

Mathematics.

Don't use a calculator.

1. Factorize $25x^2 - 81$. (Write the expression as a product.)
2. Factorize $1 - 4x + 4x^2$.
3. Factorize $4 - 64x^4$.
4. Factorize $ka^2 + 9k + 6ak$.
5. Factorize $12a^3 - 108a$.
6. Factorize $-x^4 + 2x^3y - x^2y^2$.
7. Factorize $2x^2 - 2x + 0.5$.
8. Simplify $(\sqrt{x+h} + \sqrt{x})(\sqrt{x+h} - \sqrt{x})$
9. Simplify $\frac{a^3 - 1}{a - 1} - a^2$
10. Simplify $\frac{1}{a - 2} + \frac{2a}{4 - a^2}$
11. Simplify $\left(1 + \frac{a}{b}\right)\left(\frac{a}{a+b} - 1\right)$
12. Simplify $\frac{2}{x-1} - \frac{1}{x} - \frac{4}{x^2 - 1}$
13. Simplify $\frac{\frac{1}{a} - \frac{1}{b}}{a - b}$
14. Simplify $\frac{\frac{1}{y} + 1}{3 + \frac{3}{y}}$
15. Simplify $\frac{\frac{3}{a} - 2 + \frac{a}{3}}{1 - \frac{a}{3}}$
16. Factorize $x^3 - x^2 + x - 1$.
17. Simplify $\frac{x^3 - x^2 + x - 1}{x^2 + 1}$
18. Solve the equation $x^2 + 2x - 3 = 0$.

19. Solve the equation $2 + x - x^2 = 0$.
20. Solve the equation $5 - 3x - 2x^2 = 0$.
21. Solve the equation $7x^2 + 13x - 2 = 0$.
22. Solve the equation $5x^2 - 6x - 2 = 0$.
23. Solve the equation $\sqrt{3x+1} = x-1$
24. Solve the equation $45 + 8x - 4x^2 = 0$
25. Factorize the polynomial $p(x) = 45 + 8x - 4x^2$
26. Solve the inequality $12 - 3x \geq 6$.
27. Solve the inequality $45 + 8x - 4x^2 \geq 0$.
28. Solve the inequality $x^2 - 5x + 6 < 0$.
29. Solve the inequality $8 + 2x - x^2 \leq 0$.
30. Solve the inequality $\frac{2x(x-3)}{x+1} \leq 0$.
31. Solve the inequality $\frac{2x-3}{x-1} \geq 3-x$
32. Solve the inequality $\frac{1}{2-x} \leq \frac{1}{x}$.
33. Solve the inequality $\frac{1}{x} \leq \frac{1}{x-2}$.
34. Solve the inequality $\frac{1}{2-x} \leq \frac{1}{x} \leq \frac{1}{x-2}$. That is to find the x-values solving the two inequalities

$$\frac{1}{2-x} \leq \frac{1}{x} \quad \text{and} \quad \frac{1}{x} \leq \frac{1}{x-2}.$$
35. Factorize $x^3 - x^2 + x - 1$.
36. Solve the inequality $x^3 < x^2 - x + 1$.
37. Simplify $\sqrt{2} \cdot \sqrt{8}$. (Remember; do not use a calculator.)
38. Simplify $9^{-0,5}$.
39. Simplify $\sqrt{49} + \sqrt{2} \cdot \sqrt{98}$.
40. Simplify $0,25\sqrt{32}$.
41. Simplify $32^{2/5}$.
42. Simplify $\frac{32^{0,4}}{16^{-7/4}}$.

43. Simplify $(0,04)^{-0,25}$
44. Simplify $\sqrt[3]{\sqrt{10}}$.
45. Simplify $\frac{(\sqrt{2})^{1,2} \cdot \sqrt[10]{8}}{(2^{0,3})^3}$.
46. Simplify $(3^x + 3^x + 3^x)^2$.
47. Solve the equation $(2^x + 2^{x+1} + 2^{x-1})^2 = 98$.
48. Simplify $\frac{(x^{0,4})^3 \cdot \sqrt[5]{x^2}}{(x^3)^{1/5}}$.
49. Simplify $\frac{(x^{3a})^3 \cdot x^{-b}}{x^{3a+2b}}$.
50. Solve the equation $x^h - 1 = h$.
51. Solve the equation $ax(ax+b)^{-2/3} + (ax+b)^{1/3} = 0$
52. Simplify $\ln 8 + \ln 2$ (Kom ihåg att inte använda räknare.)
53. Simplify $\ln 8 - \ln 2$
54. Simplify $\ln 8 - \ln \sqrt{8} - \ln 2 + \ln \frac{1}{\sqrt{2}}$
55. Simplify $\ln \frac{17}{18} + \ln \frac{18}{17}$
56. Simplify $\ln \frac{17}{18} + \ln \frac{9}{17}$
57. Simplify $\ln x - \ln x^2 + \ln 2x$
58. Find $\frac{x}{y}$ when $\ln x = 1 + \ln y$
59. Solve the equation $\ln x = 1 - 3\ln 2$
60. Solve the equation $2\ln x = \ln 2x$
61. Solve the equation $2\ln x = \ln 3 + \ln 12$
62. Solve the equation $\ln x^2 = 1 + \ln x$
63. Solve the equation $\ln(1-x) + \ln(1+x) = \ln 0,75$.
64. Solve the equation $\sqrt{3x+1} = x-1$
65. Find the equation for the straight line passing through the points $(-2; 4)$ and $(6; -2)$.
66. Find the equation for the straight line passing through the point $(4; 1)$ and is parallel the line to the line $2y + 3x + 4 = 0$

67. Solve the system of equations $\begin{cases} 7x + 2y = 13 \\ 3x - 4y = -9 \end{cases}$

68. Solve the system of equations $\begin{cases} 230p + 450q = 1810 \\ 100p + 150q = 650 \end{cases}$

69. Solve the system of equations $\begin{cases} 2x + 3y - z = 1 \\ -x + y + 3z = 5 \\ x - y + 2z = 2 \end{cases}$

70. Find the derivative of $f(x) = 7x^9$

71. Find the derivative of $f(x) = x^n$

72. Find the derivative of $f(x) = \sqrt{x}$

73. Find the derivative of $f(x) = \sqrt[5]{x}$

74. Find the derivative of $f(x) = x^\alpha$

75. Find the derivative of $f(x) = e^x$

76. Find the derivative of $f(x) = 2e^{-3x}$

77. Find the derivative of $f(x) = 4x^5 + 3e^{-6x}$

78. Find the derivative of $f(x) = 7e^{-8x+3x^3}$

79. Find the derivative of $f(x) = (x+5)^{17}$

80. Find the derivative of $f(x) = (x+5)^{-17}$

81. Find the derivative of $f(x) = (x^2 + 5)^{17}$

82. Find the derivative of $f(x) = (x^2 + 5)^{-17}$

83. Find the derivative of $f(x) = \sqrt[17]{x^2 + 5}$

84. Find the derivative of $f(x) = \ln x$

85. Find the derivative of $f(x) = x^7 + \ln(x+1)$

86. Find the derivative of $f(x) = \ln(1-x^2)$

87. Find the derivative of $f(x) = \ln(1+x^2+x^4)$

88. Find the derivative of $f(x) = -x + x \cdot \ln x$

89. Find the derivative of $f(x) = \sqrt[3]{1+2x^4}$

90. Find the derivative of $f(x) = \ln \sqrt[3]{1+2x^4}$

91. Find the derivative of $f(x) = e^{-3x^2} \cdot \ln(1+x^2)$
92. Find the derivative of $f(x) = \frac{x}{1+x^2}$
93. Find the derivative of $f(x) = \frac{x^2-1}{x^2+1}$
94. Find the derivative of $f(x) = \frac{4x}{(x^2+1)^2}$
95. Consider the function $f(x) = -x^2 + x + 6$ having the domain $-3 \leq x \leq 2$
- Give the global maximum and minimum of the function.
 - Give the range of the function.
96. Consider the function $f(x) = xe^{-x}$ having the domain $-1 \leq x \leq 2$
- Give the global maximum and minimum of the function.
 - Give the range of the function.
97. Consider the function $f(x) = -x^2 + x + 6$
- Give the largest possible domain to the function.
 - Find any zeros to the function, i.e. x-values where $f(x) = 0$.
 - Find the functions stationary points, i.e. x-values where where the derivative is zero.
 - Find the intervals where the function is increasing and where it is decreasing.
 - Classify the stationary points, i.e find any local minimum or maximum.
 - Find the intervals where the function is concave and convex.
 - Give any points of inflection.
 - Give the range of the function.
 - Sketch the graph.
98. Consider the function $f(x) = x^3 + 6x^2 + 9x$ and answer questions 97. a) – i).
99. Consider the function $f(x) = xe^{-x}$ and answer questions 97. a) – i).
100. Consider the function $f(x) = \frac{(x-1)^2}{1+x^2}$ and answer questions 97. a) – i).
101. Consider the function $f(x) = \frac{\ln x}{x^2}$ and answer questions 97. a) – i).

Answers.

1. $(5x+9)(5x-9)$
2. $(1-2x)^2$
3. $4(1+4x^2)(1+2x)(1-2x)$
4. $k(a+3)^2$
5. $12a(a+3)(a-3)$
6. $-x^2(x-y)^2$
7. $0,5(2x-1)^2$
8. h
9. $a+1$
10. $-\frac{1}{a+2}$
11. -1
12. $\frac{x-1}{x(x+1)}$
13. $-\frac{1}{ab}$
14. $\frac{1}{3}$
15. $\frac{3-a}{a}$
16. $(x-1)(x^2+1)$
17. $x-1$
18. $x_1=1 \quad x_2=-3$
19. $x_1=-1 \quad x_2=2$
20. $x_1=1 \quad x_2=-2,5$
21. $x_1=1/7 \quad x_2=-2$
22. $x_1=\frac{3+\sqrt{19}}{5} \quad x_2=\frac{3-\sqrt{19}}{5}$
23. $x=5$
24. $x_1=4,5 \quad x_2=-2,5$
25. $p(x)=-4(x-4,5)(x+2,5)$
26. $x \leq 2$
27. $-2,5 \leq x \leq 4,5$
28. $2 < x < 3$
29. $x \leq -2 \quad x \geq 4$
30. $x < -1 \quad 0 \leq x \leq 3$
31. $0 \leq x < 1 \quad x \geq 2$
32. $0 < x \leq 1 \quad x > 2$
33. $x < 0 \quad x > 2$
34. $x > 2$
35. $(x-1)(x^2+1)$
36. $x < 1$
37. 4
38. $1/3$
39. 21
40. $\sqrt{2}$
41. 4
42. 2^9
43. $\sqrt{5}$
44. $10^{1/6}$
45. 1
46. 3^{2x+2}
47. $x=1,5$
48. x
49. x^{6a-3b}
50. $x=(1+h)^{1/h}$
51. $x=-b/2a$
52. $4\ln 2$
53. $2\ln 2$
54. 0
55. 0
56. $-\ln 2$
57. $\ln 2$
58. e
59. $e/8$
60. $x=2$
61. $x=6$
62. $x=e$
63. $x_1=0,5 \quad x_2=-0,5$
64. $x=5$
65. $y=-0,75x+2,5$
66. $y=-1,5x+7$
67. $\begin{cases} x=1 \\ y=3 \end{cases}$
68. $\begin{cases} p=2 \\ q=3 \end{cases}$
69. $\begin{cases} x=0 \\ y=0,8 \\ z=1,4 \end{cases}$

70. $f'(x) = 63x^8$
71. $f'(x) = nx^{n-1}$
72. $f'(x) = 0,5x^{-0,5} = \frac{1}{2\sqrt{x}}$
73. $f'(x) = 0,2x^{-0,8}$
74. $f'(x) = \alpha x^{\alpha-1}$
75. $f'(x) = e^x$
76. $f'(x) = -6e^{-3x}$
77. $f'(x) = 20x^4 - 18e^{-6x}$
78. $f'(x) = 7(-8 + 9x^2)e^{-8x+3x^3}$
79. $f'(x) = 17(x+5)^{16}$
80. $f'(x) = -17(x+5)^{-18}$
81. $f'(x) = 17 \cdot 2x(x^2 + 5)^{16}$
82. $f'(x) = -17 \cdot 2x(x^2 + 5)^{-18}$
83. $f'(x) = \frac{1}{17} \cdot 2x(x^2 + 5)^{-16/17} = \frac{2x}{17(x^2 + 5)^{16/17}}$
84. $f'(x) = \frac{1}{x}$
85. $f'(x) = 7x^6 + \frac{1}{x+1}$
86. $f'(x) = -\frac{2x}{1-x^2}$
87. $f'(x) = \frac{2x+4x^3}{1+x^2+x^4}$
88. $f'(x) = \ln x$
89. $f'(x) = \frac{1}{3} \cdot 8x^3(1+2x^4)^{-2/3}$
90. $f'(x) = \frac{1}{3} \cdot 8x^3(1+2x^4)^{-2/3} \cdot \frac{1}{\sqrt[3]{(1+2x^4)}} = \frac{8x^3}{3(1+2x^4)}$
91. $f'(x) = -6xe^{-3x^2} \cdot \ln(1+x^2) + e^{-3x^2} \cdot \frac{2x}{1+x^2}$
92. $f'(x) = \frac{1-x^2}{(1+x^2)^2}$
93. $f'(x) = \frac{4x}{(1+x^2)^2}$
94. $f'(x) = \frac{4-12x^2}{(1+x^2)^3}$
95. Maximum: $f(0,5) = 6,25$ Minimum: $f(-3) = -6$ Range: $-6 \leq f(x) \leq 6,25$
96. Maximum: $f(1) = \frac{1}{e}$ Minimum: $f(-1) = -e$ Range: $-e \leq f(x) \leq \frac{1}{e}$
97. a) Largest possible domain to the function is the real line.
 b) Zeros: $x_1 = 3$ $x_2 = -2$
 c) Stationary point: $x = 0,5$
 d) $x = 0,5$ gives a maximum.
 e) $f(x)$ is increasing in the interval $x < 0,5$ $f(x)$ is decreasing in the interval $x > 0,5$
 f) $f(x)$ is concave in the domain.
 g) No inflection point.
 h) Range: $f(x) \leq 6,25$
98. a) Largest possible domain to the function is the real line.
 b) Zeros: $x_1 = -3$ $x_2 = 0$
 c) Stationary points: $x_1 = -3$ $x_2 = -1$
 d) $x = -3$ gives a local maximum. $x = -1$ gives a local minimum.
 e) $f(x)$ is increasing in the intervals $x < -3$ $x > -1$
 $f(x)$ is decreasing in the interval $-3 < x < -1$
 f) $f(x)$ is concave in the interval $x < -2$ $f(x)$ convex in the interval $x > -2$
 g) Inflection point: $x = -2$
 h) Range: the real line.

99. a) Largest possible domain to the function is the real line.
 b) Zero: $x = 0$
 c) Stationary point: $x = 1$
 d) $x = 1$ gives a maximum.
 e) $f(x)$ is increasing in the interval $x < 1$ $f(x)$ is decreasing in the interval $x > 1$
 f) $f(x)$ is concave interval $x < 2$ $f(x)$ is convex in the interval $x < 2$
 g) Inflection point: $x = 2$
 h) Range: $f(x) \leq \frac{1}{e}$

100. a) Largest possible domain to the function is the real line.
 b) Zeros: $x = 1$
 c) Stationary points: $x = -1$ och $x = 1$
 d) $x = -1$ gives a maximum. $x = 1$ gives a minimum.
 e) $f(x)$ is increasing in the intervals $x < -1$ $x > 1$
 $f(x)$ is decreasing in the interval $-1 < x < 1$
 f) $f(x)$ concave in the intervals $-\sqrt{3} < x < 0$ $x > \sqrt{3}$
 $f(x)$ convex in the intervals $x < -\sqrt{3}$ $0 < x < \sqrt{3}$
 g) Inflection points: $x = -\sqrt{3}$ $x = 0$ $x = \sqrt{3}$
 h) Range: $0 \leq f(x) \leq 2$

101. a) Largest possible domain to the function: $x > 0$
 b) Zeros: $x = 1$
 c) Stationary point: $x = \sqrt{e}$
 d) $x = \sqrt{e}$ gives a maximum.
 e) $f(x)$ is increasing in the intervall $0 < x < \sqrt{e}$
 $f(x)$ is decreasing in the intervall $x > \sqrt{e}$
 f) $f(x)$ concave in the interval $0 < x < e^{5/6}$ $f(x)$ convex in the interval $x > e^{5/6}$
 g) Inflection point: $x = e^{5/6}$
 h) Range: $f(x) \leq \frac{1}{2e}$