The Cross-Section of Risk and Return

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Introduction

- Multifactor models of the sdf posit that:

$$m^* = a + \mathbf{b}'\mathbf{f}^* \quad \text{with} \quad \mathbb{E}[m^* r_i] = 0$$

  for *any* excess return r_i and a set of traded "factors" $\mathbf{f}^*$ that span the MVE portfolio.

- implying that

$$\mathbb{E}[r_i] = \boldsymbol{\beta}_i \boldsymbol{\lambda}$$

  where $\boldsymbol{\lambda}$ is the price of risk, and $\boldsymbol{\beta}_i$ is (the vector of) projection coefficients of r_i onto $\mathbf{f}^*$.

- …which is motivation for time series regressions like:

$$(R_{i,t} - R_{f,t}) = \alpha_i + \beta_{i,m} \cdot (R_{m,t} - R_{f,t}) + \beta_{i,SMB} \cdot \text{SMB}_t + \beta_{i,HML} \cdot \text{HML}_t + \epsilon_t$$

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Search for $\mathbf{f}^*$ in in the Space of Returns

- Search in the space of returns for $\mathbf{f}^*$. But how?
- Timeline:
  1. Chen, Roll, and Ross (1986) economic factors:
     - Evidence of that there were premia associanted with innovations in macroeconomic variables, but the Sharpe ratios associated with these portfolios were small.
  2. Connor and Korajczyk (1988) statistical factors using PCA:
     - effective in explaining the covariance structure, but all but the first PC—which looks like the market—did not carry much of a premium.
  3. Fama and French (1993) characteristic sorted portfolios:
     - "The 3-factor model does a good job in explaining the cross-section of average returns."

CPs (Characteristic Portfolios)

- As in Fama and French (1993), sorting on characteristics to form *characteristic portfolios* (CPs) has become standard in the empirical asset pricing literature.
- That is, find a characteristic that is associated with expected returns, e.g. book-to-market, and create a corresponding characteristic portfolio by sorting on this characteristic.
  - The resulting characteristic portfolio goes long high- and short low-characteristic stocks.
- *Examples:* SMB, HML, RMW, CMA; UMD; WML; LIQ; ISU; QMJ, etc.
  - Fama and French (1993, 2015); Carhart (1997); Daniel and Moskowitz (2016); Pástor and Stambaugh (2003); Daniel and Titman (2006); Asness, Frazzini, and Pedersen (2013); Lustig, Roussanov, and Verdelhan (2011)

CPs are inefficient

- PCA ignores information about expected returns that comes from characteristics
- Characteristic sorts ignore information about the covariance structure that come historical individual firm's return covariances.

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Can characteristic portfolios be improved?

- These characteristic portfolios can only explain the cross-section of returns if they span the mean variance efficient (MVE) portfolio
- We argue that characteristics are likely to be correlated with *un*priced factor risk
  - implying that the CPs will inefficient
  - ie., they won't span the MVE portfolio, or price the cross-section of average returns.
- We propose a methodology to hedge out *un*priced risk ...
  - ... using *hedge portfolios* formed using ex-ante forecasts of the covariance structure.
  - The combinination of the CP and the hedge portfolios are CEPs (*Characteristic Efficient Portfolios*)

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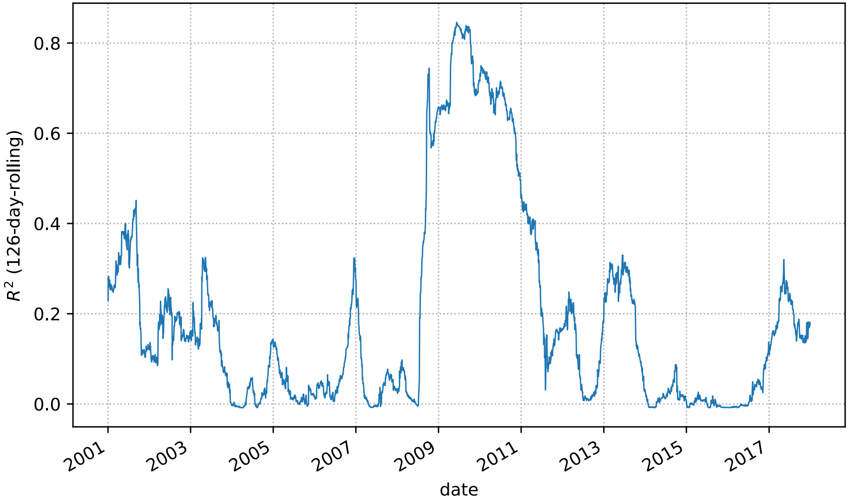
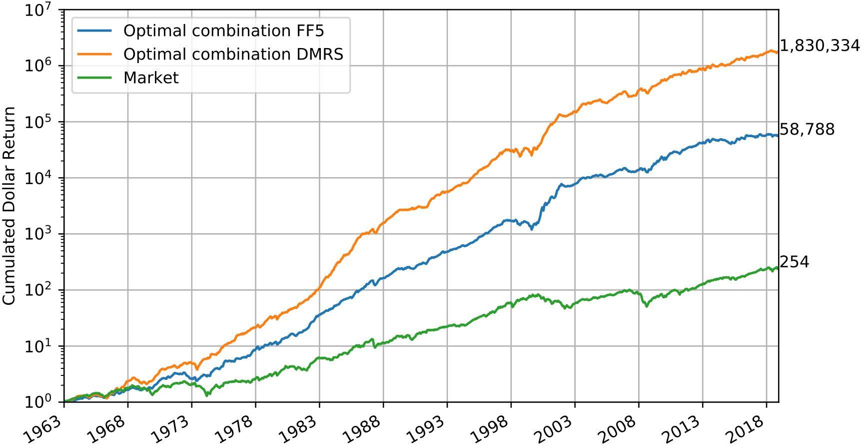
Money Industry R^2



R^2 of 126-day rolling regressions of HML on Money industry

Hedging *un*priced Risk Results in More Efficient CPs



† All portfolios are scaled to have the same annualized volatility as the market ($\sigma = 15\%$)

Why do we care?

- Our approach significantly improves the efficiency of standard characteristic portfolios
  - SR^2 of optimal combination of five Fama and French (2015) factors increases from 1.16 to 2.13 (annualized).
    - Raises the hurdle for asset pricing models..
    - Suggests either much higher σ_m, or much larger frictions.
- CEPs provide a lens through which we can learn about the economic models for sources of premia in asset returns.
- CEPs can be used as efficient benchmarks for performance evaluation.
- Improved Sharpe-ratios for quant-strategies/smart-beta strategies

Outline

1. Theory:
   1. A simple example
   2. Generalization
2. Empirical Approach
   1. How to construct the hedge portfolios
   2. How to construct the characteristic *efficient* portfolios
   3. Empirical Results

Basic Setup

Consider a standard setting with no arbitrage.

- Excess returns are determined by a two-factor structure, one priced and one unpriced factor:

$$r_i = \beta_i (f + \lambda) + \gamma_i g + \varepsilon_i \tag{1}$$

  - f is a priced factor with premium λ
  - g is an *un*priced factor,
  - $\mathbb{E}[f] = \mathbb{E}[g] = \mathbb{E}[\varepsilon_i] = 0$
  - $f \perp g \perp \varepsilon_i \;\; \forall i$, and $\varepsilon_i \perp \varepsilon_j \;\; \forall i \neq j$.

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Characteristic x as a Proxy for Expected Returns

- We do not observe β_i or λ (or f or g directly).
- However, suppose there exists an observable characteristic x_i that lines up perfectly with expected returns:

$$\boldsymbol{\mu} \equiv \mathbb{E}[\boldsymbol{r}] = \boldsymbol{x}\lambda_c \tag{2}$$

  - See, e.g., Fama and French (1993) & Daniel and Titman (1997).
- $\Rightarrow$ characteristic is perfect proxy for priced factor loading:

$$\beta_i = \frac{\lambda_c}{\lambda} x_i \tag{3}$$

- Suppose that we form a characteristic portfolio by buying high x assets and selling low x assets. *Will the projection of f in the space of returns be in the span of the resulting portfolio?*

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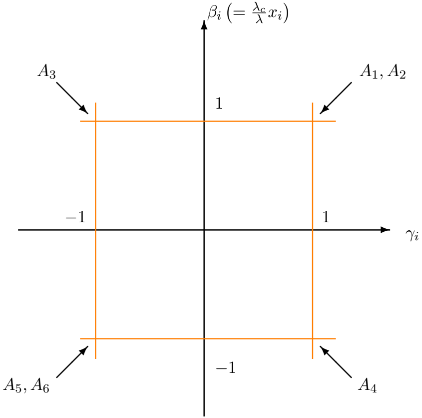

- In addition, we assume that:
  - Market capitalizations of all assets are identical
  - Assets have equal residual variance.

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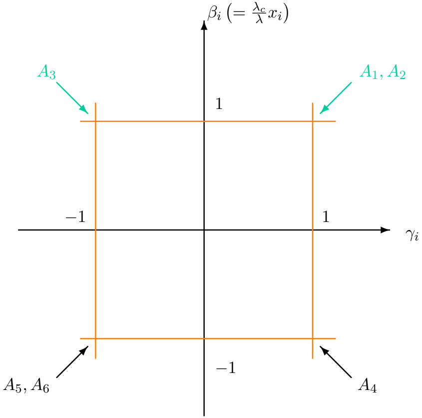

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$$r_c = \frac{1}{3} \times (r_1 + r_2 + r_3) - \frac{1}{3} \times (r_4 + r_5 + r_6)$$

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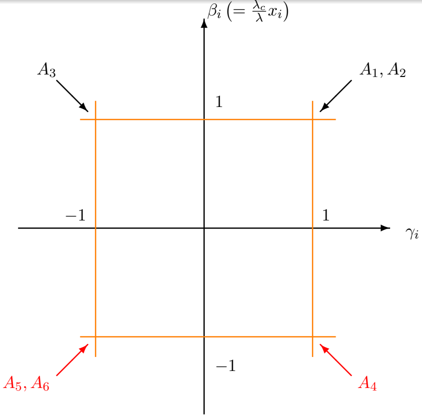

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$$\Rightarrow \beta_c = 2, \quad \gamma_c = 2/3$$

CP is not MVE

- r_c is ***not*** mean-variance-efficient
  - It loads on both the priced (f) and unpriced (g) factors.
  - $\Rightarrow$ cannot be the projection of the stochastic discount factor on the space of returns
- How can we improve r_c?
  - Construct a *hedge portfolio* with weights $\boldsymbol{w}_h$ that has
    - zero expected return $\implies \beta_h = 0$
    - strong correlation with $r_c \implies$ large γ_h, low σ^2_ϵ
  - Combine r_c and r_h to get
    - same expected return
    - lower volatility

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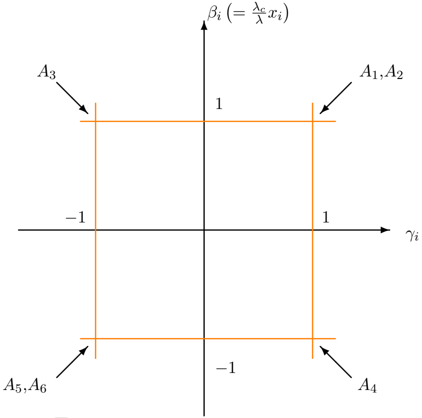
Constructing a Hedge Portfolio



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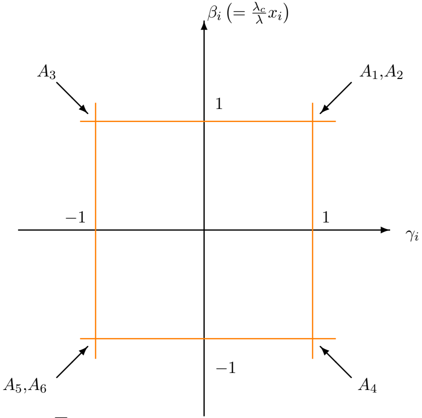

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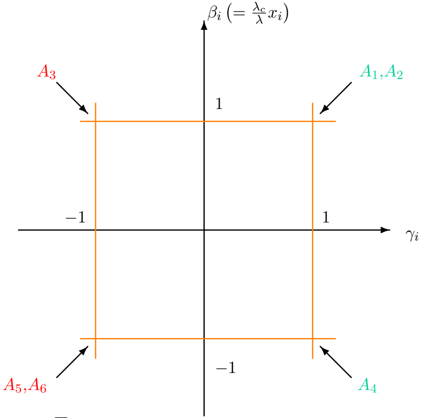

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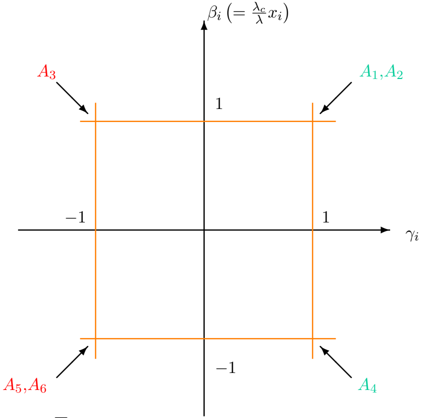

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Improved CP

- Improved CP is a combination of the CP and the hedge portfolio:

$$r_c^* = r_c - \delta r_h$$

or, rearranging:

$$r_c = \delta r_h + r_c^*$$

- Optimal hedge ratio:

$$\min_{\delta} \mathsf{var}\left(r_c^*\right) \quad \Rightarrow \quad \delta^* = \frac{\mathsf{cov}\left(r_c, r_h\right)}{\mathsf{var}\left(r_h\right)} = \rho_{c,h} \frac{\sigma\left(r_c\right)}{\sigma\left(r_h\right)}$$

- Sharpe ratio improvement:

$$\frac{\mathsf{SR}_c^*}{\mathsf{SR}_c} = \frac{1}{\sqrt{1 - \rho_{c,h}^2}} > 1$$

- In this example, this hedge portfolio is maximally correlated with the CP, so the resulting hedged CP is a *Characteristic Efficient Portfoio* (CEP).

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General Case: Multiple Characteristics

Empirically relevant case with arbitrary factor structure, and with M characteristics that drive expected excess returns

- M-characteristics

$$\boldsymbol{\mu} = X\boldsymbol{\lambda}_c, \tag{A1}$$

  - X is $(N \times M)$ matrix of characteristics
  - $\boldsymbol{\lambda}_c$ is an $(M \times 1)$ vector of characteristic premia

- We show how to form M optimal hedge portfolios (W_H^*) which, combined with the inefficient CPs, form a set of Characteristic-Efficient Portfolios (CEPs), which span the MVE.

Benchmark Factor Model

- We use the Fama and French (2015) five factors as our benchmark factor-portfolios
  - HML: book-to-market
  - RMW: profitability
  - CMA: investment
  - SMB: size
  - $Mkt - R_f$

Ingredients

- Recap — *h*edge-portfolio:
  - Zero-expected return
  - Maximum loading on the CPs
- We do not observe:
  - f_t, g_t, or β or γ
- But, we do observe:
  - Characteristics: $x_{i,t} \left(= \frac{\lambda_{t-1}}{\lambda_c}\beta_i\right)$
  - Historical returns: ex-ante forecast of $b_m (= k_1\beta + \boldsymbol{k}_2\boldsymbol{\gamma})$
- Thus, controlling for the characteristic, any remaining variation in $b_{m,i}$ must come from variation in δ_i.

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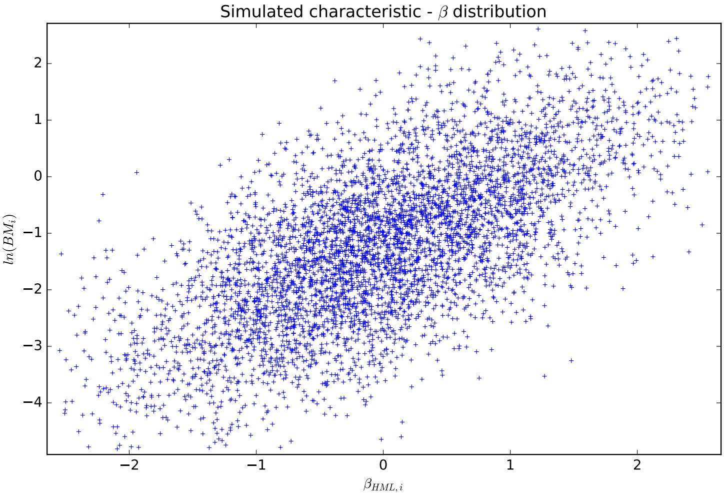
Sorting Procedure



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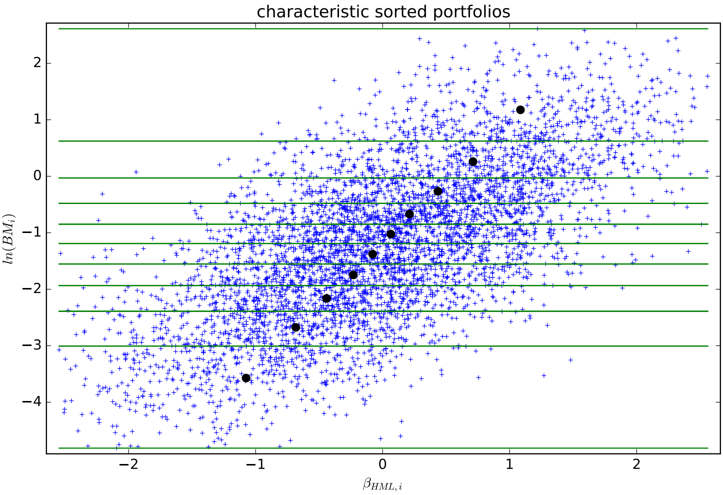

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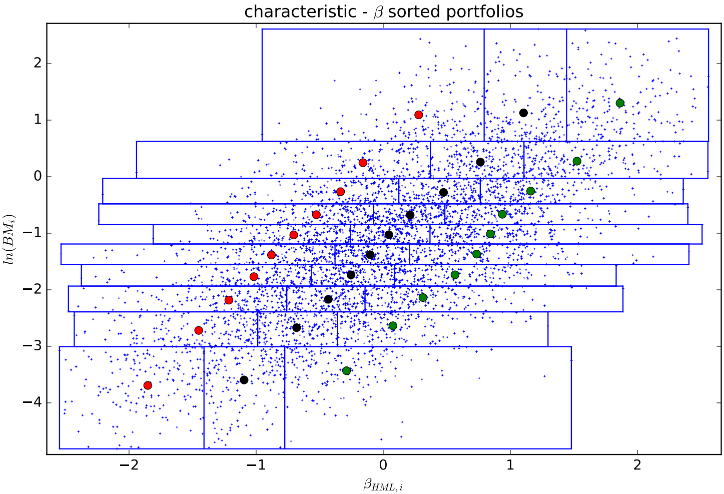

Hedge Portfolio Formation

- To forecast betas, we use daily returns, and different horizons for estimating correlations and volatilities.
  - However, note again that we form h only once/year (on June 30).
- Each June 30th, form **five** hedge portfolios
- Sort stocks into 3×3 buckets according to size and characteristic (BEME, OP or INV)
  - For MktRF and SMB, we do it with all 3 characteristics
- Form a zero investment portfolio
  - going long the low-forecast-beta portfolios
  - and short the high-forecast-beta portfolios

Pricing the *h*edge portfolio

If the characteristics line up well with expected returns and we did a good job estimating b's, each hedge portfolio should have:

- Zero expected return (approximately)
- Strong negative loading on the corresponding factor-portfolio
- Positive α w.r.t. the FF five-factor model

Monthly Time Series Regressions (07/1963 - 06/2019)

$$r_{h,m} = \alpha + b_{MktRF} r_{MktRF} + b_{SMB} r_{SMB} + b_{HML} r_{HML} + b_{CMA} r_{CMA} + b_{RMW} r_{RMW} + \epsilon_t$$

Hedge-Portfolio	Avg.	α	b_{Mkt-RF}	b_{SMB}	b_{HML}	b_{RMW}	b_{CMA}	R^2
$r_{h,MktRF}$	0.10	-0.18	0.41	0.40	0.05	-0.17	-0.06	0.66
	(0.80)	(-2.44)	(22.39)	(15.18)	(1.48)	(-4.68)	(-1.15)	
$r_{h,SMB}$	0.17	0.03	0.17	0.56	-0.01	-0.15	-0.16	0.72
	(1.74)	(0.50)	(12.27)	(28.28)	(-0.33)	(-5.57)	(-3.95)	
$r_{h,HML}$	0.07	-0.11	0.03	-0.05	0.80	0.20	-0.54	0.61
	(0.74)	(-1.86)	(1.80)	(-2.34)	(28.21)	(6.68)	(-12.03)	
$r_{h,RMW}$	0.08	-0.21	-0.05	0.04	0.31	0.69	0.11	0.65
	(0.86)	(-3.66)	(-3.27)	(1.96)	(11.69)	(24.80)	(2.51)	
$r_{h,CMA}$	-0.04	-0.20	0.04	0.02	-0.31	0.09	0.96	0.43
	(-0.52)	(-3.39)	(2.60)	(1.10)	(-10.95)	(2.90)	(21.13)	
EW3	0.04	-0.17	0.01	0.00	0.27	0.32	0.17	0.70
HML,RMW,CMA	(0.64)	(-5.45)	(0.83)	(0.39)	(17.52)	(20.56)	(7.30)	
EW4	0.05	-0.18	0.11	0.10	0.21	0.20	0.12	0.58
EW3+MktRF	(1.17)	(-5.92)	(14.60)	(9.75)	(15.08)	(13.71)	(5.18)	
EW5	0.07	-0.15	0.10	0.15	0.19	0.17	0.08	0.57
EW4+SMB	(1.57)	(-5.01)	(14.08)	(14.86)	(13.60)	(11.65)	(3.83)	

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EW3	0.04	-0.17	0.01	0.00	0.27	0.32	0.17	0.70
HML,RMW,CMA	(0.64)	(-5.45)	(0.83)	(0.39)	(17.52)	(20.56)	(7.30)	
EW4	0.05	-0.18	0.11	0.10	0.21	0.20	0.12	0.58
EW3+MktRF	(1.17)	(-5.92)	(14.60)	(9.75)	(15.08)	(13.71)	(5.18)	
EW5	0.07	-0.15	0.10	0.15	0.19	0.17	0.08	0.57
EW4+SMB	(1.57)	(-5.01)	(14.08)	(14.86)	(13.60)	(11.65)	(3.83)	

Monthly Time Series Regressions (07/1963 - 06/2019)

$$r_{h,m} = \alpha + b_{MktRF} r_{MktRF} + b_{SMB} r_{SMB} + b_{HML} r_{HML} + b_{CMA} r_{CMA} + b_{RMW} r_{RMW} + \epsilon_t$$

Hedge-Portfolio	Avg.	α	b_{Mkt-RF}	b_{SMB}	b_{HML}	b_{RMW}	b_{CMA}	R^2
$r_{h,MktRF}$	0.10	-0.18	0.41	0.40	0.05	-0.17	-0.06	0.66
	(0.80)	(-2.44)	(22.39)	(15.18)	(1.48)	(-4.68)	(-1.15)	
$r_{h,SMB}$	0.17	0.03	0.17	0.56	-0.01	-0.15	-0.16	0.72
	(1.74)	(0.50)	(12.27)	(28.28)	(-0.33)	(-5.57)	(-3.95)	
$r_{h,HML}$	0.07	-0.11	0.03	-0.05	0.80	0.20	-0.54	0.61
	(0.74)	(-1.86)	(1.80)	(-2.34)	(28.21)	(6.68)	(-12.03)	
$r_{h,RMW}$	0.08	-0.21	-0.05	0.04	0.31	0.69	0.11	0.65
	(0.86)	(-3.66)	(-3.27)	(1.96)	(11.69)	(24.80)	(2.51)	
$r_{h,CMA}$	-0.04	-0.20	0.04	0.02	-0.31	0.09	0.96	0.43
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Optimal Hedge Ratio $\boldsymbol{\delta}_m$

- Constructing improved or hedged factor portfolios:

$$r^*_{c,m,t} = r_{c,m,t} - \boldsymbol{r}_{h,t}\hat{\boldsymbol{\delta}}_{m,t-1}$$

$$\text{where} \quad m \in \{HML, RMW, CMA, SMB, MktRF\}$$

Optimal Hedge Ratio $\boldsymbol{\delta}_m$

$$r^*_{c,m,t} = r_{c,m,t} - \underbrace{\boldsymbol{r}_{h,t}}_{5\times 1} \hat{\boldsymbol{\delta}}_{m,t-1}$$

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Optimal Hedge Ratio $\boldsymbol{\delta}_m$

$$r^*_{c,m,t} = r_{c,m,t} - \underbrace{\boldsymbol{r}_{h,t}}_{5\times 1} \hat{\boldsymbol{\delta}}_{m,t-1}$$

$$\text{where} \quad m \in \{HML, RMW, CMA, SMB, MktRF\}$$

- $\hat{\boldsymbol{\delta}}_{m,t-1}$ is estimated *ex-ante*, from the regression:

$$\begin{aligned} r_{c,m,t} &= \boldsymbol{\delta}_{m,t-1}\boldsymbol{h}_t + \epsilon_{k,t} \\ r^*_{c,m,t} &= \epsilon_{m,t} \end{aligned}$$

  - That is, the CEP returns are the residuals from these regressions.

Estimating $\hat{\delta}_m$

Same basic procedure as for estimating individual firm b's.

- Estimation is out-of-sample, using:
  - daily pre-formation return regressions
  - different horizons for correlation and volatility estimations (60 months/12 months).
  - "fixed-weight" portfolios, both for $r_{c,t}$ and $\boldsymbol{r_{h,t}}$

- We also calculate *industry hedged* portfolios $r_{(c-ind)m,t}$, which uses the same estimation technique to orthogonalize the FF-portfolios to industry risk.
  - This allows us to assess the hypothesis that what we are picking up with our hedging procedure is just industry risk.

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CEP vs. industry-neutral portfolios

	r_c	r_c^*	r_{c-ind}
HML			
Mean	3.68	2.43	2.61
Vol	9.60	5.87	5.15
SR^2	0.15	0.17	0.26
RMW			
Mean	3.22	2.65	2.29
Vol	7.79	5.06	5.80
SR^2	0.17	0.27	0.16
CMA			
Mean	2.63	2.33	2.12
Vol	6.51	4.31	3.97
SR^2	0.16	0.29	0.28
SMB			
Mean	2.89	2.00	2.90
Vol	10.27	6.52	8.29
SR^2	0.08	0.09	0.12
MktRF			
Mean	6.52	5.96	-
Vol	15.14	10.51	-
SR^2	0.19	0.32	-

ex-post Optimal Combinations

	r_c	r_c^*	r_{c-ind}
In-sample optimal combination			
Mean	3.49	2.82	2.57
Vol	3.23	1.92	2.20
SR^2	1.16	2.16	1.37

Three Things to Keep in Mind:

1. All information used is readily ex-ante available information
2. We do not need to identify the unpriced common source of variation
3. Conservative portfolio construction:
    - rebalanced once/year
    - components of the hedge portfolio are all value weighted.

Conclusions

- CPs formed on the basis of characteristics sorts alone are unlikely to span the MVE portfolio
  - FF5 model is easily rejected ($t = -5.86$)
- We improve those CPs by hedging out *un*priced risk
  - using ex-ante information on the covariance structure
- Presents a greater challenge to asset pricing models
  - SR^2 of optimal FF5-combination increases from 1.16 to 2.13
- Procedure should work for *any* set of CPs
- FF5 CEPs returns can be downloaded: `www.kentdaniel.net/data.php`.

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