The Cross-Section of Risk and Returns

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Main Idea

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4. You hedge the most risk by choosing $\max_v \mathbb{C}\text{orr}[p, v^\top r]^2$ (and signing it right)

Context

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Context

1. You are given a portfolio $p = w^\top r$ with $\mathbb{E}[p] > 0$
   - These are characteristic-sorted portfolios (CPs) like HML, SMB, UMD
   - We hope to use them to span the MVE frontier
   - They seem promising because they have sizable risk premia
2. You want to hedge p's risk for cheap
   $\rightarrow$ cheap $=$ lose no expected return
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Context

1. You are given a portfolio $p = w^\top r$ with $\mathbb{E}[p] > 0$

2. You want to hedge p's risk for cheap
   $\rightarrow$ cheap $=$ lose no expected return
   - But are these CPs MVE?

3. The space of cheap portfolios is $\{v : \mathbb{E}[v^\top r] = 0\}$

4. You hedge the most risk by choosing $\max_v \mathbb{C}\text{orr}[p, v^\top r]^2$

Context

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3. The space of cheap portfolios is $\{v : \mathbb{E}[v^\top r] = 0\}$
   - If one of these portfolios is correlated with p, then p contained unpriced risk
   - If you *can* hedge their risk for cheap, then they aren't MVE!

4. You hedge the most risk by choosing $\max_v \mathbb{C}\text{orr}[p, v^\top r]^2$

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4. You hedge the most risk by choosing $\max_v \mathbb{C}\text{orr}[p, v^\top r]^2$
   - The hedged portfolio gives the same expected return for lower variance – it's closer to MVE

What was the problem?

Let X be the $(N \times M)$ matrix of the N assets' M characteristics.
Let Σ be the $(N \times N)$ return covariance matrix

Problem: CP are built looking only at X. We ignored Σ.

MVE comes from $\min_w w^\top \Sigma w$ and hence involves Σ.

Paper says: The efficient weights are

$$\Sigma^{-1} X \left(X^\top \Sigma^{-1} X \right)^{-1}$$

This leads to an issue

N is big and Σ is hard to estimate – the paper avoids the issue.

- Instead, it assumes a model where characteristics drive risk premia:

$$\mathbb{E}[r] = X\lambda_c.$$

- Then the space of cheap portfolios is

$$\{v : X^\top v = 0\},$$

- and we pick the cheap v that

$$\max_v v^\top b$$

for b the regression slope of each asset on a CP p
because this maximizes the $\mathbb{C}\text{orr}(v^\top r, p)^2$.

$v^\top b$ is our hedge portfolio

Punchline

"Our paper connects to the recent vintage of papers ...[taking] as their starting point a set of characteristics that explains average excess returns. Our focus instead is on improving the efficiency of the characteristic portfolios by using individual asset loadings on the CPs.

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Let's get into that...

Model

KPS: Conditional beta

$$r_{i,t+1} = (\beta_{i,t})^\top f_{t+1} + \epsilon_{i,t+1}$$

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$$r_{i,t+1} = (\Gamma X_{i,t})^\top f_{t+1} + \epsilon_{i,t+1}$$

Model

KPS: Conditional beta = characteristic-instrumented factor exposures

$$r_{i,t+1} = (\Gamma X_{i,t})^{\top} f_{t+1} + \epsilon_{i,t+1}$$

DMRS: "exposure to f is a linear combination of the M characteristics that describe expected excess returns"

Model

DMRS: Max $v^\top b$ for b regression coefficient of each element of r on p
$\Rightarrow$ detailed construction of b to *predict* future covariance

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$$f_{t+1} = w_t r_{t+1}$$

Model

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$\Rightarrow$ detailed construction of b to *predict* future covariance

KPS:

$$\begin{aligned} f_{t+1} &= w_t r_{t+1} \\ &= (\beta_t^\top \beta_t)^{-1} \beta_t r_{t+1} \end{aligned}$$

Model

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KPS:

$$
\begin{aligned}
f_{t+1} &= w_t r_{t+1} \\
&= (\beta_t^\top \beta_t)^{-1} \beta_t r_{t+1} \\
&= (\Gamma^\top X_t^\top X_t \Gamma)^{-1} \Gamma^\top X_t^\top r_{t+1}
\end{aligned}
$$

Model

DMRS: Max $v^\top b$ for b regression coefficient of each element of r on p
$\Rightarrow$ detailed construction of b to *predict* future covariance

KPS:

$$f_{t+1} = w_t r_{t+1}$$

$$= (\Gamma^\top X_t^\top X_t \Gamma)^{-1} \Gamma^\top X_t^\top r_{t+1}$$

$$vec(\Gamma^\top) = \left[\sum_{t=1}^{T} Y_{t+1} Y_{t+1}^\top\right]^{-1} \left[\sum_{t=1}^{T} Y_{t+1}^\top r_{t+1}\right]$$

$$\text{for } Y_{t+1} \equiv \left(X_t \otimes f_{t+1}^\top\right)$$

Weight w_t involves both characteristic information X_t and covariance information Γ

from Kelly, Moskowitz, Pruitt (2020 WP)

Predicted beta for month t	$\beta_{i,t} = X_{i,t}\Gamma$
Daily factor and stock return for days **after** month t	$r_{i,d}, f_d$
Realized beta	$RealBeta_{t+1}^{OOS} = \left(\sum_d f_d f_d^\top\right)^{-1} \sum_d f_d r_{i,d}$

Is $\beta_{i,t}$ predicting $RealBeta_{t+1}^{OOS}$?

Constant	0.00	−0.01	0.00	−0.00	−0.00
(t-stat)	(0.45)	(−2.90)	(1.19)	(−0.04)	(−1.04)
Slope	1.00	1.02	1.01	1.00	1.01
(t-stat)	(388.46)	(134.51)	(133.23)	(136.79)	(122.62)
$[t : \beta = 1]$	[−0.25]	[2.20]	[1.22]	[0.16]	[1.03]
R^2 (%)	25.86	7.15	4.97	7.25	4.38

Standard errors clustered by month and firm. Usual t-statistics (of the null that the parameter equals zero) are reported in parentheses. For slope coefficients, we also report in rows labeled "$[t : \beta = 1]$" t-statistics of the null that the parameter equals 1.

Seem to be capturing that future covariance information, not ignoring it

The regimen lives!

- KPS:
  - Given characteristics,
  - find f that maximally explain $\mathbb{V}(r)$,
  - then hope (by an APT logic) that they also explain $\mathbb{E}(r)$.
- DMRS:
  - Given f,
  - find a maximally-correlated hedge that has $\mathbb{E}(h) = 0$,
  - and combine it with f to get closer to MVE.

Add together

- The DMRS insight applies: for CPs but also moment-based estimators like KPS
- Additional moment restriction to be used

Conclusion

- Empirically: their hedged portfolios are closer to MVE than CPs
- Theoretically: a factor improvement regimen

Should be quite influential