$$

- Compensation threshold is determined as usual

$$\begin{aligned} F^i_W\left(\overline{W}^i\right) &= -1, \\ F^i_{WW}\left(\overline{W}^i\right) &= 0. \end{aligned}$$

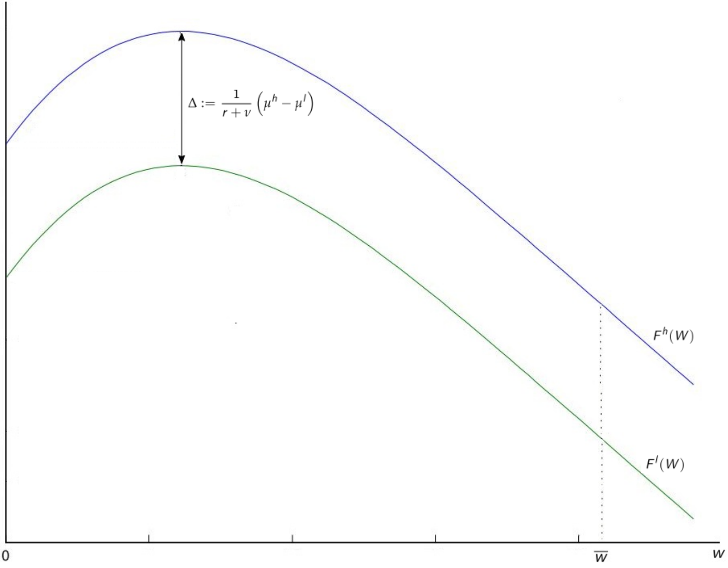
Principal's Value Function

- Applying the change of variable formula (noting that $dN^g dN^b = 0$), the principal's value function satisfies the **coupled** HJB

$$
\begin{aligned}
rF^i &= \mu^i - I + \left[\rho W - \nu p(I)\,\psi^g - \nu\,(1-p(I))\,\psi^b\right] F^i_W + \frac{1}{2}\lambda^2\sigma^2 F^i_{WW} \\
&\quad + \nu\left[F^h(W+\psi^g) - F^i(W)\right] + \nu\left[F^l(W+\psi^b) - F^i(W)\right].
\end{aligned}
$$

- Note that all key parameters are **independent** of the state μ^i:
  - Investment technology with
    - marginal benefits $\nu p' \Delta$,
    - marginal costs -1.
  - Underlying agency problem with
    - shirking benefits λ,
    - discount rates r and ρ,
    - replacement costs k.

Parallel Shift of the Value Function(s)



Define $F(W) := F^l(W)$ and use that $F^h = F(W) + \Delta$

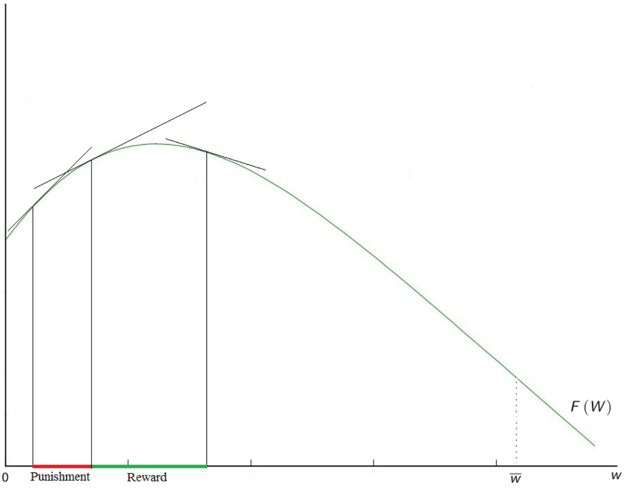
Costs of Rewards and Punishments

- Contract optimally trades off costs of replacement (k) and costs from paying the agent in the future ($\rho > r$).
  - By the same logic as in *Reward for Luck*, it would be optimal to keep marginal costs of compensation $F_W(W)$ constant if there is a technology shock.
  - With parallel value functions, this would imply to keep W constant, that is,
$$\Psi^i = 0.$$
- However, by Incentive compatibility, the agent has to be rewarded for a success and punished for a failure:
$$\Psi^g > 0 > \Psi^b.$$

Costs of Rewards and Punishments

- Contract strikes optimal trade off between costs of replacement (k) and costs from paying the agent in the future ($\rho > r$).
- Rewarding agent by $\Psi^g > 0$ for success distorts optimal trade off:
  - Too **high** future pay and too **low** firing threat after success.
- Analogous distortion from punishing agent by $\Psi^b < 0$ for failure:
  - Too **low** future pay and too **high** firing threat after failure.

Costs of Rewards and Punishments



No "smoothing": $F_W(W+\psi^b) > F_W(W) > F_W(W+\psi^g)$

Optimal Rewards and Punishments

- Providing incentives for investment implies that marginal compensation costs can not be kept constant in the event of a technology shock.
- The best that can be achieved is to keep marginal costs constant in expectation (keep expected distortion equal to zero)

$$0 = p(I) \underbrace{[F_W(W + \psi^g) - F_W(W)]}_{\text{distortion after success}} + [1 - p(I)] \underbrace{[F_W(W + \psi^b) - F_W(W)]}_{\text{distortion after failure}}.$$

- $\psi^g \to 0$ if $p(I) \to 1$ and $\psi^b \to 0$ if $p(I) \to 0$.
- $\psi^b \to 0$ if $w \to 0$ and $\psi^b \to 0$ if $W \to \overline{W}$.

Optimal Investment

- Optimal Investment is determined by FOC

$$\nu p'(I)\,\Delta - 1 - MAC(I) = 0$$

where $MAC(I)$ consist of

1. Due to Incentive compatibility $\psi^g - \psi^b$ has to increase

$$\left(\frac{\lambda}{\nu}\frac{-p''(I)}{p'(I)^2}\right) p(I)\,(1-p(I))\,\nu\left[F_W(W+\psi^b) - F_W(W+\psi^g)\right] \geq 0$$

This term vanishes for $p(I) \to 1$ and for $p(I) \to 0$

Optimal Investment

- Optimal Investment is determined by FOC

$$\nu p'(I)\,\Delta - 1 - MAC(I) = 0,$$

where $MAC(I)$ consist of:

2. Investment **success** triggering **reward** becomes **more** likely

$$\nu p'(I)\underbrace{\Big[F(W) + \psi^{g} F_{W}(W) - F(W + \psi^{g})\Big]}_{\rightarrow 0 \text{ for } p(I)\rightarrow 1} \geq 0.$$

3. Investment **failure** triggering **punishment** becomes **less** likely

$$-\nu p'(I)\underbrace{\Big[F(W) + \psi^{b} F_{W}(W) - F(W + \psi^{b})\Big]}_{\rightarrow 0 \text{ for } p(I)\rightarrow 0} \leq 0.$$

Optimal Investment

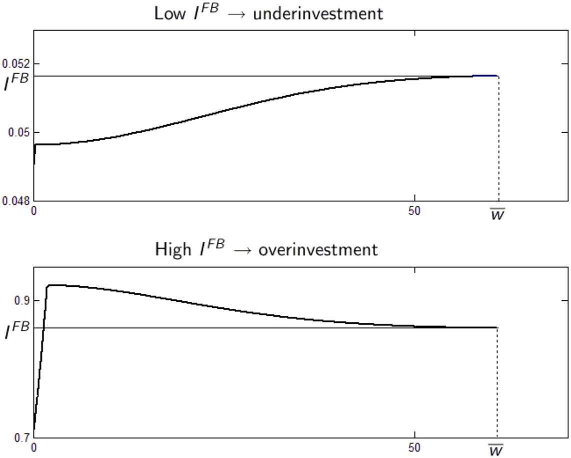
- Optimal Investment is determined by FOC

$$\nu p'(I)\,\Delta - 1 - MAC(I) = 0$$

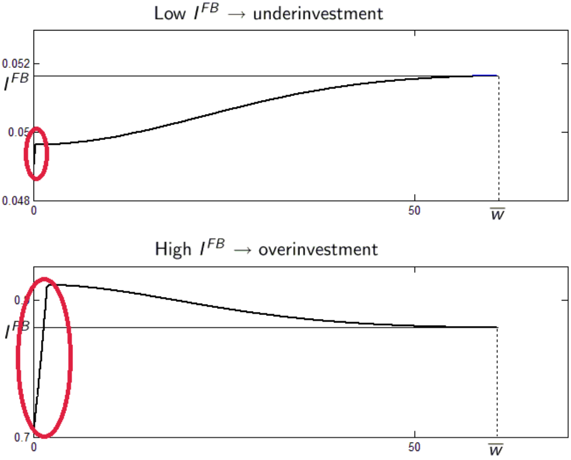
$\Longrightarrow$ $MAC(I)$ will be positive for low I and negative for high I

- If, in equilibrium,
  - $MAC(I) = 0$, then $I(W) = I^{FB}$,
  - $MAC(I) > 0$, then $I(W) < I^{FB}$,
  - $MAC(I) < 0$, then $I(W) > I^{FB}$.
- Compare situations with different returns to investment (measured by Δ).

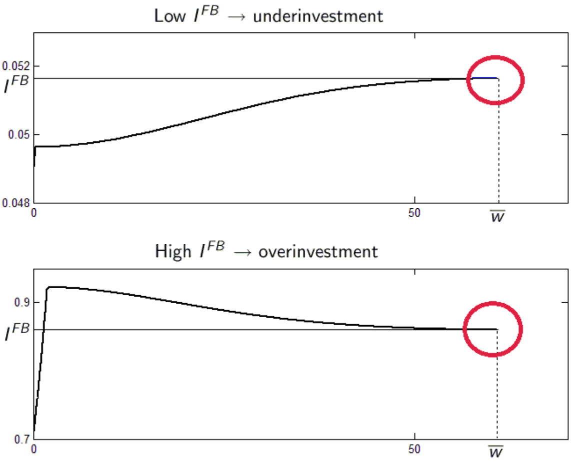
Investment Distortions



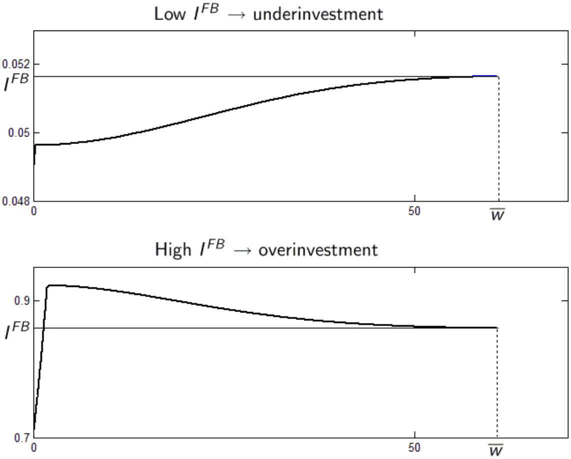
Underinvestment if Agent is Too Poor to be Punished



First-Best Investment at Payout Boundary

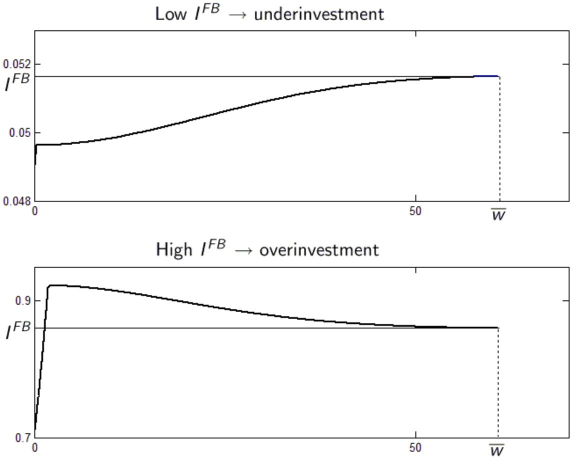


Investment Depends on Past Cash Flows



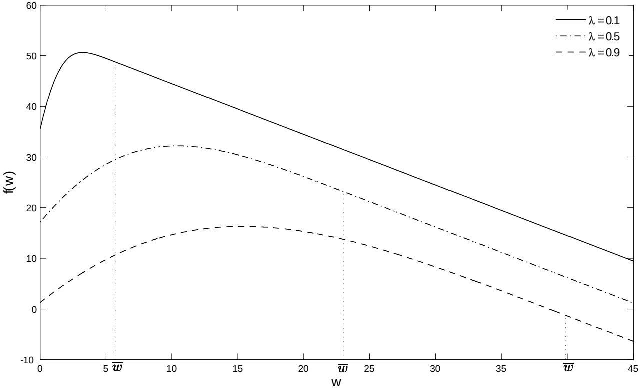
$$dW = \rho W dt + \lambda \sigma dZ + \psi^g \left[dN^g - \nu p dt \right] + \psi^b \left[dN^b - \nu \left(1 - p\right) dt \right]$$

Investment Depends on Past Investment Outcome

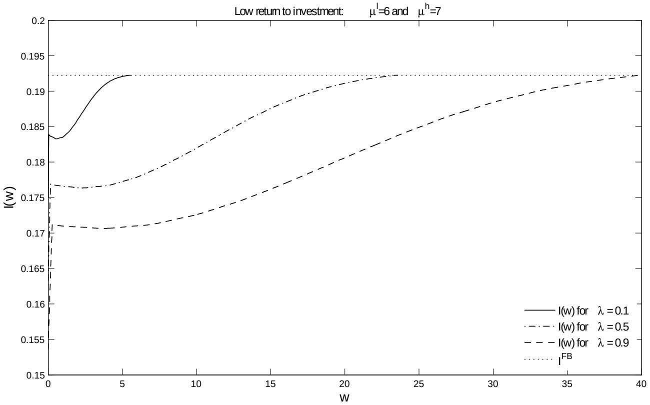


$$dW = \rho W dt + \lambda \sigma dZ + \psi^{g} \left[dN^{g} - \nu p dt \right] + \psi^{b} \left[dN^{b} - \nu \left(1 - p\right) dt \right]$$

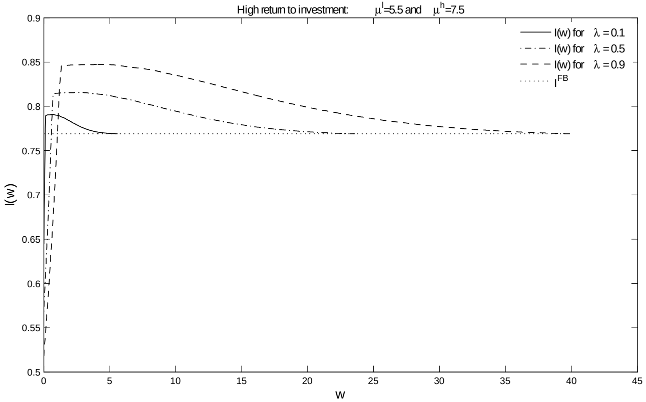
Implications of Changes in Corporate Governance



Implications of Changes in Corporate Governance



Implications of Changes in Corporate Governance



Extensions and Applications V

Piskorski and Westerfield (2011):
Optimal Dynamic Contracts with Moral Hazard and Costly Monitoring

Motivation

So far:

- Ex-post incentive mechanism in the form of managerial compensation.
- Reward or punish manager based on realized corporate performance (with predefined scheme).
- (Promised) compensation and firing threat to provide incentives.

However, investors can also decide to invest resources to actively reduce agency problems:

- Investors can monitor manager to reduce scope for shirking.
- E.g. continuous or repeated audits or direct involvement of the principal in operations.
- Active monitoring provides an additional incentive device for the principal and allows to reduce the likelihood of costly termination.

Basic Setting

Similar to DeMarzo and Sannikov (2006)

- Time is continuous with $t \in [0, \infty)$
- All players are risk-neutral
- Agent has limited liability and limited wealth, so principal has to cover operating losses and initial set up costs K

BUT additionally:

- Principal has access to a (costly and stochastic) monitoring technology allowing him to detect shirking (cf. CSV literature).

Basic Setting

- Cash flows evolve according to

$$dY_t = (\mu - a_t)\, dt + \sigma dZ_t$$

where $a_t \geq 0$ denotes the agent's shirking process

- Agent gets private benefit from harmful hidden action (diversion, asset misuse, etc.) at rate a_t ($\lambda = 1$).
- Simple monitoring technology:
  - Principal chooses level of monitoring $m_t \geq 0$ at cost θm_t,
  - Monitoring gives access to a signal N indicating whether the agent has shirked,
  - N follows a Poisson process with intensity

$$\nu(m_t, a_t - a_t^r) = m_t \max\{0, a_t - a_t^r\},$$

where a_t^r denotes the recommended level of shirking at t.

Basic Setting

- Specific monitoring technology:
  - Pay now to observe contemporaneous shirking, no "looking back",
  - Probability of detection proportional to amount of shirking and monitoring,
  - No false positives.
- Agent's outside value:
  - If fired following bad performance: R.
  - If fired following discovery of shirking through monitoring: $0 \leq W_F \leq R$.

Contracting Problem

- The principal offers the agent an incentive compatible contract specifying:
  - cash payments $\{C_t, t \geq \tau\}$,
  - recommended shirking $\{a_t^r, t \geq \tau\}$,
  - monitoring $\{m_t, t \geq \tau\}$,
  - and stopping times τ^d (under-performance) and τ^f (detection of shirking), with $\tau = \min\left\{\tau^d, \tau^f\right\}$.
- Optimal contract maximizes

$$E^{a=a^r}\left[\int_0^{\tau} e^{-rt}\left(dY_t - dC_t - \theta m_t dt\right) + e^{-r\tau} L\right],$$

where $a = \{a_t, t \geq \tau\}$ maximizes the agent's expected utility

$$E^a\left[\int_0^{\tau} e^{-\rho t}\left(dC_t + a_t dt\right) + e^{-\rho\tau^d} R + e^{-\rho\tau^f} W_F\right].$$

Agent's Continuation Value and Incentive Compatibility

- If $a = a^r$, the agent's continuation value evolves according to

$$dW_t = \underbrace{\rho W_t - dC_t - a_t^r dt}_{promise\ \ keeping} + \underbrace{\Gamma_t \left(dY_t - \left(\mu - a_t^r\right) dt\right)}_{pay\ \ for\ \ performance} - \underbrace{\Psi_t dN_t}_{punishment\ \ for\ \ shirking}.$$

  - Intensity of N_t is zero as monitoring creates no false positives.

- The contract is incentive compatible iff

$$\begin{array}{cl} \Gamma_t \geq 1 - \Psi_t m_t & if\ \ a_t^r = 0, \\ \Gamma_t \in \{1 - \Psi_t m_t, 1\} & if\ \ a_t^r > 0. \end{array}$$

- Intuition: If the agent diverts an additional amount edt,
  - he enjoys a private benefit of edt,
  - his continuation value is reduced by $e\Gamma_t dt$,
  - and expected punishment is $e\Psi_t m_t dt$.
- If he diverts edt less, he loses edt and W increases by $e\Gamma_t dt$.

Derivation of HJB for principal's value function

- The problem can be simplified by noting that:
  - Wlog we can focus on contracts with $a^r = 0$ (recommended shirking can be replaced by consumption),
  - Choose $\Psi_t = \psi(W_t) = W_t - W_F$ (punish as hard as possible, "out of equilibrium"),
  - Choose $\Gamma_t = \gamma(W_t) = 1 - \Psi_t m_t$ (minimize volatility of W_t),
  - As usual cash compensation is deferred till a threshold $\overline{W}$ is reached.
- For $W \in \left[R, \overline{W}\right]$, $F(W)$ has to satisfy the HJB equation

$$rF(W) = \max_{m \geq 0} \left\{ \mu - \theta m + \rho W F'(W) + \frac{1}{2} \underbrace{\left(1 - \left(W - W_F\right) m\right)^2}_{=\gamma(W,m)^2 = (1-\psi(W)m)^2} \sigma^2 F''(W) \right\},$$

with the usual boundary conditions.

Optimal Monitoring

- From the HJB

$$rF(W) = \max_{m \geq 0} \left\{ \mu - \theta m + \rho W F'(W) + \frac{1}{2} \left(1 - (W - W_F)\, m\right)^2 \sigma^2 F''(W) \right\},$$

the principal chooses to monitor at rate

$$m = \frac{\theta}{(W - W_F)^2\, \sigma^2 F''(W)} + \frac{1}{(W - W_F)},$$

whenever

$$F''(W) < -\frac{\theta}{\sigma^2 (W - W_F)}.$$

- So, the optimal "pay-for performance sensitivity" given by

$$\gamma(W_t) = 1 - \psi(W_t) m_t = -\frac{\theta}{(W_t - W_F)\, \sigma^2 F''(W_t)}.$$

$\rightarrow$ Monitoring allows to reduce performance based incentives and, thus, termination probability.

Optimal Monitoring

- From

$$\gamma(W_t) = 1 - \psi(W_t)m_t = -\frac{\theta}{(W_t - W_F)\,\sigma^2 F''(W_t)},$$

  we have more monitoring/less pay-for-performance if:

  - monitoring costs θ are low,
  - monitoring is effective ($W_t - W_F$ is high),
  - aversion to volatility in W_t is high ($F''(W_t)$).

- Timing and intensity of monitoring is shaped by two competing forces: "risk of termination" and the agent's "inside stake".

  - When W decreases the risk of termination increases, while the agent's inside stake decreases,
  - Quantitative assessment needed.

Optimal Monitoring

- The monitoring function $m(W)$ can have two shapes depending on the cost effectiveness of monitoring:
  1. if $\psi(R) = R - W_F$ is sufficiently high relative to θ, $m(W)$ is decreasing,
  2. if $\psi(R) = R - W_F$ is small relative to θ, $m(W)$ is hump-shaped
- The pay-for-performance sensitivity $\gamma(W)$ accordingly is either increasing or U-shaped.
- The principal will replace termination entirely by (maximal) monitoring if the costs of inefficient termination are sufficiently high:
  - $\exists L^*$ such that the contract exhibits termination iff $L > L^*$.
  - For $L < L^*$, there is full monitoring at $W = R$ $(m = 1/\psi(R))$, such that $F(W = R) = \max\{L, L^*\}$.
  - The threshold L^* is (weakly) decreasing in W_F and θ.

Comparative Statics