Traders vs. Relationship Managers: Reputational Conflicts in Full-Service Investment Banks*

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ABSTRACT

We present a model that explains why investment bankers have struggled in recent years to manage conflicts of interest. The model captures two conflicting dimensions of reputation. On the one hand, banks can build a type reputation for technical competence by performing complex deals that may not serve their clients' interest; on the other hand, bankers can sustain a behavioral reputation by refraining from doing so. Unproven banks favor type reputation over behavioral reputation; being ethical in our model is a luxury reserved for banks that have proven their abilities. The model also sheds light on conflicts between the trading and advisory divisions of investment banks, as well as the consequences of technological change for time variation in the relative strength of behavior- and type- reputation concerns.

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I. Introduction

Traditionally, investment bankers advised client firms on capital raising transactions and represented the quality of their securities to prospective investors. Neither function was particularly susceptible to formal contracting because the quality and veracity of advice was not easily verified in court. As a consequence, bankers had an incentive to build reputations for being trustworthy. The long-standing, often exclusive, banking relationships that characterized much of the 20th century suggest that such efforts served bankers’ interests as well. This central function of investment banking has changed little over time. But as banks have increased the scale, scope, and complexity of their operations, they have come under increasing criticism for failure to manage the conflicts of interest inherent in their businesses.¹ From this perspective, it is less clear that reputation concerns have remained an effective governance mechanism.

In this paper we study a model designed to explain why investment bankers have struggled in recent years to manage conflicts of interest. Our argument rests on the idea that different dimensions of a bank’s reputation may conflict with one another and need not be of equal value to each of a bank’s operating units. The analysis contributes to a growing literature that demonstrates how an agent’s concern for his own reputation can lead him to take actions that harm his clients.² For example, a junior banker wishing to showcase his abilities as a deal maker might advise his clients to undertake an unnecessary takeover, or to perform an unnecessarily complex structured financing. Alternatively, a trader might pursue a reputation for great skill by breaching so-called “Chinese Walls” within banks that are intended to prevent misuse of sensitive client information.

Reputation is pursued at the expense of clients when there is uncertainty over an individual’s type. Chen, Morrison and Wilhelm (2012) demonstrate that the incentive distortion associated with personal reputation concerns can be countered by situating the agent in a long-lived firm that can monitor the agent’s activities and so prevent it from engaging in socially damaging reputation building. When clients anticipate that the firm will perform this type of monitoring they are prepared to pay more for its services, because they do not

¹Noteworthy examples include the 2003 “Global Settlement” of alleged conflicts of interests between investment bankers and research analysts in 10 prominent banks; allegations that Goldman Sachs and Citigroup, among others, bet against their clients in transactions tied to subprime mortgages; and criticism of Goldman for representing both parties to Kinder Morgan’s proposed acquisition of El Paso Corporation. See “Goldman, on Both Sides of Deal, is Now in Court,” Steven M. Davidoff, The Deal Professor, New York Times Dealbook, February 7, 2012, http://dealbook.nytimes.com/2012/02/07/goldman-on-both-sides-of-a-deal-is-now-in-court/.

²See for example Ely and Valimäki (2003), Ely, Fudenberg and Levine (2008), and Morris (2001). In these models, as in ours, the agent knows better than the client how to best serve the client’s needs. Although our models are very different, Bolton, Freixas and Shapiro (2007) study the conflict between banks advising clients and selling them products to meet their needs. Their focus on the interaction between (even small) reputation concerns and competition among banks yields the novel conclusion that independent advisory firms need not be the only credible source of unconflicted advice.
anticipate having to subsidize the creation of individual reputations. Hence, the desire of the firm’s owners to sustain its institutional reputation can serve to counter the bad incentives associated with individual reputation building.\(^3\)

In our model banks have an incentive to build a type reputation for unique skill that will be valuable in the future by taking actions that may not best serve the interests of their clients. On the other hand, banks can charge higher fees if they earn a reputation for resisting the urge to behave in this fashion. Thus there is a conflict between a type reputation that reflects client beliefs concerning a bank’s abilities and a behavioral reputation that reflects the bank’s equilibrium actions, and is supported by the equilibrium beliefs of its clients. Behavioral reputations are closely associated with the type of client-focused behavior that is usually characterized by market observers as “ethical.” While one can think of a firm whose equilibrium behavior sustains a behavioral reputation as an ethical firm, we acknowledge that ethical actions involve more than a desire to maintain a profitable behavioral reputation. Nevertheless, we suspect that this is the basis of much market behavior that attracts the “ethical” soubriquet.

In the basic model, the bank has a single division and knows both its type and the state of the world when it takes an action on behalf of the client. Clients observe their payoffs but neither the state of the world nor the bank’s action. As a consequence, the client receives only a very noisy signal regarding whether the bank’s action was self-interested. Our main result is that, when type-reputations are long-lived, behavioral reputational incentives are insufficiently strong to overcome the incentive to build a type reputation even when banks are very patient.\(^4\) It is therefore unrealistic to expect an unproven bank with a weak type reputation to adopt ethical behavioral patterns; being ethical in our model is a luxury reserved for banks that have proved their abilities.

When clients expect unproven banks to focus on enhancing their type reputations they pay them accordingly, and do not punish them for their actions. We exhibit an equilibrium for our model in which unproven banks engage in a type-reputation-building phase before entering a phase where they behave, and are expected to behave, ethically. In that phase the consequence of a transgression is a permanent loss of faith, and a reversion to reputation-building behavior.

If financial innovations are evidence of individual skill they could serve as a means of establishing type reputations.\(^5\) Our reasoning therefore suggests that less well-established

\(^3\)Conversely, because they can profit immediately from a share of the superior returns a senior agent reaps from his individual reputation, very impatient owners may lack incentive to build an institutional reputation.

\(^4\)In contrast to Ely and Välimäki (2003) and Chen et al. (2012), knowledge is transferred between junior and senior workers. As a result, bank types do not change with each new generation of workers, so that the gains from type reputation formation are long-lived. Hence, even a very patient bank has something to gain from building a type reputation, and institutional reputation is correspondingly harder to sustain.

\(^5\)Based on discussions with bankers responsible for new product development at four major banks, Tufano
banks without a behavioral reputation to lose will be a more common source of financial innovation. But this also implies that at least some innovative activity will be inefficient.\textsuperscript{6} Similarly, banks with strong reputations for placing their clients’ interests’ first should be more likely to adopt a “fast follower” policy.\textsuperscript{7}

We next present an extension of the model intended to reflect conflicts in reputation concerns across bank business units. As we suggested earlier, the advisory focus of traditional investment-banking functions remains relatively less susceptible to formal contract and thus dependent on a behavioral reputation for placing client interests first. In contrast, trading and brokerage functions are relatively more susceptible to performance measurement, formal contract, and the signaling of ability. As a consequence, we might expect these functions to place type-reputation concerns before concerns for behavioral reputation.

We formalize the conflict by assuming that the bank has independent execution and advisory divisions. The advisory division observes the state of the world and advises the client on the action it should request from the execution division. It does not know the execution division’s type. The execution division knows its type but does not observe the state of the world. In this setting, the advisory division concentrates upon behavioral reputation, and so it advises the client of the appropriate action to request from the execution division. This facilitates client monitoring of the execution division and, as a result, we demonstrate that a socially efficient equilibrium without a type-reputation-building phase exists. One way to interpret this result is as a justification for Chinese Walls designed to limit the flow of information within banking organizations.

However, even with this institutional setup this is not the only possible outcome; an alternative equilibrium with a short type-reputation-building phase exists, and may be preferred by the execution division to the socially optimal one. Moreover, close contact between the execution and the advisory business, and in particular sharing of information about the execution division’s type, destroys the socially optimal equilibrium.

The inefficient equilibrium exists under conditions that could arise when a strong advisory function operates alongside a less well-established execution function or one that has lost key people with whom the firm’s type reputation is associated. The flurry of commercial bank acquisitions of investment banks during the late 1990s joined a number of

\textsuperscript{6}(1989, p. 235) suggests that “Bankers believe that innovating signals their intangible and unique abilities better than advertising.” His empirical analysis of 58 financial innovations from 1974-1986 reveals a strong association between innovation and market share. Carrow (1999) obtains similar results.

\textsuperscript{7}On a similar note, see Lowery (2012) for a model in which investment in financial expertise is socially wasteful, and Lerner and Tufano (2011) and Litan (2010) for recent attempts to evaluate welfare implications of financial innovations.

\textsuperscript{7}Ellis (2009, ch.10) identifies Goldman Sachs having adopted this strategy as an explicit byproduct of John Whitehead’s 1956 study of the firm’s efforts to generate new business. The study ultimately led to the separation of relationship management from execution functions.
well-established advisory operations with the nascent execution divisions of their acquirers.\textsuperscript{8} Similarly, bankers routinely spent their entire careers with a single firm prior to 1970 but mobility ratcheted up over the next several decades to the point where by the late 1990s entire teams were changing banks.\textsuperscript{9} From this perspective, it is perhaps not surprising that advisory clients expressed growing concerns that private information about their operations or transaction strategy might be shared with competitors or with the bank’s own trading operations nor is the appearance of specialized or “boutique” advisory firms that appeal to such concerns by emphasizing their freedom from conflicts.\textsuperscript{10}

Finally, we consider the effect upon our results of periodic technological shocks that reset the bank’s type. These shocks alter the dynamics of reputation formation in our model, because they give the bank repeated opportunities to announce its type, and so to build a behavioral reputation for truth-telling. This reputation is so valuable in the long run that first best can be achieved. However, we also show that, if a technological shock carries with it a significant risk of bank failure, the consequential attenuation of the long-run value of a truth-telling reputation prevents it from supporting a first-best equilibrium.

The role of technological change in our model sheds further light on several long-run patterns in the financial markets. For example, until the middle of the 20th century, bank/client relationships were quite stable and often exclusive.\textsuperscript{11} Relationship stability is consistent with banks having both strong type- and behavioral- reputations. In other words, clients should have little incentive to switch banks if they perceive their bank as having acted in their best interest in the past and maintaining the skills necessary for the transaction at hand. We present new evidence of the correspondence between weakening client relationships and technological and regulatory shocks that swept the industry beginning in the early 1960s.\textsuperscript{12}

In addition to weakening client relationships, the industry witnessed a steady decline in the average tenure of investment bank partners.\textsuperscript{13} If behavioral reputation stems from consistently client-centric behavior on the part of individual bankers, then its preservation

\textsuperscript{8}In 1997 alone, NationsBank acquired Montgomery Securities, ING Group acquired Furman Selz, Bankers Trust acquired Alex. Brown, CIBC acquired Oppenheimer, BankAmerica acquired Robertson Stephens (and sold it to BankBoston in 1998), and SBC Warburg acquired Dillon Read. See Morrison and Wilhelm (2007, Ch.10) for details.

\textsuperscript{9}See Morrison and Wilhelm (2007, Ch.9) and Chemmanur, Ertugrul and Krishnan (2012).

\textsuperscript{10}Using post-1975 data, Asker and Ljungqvist (2010) find that large firms avoid developing relationships with banks that represent product-market competitors. This finding stands in sharp contrast to the high level of industry specialization among investment banks during the first half of the 20th century (see Morrison and Wilhelm (2007, Ch.7) and Carosso (1970) ). The academic evidence on conflicts related to the provision of advisory services is mixed. For example, Bodnaruk, Massa and Simonov (2007) report evidence of banks taking positions in the targets of bidding firms which they advise. In contrast, Griffin, Shu and Topaloglu (2012) find no evidence that banks advising in corporate takeovers share client information with institutional investors.

\textsuperscript{11}See Morrison and Wilhelm (2007, Ch.8).

\textsuperscript{12}See Morrison and Wilhelm (2007, 2008).

\textsuperscript{13}See Morrison and Wilhelm (2007, Ch.9).
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should, in part, rest on banks’ ability to sustain intergenerational transfer of client relationships. But Morrison and Wilhelm (2008) suggest that their ability to do so declined with increasing scale during the 1970s and 1980s. If behavioral reputation concerns consequently weakened, our model suggests that banks would be more inclined to engage in activities that might be perceived as a threat to their clients’ interests. In contrast, Goldman Sachs was relatively slow to join the race to achieve greater scale and scope and the average tenure of its partners declined less rapidly than for many of its peers. It is perhaps not surprising then that, as we show in Section VIII, Goldman openly avoided representing bidders during the 1980s hostile takeover movement in deference to concerns for their client relationships.

The rest of the paper is organized as follows. Section II reviews some of the relevant economic literature on reputation and relates it to our work. Section III presents our model and Section IV defines a model equilibrium. Section V shows how reputation is managed in equilibrium, and Section VI shows how the creation of an independent advisory division within the bank can improve outcomes. Section VII presents our results on technological shocks. Section VIII provides a broad overview of the evolution of the investment-banking industry during the last half century through the lens of our model. Section IX concludes.

II. Literature Review

The traditional approach to reputation modelling focused on type reputation models. Early models involved the introduction of “commitment types:” agents who always took actions that led to surplus-maximizing outcomes, even when those actions were irrational. Kreps, Milgrom, Roberts and Wilson (1982) demonstrated even a small probability of commitment types was sufficient to induce cooperation in a finitely repeated prisoner’s dilemma game. Fudenberg and Levine (1992) demonstrate that when there is a small probability of commitment types, an infinitely-lived rational and self-interested agent who plays the prisoner’s dilemma with a series of short-lived agents can overcome his moral hazard problem by pretending to be a commitment type: that is, by building a reputation for cooperation.

Fudenberg and Levine’s insight is the basis for a number of finance papers, in which type could reflect the standard preference for cooperation (Boot, Greenbaum and Thakor (1993), Fulghieri, Strobl and Xia (2010), Winton and Yerramilli (2011)), or ability (Diamond (1989), Chemmanur and Fulghieri (1994a, 1994b)). However, while type reputation concerns resolve moral hazard problems in all of these models, reputation effects are short-lived (Cripps, Mailath and Samuelson, 2004). Hence, this type of model seems a poor choice for examining institutional reputation.

14 Also see Hartman-Glaser (2012) for a model in which there is tension between a bank’s reputation for honest representation of the type of an asset used in a securitization and the bank’s ability to signal asset type by retaining a fraction of the asset. While reputation concerns can increase the bank’s equilibrium payoffs by reducing retention, this does not imply that the bank is more likely to perfectly reveal information.
The behavioral approach that we adopt for institutional reputation is introduced by Kreps (1990) in an overlapping generation model in which there is moral hazard but no adverse selection. “Reputation” in this type of model refers to an equilibrium belief by clients that an agent will take specific actions; it is sustained by the consequences of the beliefs that the clients will adopt if the wrong actions are taken. Clients monitor actions imperfectly in this model, as in ours. If the monitoring is sufficiently informative then a general folk theorem applies, in which future punishment and rewards provide the incentives needed to sustain the equilibria (Fudenberg, Levine and Maskin, 1994).

Later work extended Kreps’ approach by combining both type reputation (adverse selection) and behavioral reputation (moral hazard) (see Tadelis (1999, 2002)). Tirole (1996) examines collective reputations in this way, and shows that bad equilibria can persist because a moral hazard in teams problem renders individuals unwilling to improve the collective reputation. Morrison and Wilhelm (2004) examine professional services firms as entities that certify agents of uncertain type and that use a collective reputation to incentivize senior agents to train juniors.

In our model there is non-trivial learning in equilibrium. This is a difficult problem because such games do not admit the recursive equilibrium structure of Abreu et al. (1990). Fudenberg and Yamamoto (2010) address this type of problem by addressing a simpler set of equilibria in which a player’s best response does not depend upon his beliefs (and, hence, upon his learning), and so are able to re-introduce a recursive equilibrium structure. Fudenberg and Yamamoto are able to prove a folk theorem for their restricted class of equilibria. But none of the papers in this strand of the literature examines reputational conflict.

Finally, our model is related to the relational contract literature, particularly in Section VI, where we allow banks to announce their types. In contrast to earlier work in this area (see for example Levin (2003), Athey, Bagwell and Sanchirico (2004), Athey and Bagwell (2008)), which considers only long-run players, we consider the case where clients are short-lived. This has two important consequences. First, the undesirable incentive to build a personal reputation is particularly important because fees are based entirely on the bank’s type reputation. In contrast, if clients are long-lived, there may be equilibria in which the bank’s payoff is independent of its type reputation, so that the bad reputation incentive is reduced (see Athey and Bagwell (2008)). Second, the relational contract is more limited than Levin’s (2003), in which contracting parties sometimes make large punishment payments. Such contracts are impossible in our setting because there is no mechanism to induce our short-lived clients to make such payments. Moreover, they will pay a higher fee if they anticipate a large payment from the bank, and so will destroy the incentive effects of such

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15 The seminal treatment of this topic is due to Abreu, Pearce and Stacchetti (1990).
payments.

III. Model

We consider an infinitely lived bank that is created at time 0. Time is indexed by $t = 0, 1, 2, \ldots$, and period $t$ runs from time $t - 1$ to $t$. In each period the bank has a one owner, one worker, and a single client. All agents in our model are risk neutral, and they have a common per-period discount factor $\delta$. The owner and the worker work for one period and retire; a new client deals with the bank in each period.

In each period the worker takes an action $A \in \{1, 2\}$ on behalf of the client. The cost of taking either action is 0 to the worker, but the payoff that the client derives from $A$ depends upon the state of nature $\omega \in \{1, 2\}$. Action $\omega$ is optimal for the client in each state $\omega$. When state 1 realizes, the client receives a payoff of 1 from action 1 and $x \in (0, 1)$ from action 2. When state 2 realizes, the client’s payoff after action 1 is $x$ or 0 with respective probabilities $q$ and $1 - q$; it is $x$ if action 2 is selected. In each period, the probability that state 1 realizes is $p > 0$.

There are two types of workers. Dumb (type D) workers are only able to perform action 1; smart (type S) workers can perform action 1 or action 2. We assign the worker’s type (S or D) to the bank, and for convenience we will refer to S-banks and D-banks. Whether or not a worker is smart depends upon his knowledge and the training that he received before starting work. The time 0 probability that the first worker is smart is $\theta_0$. Subsequent generations of workers are trained by the outgoing worker, so that the bank’s type is maintained. The intergenerational knowledge transfer between workers is observed by the new owner, who then makes a take-it-or-leave it offer to the old owner for ownership of the bank, and hence captures all of its value. It is a consequence of our assumptions on training and its observability that the bank’s type is known to its owner and its worker; we assume that this information is not revealed to the client.

We write $z \in \{0, x, 1\}$ for the client’s payoff in any period. We assume that $z$ is observable but that none of $z$, $\theta$ and $A$ can be verified in court. It follows that the only possible contract between the bank and the client requires the payment of a fixed fee by the worker in exchange for an unspecified action by the worker. We assume that there is competition among clients for the bank’s services, so that the bank captures all of the expected client surplus from its action. The bank’s revenue is shared between the worker and the owner; for simplicity, we assume an exogenous sharing rule that assigns a proportion $\rho$ of the surplus to the owner. Without loss of generality, we set $\rho = 1$. 
Figure 1. Extensive form for the stage game. Nature selects the bank’s type at time 0, and the bank retains that type. Payoffs are stage game client surpluses. The client cannot distinguish between S- and D-banks, and therefore has the information sets identified in the Figure by dashed horizontal lines.

One of the stage games is illustrated in Figure 1.¹⁶ As indicated in the Figure, the bank’s type is established at time 0. This information is not observed by the client, who therefore cannot distinguish between the elements of the information sets indicated in the Figure by horizontal dashed lines. Note that, because the worker has a one period career, he has no incentive to build an individual reputation. We can therefore focus upon the creation of the bank’s reputation. To maintain this focus, we assume that the owner can costlessly control the worker’s action, and, hence, we refer in this section to the bank’s actions, rather than to those of the worker or the owner. Finally, we assume that an S-bank’s services are more valuable to a client than a D-bank’s:

¹⁶In this game S-type banks can protect the client from the worst outcome, 0, in state 2, but cannot do better than the mediocre return \(x\). We have also examined a more complex game in which, with probability \(\lambda\), the S-type bank can achieve a superior return \(X > x\) by taking action 2, while D-type banks can mimic S-type banks with probability \(q\). The analysis is more complex, but the qualitative features of our equilibrium are unchanged. See footnote 19.
\[ x > p + (1 - p) qx. \]  

The left hand side of equation (1) is the lowest single period client surplus generated by a smart bank, and the right hand side is the expected surplus created by a dumb bank. It is immediate from equation (1) that the bank can maximize its value by building a reputation for being smart. It can accomplish this by selecting action 2 wherever possible; in particular, it will do so in state 1, in which case its reputation comes at the expense of its client.

\section*{IV. Equilibrium Definition}

We will exhibit Bayesian Nash Equilibria of our model. We start by presenting a formal definition of these equilibria. First, recall that the only publicly available datum in any period is the client’s payoff \( z \in \{0, x, 1\} \). We therefore define a \textit{time t history} to be an element \( h_t \) of \( H_t = \{0, x, 1\}^t \). We define \( H \) to be the set of all possible game histories to every possible \( t \):

\[ H = \bigcup_{0}^{\infty} H_t. \]  

Let \( A_t \in \{1, 2\} \) be the bank’s time \( t \) action. When the bank selects its time \( t \) action it knows the history \( h_t \) of client payoffs and its own type. Hence, given a time \( t \) history \( h_t \), a strategy \( \mu \) for the bank assigns a probability \( \mu_t \) of setting \( A = 1 \) when \( \omega = 1 \):

\[ \mu : H \rightarrow \mathbb{R}[0, 1] : h_t \mapsto \mu_{H_t} \equiv \mu_t. \]  

Note that strategy choice is non-trivial only for S-banks, because D-banks are technologically limited to the strategy that sets \( h_t \equiv 1 \). We write \( M \) for the set of strategies.

We define the client’s \textit{action belief} to be the probability \( \beta_t \) that she assigns at time \( t \) to the event that the bank takes action 1 given that the state \( \omega \) is 1. The client conditions her action belief upon \( H_t \):

\[ \beta : H \rightarrow \mathbb{R}[0, 1] : h_t \mapsto \beta_{H_t}. \]  

We write \( B \) for the set of action beliefs. The client’s action belief reflects the time \( t \) probability \( \theta_t \) that she assigns to the event that the bank has type S. We can think of \( \theta_t \) as the bank’s \textit{reputation}. Like the action belief, the type belief is conditioned upon the payoff history:

\[ \theta : H \rightarrow [0, 1] : h_t \mapsto \theta_{H_t} \equiv \theta_t. \]  

Let \( \Theta \) be the set of all possible type beliefs. Type beliefs and action beliefs give rise to an expected revenue function \( R \) for the bank:

\[ R : \Theta \times H \times B \rightarrow \mathbb{R} : (\theta, h_t, \beta) \mapsto (1 - \theta_t) \nu_D + \theta_t \nu_S(\beta), \]
where $\nu_D$ and $\nu_S(h_t, \beta)$ are the respective time $t$ expected client surpluses generated by D- and S- banks:

\begin{align}
\nu_D &= p + (1 - p) qx; \\
\nu_S(h_t, \beta) &= p\beta_t + p (1 - \beta_t) x + (1 - p) x \\
&= x (1 - p\beta_t) + p\beta_t. \quad (9)
\end{align}

When the action belief function $\beta$ is clear from the context, we will write $\nu_S(\beta_t) = \nu_S(h_t, \beta)$.

Now consider the period $t$ owner of an S-bank. He receives all of the period $t$ revenue that the bank generates, and sells the bank at the end of the period for its time $t$ expected value. We can therefore define the S-bank’s value function $V$:

\begin{align}
V(h_t, \beta, \mu, \theta) &= R(h_t, \beta) + \delta \mathbb{E}_{S,t} [V(\langle h_t, z_t \rangle, \beta, \mu)] \\
&= R(h_t, \beta) + \delta \{ p\mu_t V(\langle h_t, 1 \rangle, \beta, \mu) + (1 - p\mu_t) V(\langle h_t, x \rangle, \beta, \mu) \},
\end{align}

where we write $\langle h_t, z \rangle$ for the history obtained by augmenting $h_t$ with the payoff $z$.

Our equilibrium concept is presented in Definition 1:

**Definition 1:** An equilibrium comprises an action belief $\beta$, a strategy $\mu$ and a type belief $\theta$ such that:

1. For every $h_t$,
   \[ \mu_t \in \arg \max_{\mu_t} p\mu_t V(\langle h_t, 1 \rangle, \beta, \mu_{t+1}, \theta) + (1 - p\mu_t) V(\langle h_t, x \rangle, \beta, \mu_{t+1}, \theta); \]

2. For every $t$, $\beta_t = \mu_t$;

3. $\theta$ is obtained from $\theta_0$ by Bayes’ Law.

Like many other infinitely repeated games, our model admits many equilibria. We focus upon the equilibria that generate the highest payoff for S-banks.\footnote{An alternative approach would be to focus upon those equilibria that maximize social surplus. These approaches are similar for patient banks (those for which $\delta$ is close to 1).} We define $V^\ast(\hat{\theta})$ to be the maximum equilibrium bank value given bank reputation $\hat{\theta}$; the associated optimal equilibrium is the equilibrium $(\beta, \mu, \theta)$ for which the firm’s time 0 value with is $V^\ast(\hat{\theta})$ when $\theta_0 = \hat{\theta}$. Note that this equilibrium is well-defined: if the maximum value was achieved only after $T > 0$ periods then the beliefs, strategy and reputation functions applicable at time $T$ could be used at time 0 to achieve the same value. We assume that $V^\ast(\hat{\theta})$ can always be attained. Then, if $(\beta, \mu, \theta)$ is the time $t$ value-maximizing equilibrium, we must have

\begin{equation}
V^\ast(\theta(h_t)) = R(h_t, \beta_t, \mu_t) + \delta \mathbb{E} [V(\langle h_t, z_{t+1} \rangle, \beta, \mu, \theta)]. \quad (10)
\end{equation}
V. Equilibrium Reputation Management

A. Reputation Building

Recall that the client payoff is $x$ with probability 1 if an S-bank selects action 2, whereas D-banks can achieve this payoff only in state 2, and then only with probability $q$. S-banks can therefore increase their reputation early in their lives by adopting a strategy of selecting $A = 2$ for sure. Lemma 1 demonstrates that the number of periods for which an S-bank has to adopt this strategy to achieve a reputation $\bar{\theta}$ is bounded above for any given $\theta_t$.

**Lemma 1:** An S-bank can increase its reputation from $\theta_t$ to any large $\bar{\theta} < 1$ by choosing action 2 for a fixed $T (\theta_t, \bar{\theta})$ periods.

**Proof:** Given a time $t$ reputation $\theta_t$ and a realized client payoff $x$, the time $t + 1$ reputation $\theta_{t+1}$ is derived from $\theta_t$ by Bayes’ Law as follows:

$$\theta_{t+1} = \frac{\theta_t [p (1 - \mu_t) + (1 - p)]}{\theta_t [p (1 - \mu_t) + (1 - p)] + (1 - \theta_t) (1 - p) q} \tag{11}$$

Let

$$\gamma_{t+1} = \frac{\theta_{t+1} (x)}{1 - \theta_{t+1} (x)}$$

be the time $t + 1$ likelihood ratio for an S-bank, conditional upon a time $t$ client payoff of $x$. We can write

$$\gamma_{t+1} = \frac{\gamma_t}{q} \left[ \frac{(1 - \mu_t) p + (1 - p)}{1 - p} \right]$$

$$\geq \frac{\gamma_t}{q}.$$

Hence

$$\log \gamma_{t+1} \geq \log \gamma_t + \log (1/q)$$

$$\geq \log \gamma_0 + (t + 1) \log (1/q).$$

It follows that $\log \gamma_{t+T} \geq \log \gamma_t + T \log (1/q)$. The result is therefore immediate with

$$T (\theta_t, \bar{\theta}) = \left[ \frac{\log \bar{\theta} - \log \gamma_t}{-\log q} \right] + 1,$$

where $[\cdot]$ is the truncation function. Q.E.D.
B. Type Reputation, Behavioral Reputation, and Ethical banks

Lemma 1 identifies one of the dynamics that drives our results: namely, the possibility that an S-bank could engage in a period of reputation building, so as to reap the benefits of a high reputation in the future. This effect applies in classical models of reputation in which an agent’s reputation relates to his type.

An agent’s type reputation reflects its counterparties’ beliefs about its capabilities. We are concerned in this paper with the extent to which an S-firm’s incentive to build its type reputation is tempered by its desire to sustain a behavioral reputation. An agent’s behavioral reputation reflects the outcomes that its counterparties have experienced: it derives from the agent’s choices, rather than its abilities. In our model, an S-bank builds a behavioral reputation by choosing action 1 in state 1 so that clients experience high payoffs. Choices of this type are usually characterized as ethical, and we refer to an S-bank that takes them as ethical.\(^{18}\)

**Definition 2:** An S-bank’s equilibrium behavior is ethical precisely when it takes action 1 in state 1. An ethical equilibrium is one in which S-banks behave ethically.

Ethical behavior as we have defined it is an equilibrium phenomenon. Moreover, it is inconsistent with type reputation formation, which, as noted in Lemma 1, requires an S-bank repeatedly to take action 2 in state 1. S-banks therefore face a choice between building a type reputation, and behaving ethically. Since ethical behavior in our model is a self-interested phenomenon, it will arise in equilibrium when it is sufficiently profitable.

An S-bank’s equilibrium behavior will be ethical precisely when the following incentive compatibility constraint is satisfied:

\[
V (\langle h_t, 1 \rangle, \beta, \mu, \theta) \geq V (\langle h_t, x \rangle, \beta, \mu, \theta). \tag{12}
\]

Lemma 2 is an immediate consequence of Condition (12).

**Lemma 2:** If an equilibrium is optimal at time \(t\), it remains optimal at time \(t+1\) if \(z_{t+1} = 1\).

**Proof:** If the continuation equilibrium were not optimal, the \(\mu\) and \(\beta\) could be changed to increase the firm’s value when \(z_{t+1} = 1\) realized without violating Constraint (12). By Equation (10), this would increase the time \(t\) value of the firm, which would contradict the fact that the time \(t\) equilibrium was optimal. Q.E.D.

Note that Bayes’ Law (part 3 of Definition 1) gives us the following in any equilibrium.

\[
\theta (\langle h_t, 1 \rangle, \beta) = \theta (h_t, \beta). \tag{13}
\]

\(^{18}\)We acknowledge in the Introduction that ethical choices concern more than a self-interested decision to build a behavioral reputation.
Hence, the firm’s reputation is not updated after a payment of 1. We can use this fact to re-write Equation (10) as follows:

\[ V^* (\theta (h_t)) = R (h_t, \beta_t, \mu_t) + \delta \mathbb{E} [V (\langle h_t, z_{t+1}, \beta, \mu, \theta \rangle)] \leq R (h_t, 1, 1) + \delta V^* (\theta (h_t)), \]  

where the second line follows because revenue is increasing in \( \beta \) and \( \mu \), and because, by the IC constraint (12), the maximum value for \( V (\langle h_t, z_{t+1}, \beta, \mu, \theta \rangle) \) is attained when \( z_{t+1} = 1 \) and is the optimal value \( V^* (\theta (h_t)) \), because first, by Equation (13), reputation is not updated when \( z_{t+1} = 1 \), and second, by Lemma 2, an optimal equilibrium remains optimal upon payment of 1. Rearranging Equation (14) yields the following expression:

\[ V^* (\theta) = \frac{R (h_t, \beta_t = 1, \mu_t = 1)}{1 - \delta} = \frac{\phi (\theta)}{1 - \delta}, \]  

where

\[ \phi (\theta) = \theta \nu_S (1) + (1 - \theta) \nu_D. \]  

is the expected surplus that clients derive from an ethical bank with reputation \( \theta \).

Our analysis is often concerned with limiting effects as \( \delta \to 1 \). In the limit the right hand side of equation (15) is infinite. It is therefore convenient to make the following definition:

**Definition 3:** The time \( t \) standardized bank value \( v \) is \( (1 - \delta) V \).

For a bank with value \( V \), the standardized bank value is the per-period constant revenue \( v \) whose value is \( V \).

At this stage, it is convenient to introduce a simplified notation. When \( h, \beta, \) and \( \mu \) are understood from the context, we define \( V_t (\theta) = V (h_t, \beta_t, \mu_t, \theta) \) and \( v_t (\theta) = v (h_t, \beta_t, \mu_t, \theta) \). Equation (15) immediately yields Proposition 1:

**Proposition 1:** The standardized bank value \( v \) of an ethical S-bank satisfies \( v_t (\theta) \leq \theta \nu_S (1) + (1 - \theta) \nu_D \).

S-banks behave ethically in equilibrium only if it is sufficiently profitable to do so. Proposition 1 indicates that ethical behavior is not profitable for banks with low type reputation. A natural question is therefore whether or not an ethical equilibrium exists for low \( \theta \) banks. To answer this question we need the following technical result:

**Lemma 3:** Given any \( \epsilon, \psi > 0 \) there exists an integer \( K (\epsilon, \psi, \theta_0) \) such that

\[ \Pr [\# \{ t > 0 : \theta_t < 1 - \psi \} \leq K (\epsilon, \psi, \theta_0) \mid S\text{-bank}] \geq 1 - \epsilon. \]
Lemma 3 states that with high probability, an S-bank’s equilibrium reputation deviates from 1 for only a fixed number of periods. The result is proved in the Appendix; its intuition is that in expectation an S-bank reveals some information about its type with strictly positive probability in each period. Hence, over a high number of periods, its type is revealed with very high probability.

The following result is a consequence of Lemma 3.

**Lemma 4:** Let $\bar{\theta} < 1$ be any type reputation. Then a sufficiently patient S-bank has a standardized value $v$ that is bounded below by a value very close to $\bar{\theta}x + (1 - \bar{\theta})$. Formally, given any $\bar{\theta}$ and $\gamma$ there exists a bank discount factor $\delta_{\bar{\theta}, \gamma}$ such that $v_t(\theta) \geq \bar{\theta}x + (1 - \bar{\theta}) - \gamma$.

Lemmas 3 and 4 now yield Proposition 2:

**Proposition 2:** Suppose that there exists $\bar{\theta} < 1$ such that

\[
\bar{\theta}x + (1 - \bar{\theta}) > \phi(\theta_t),
\]

where $\phi(\cdot)$ is defined by equation (16). Then there exists $\bar{\delta} < 1$ such that whenever $\delta > \bar{\delta}$ there does not exist an ethical equilibrium that maximizes the S-bank’s value.

When $\theta_t$ is close to zero there exists a $\bar{\theta}$ that satisfies (17). The Proposition therefore demonstrates that there is no equilibrium in which a patient S-bank with a low type reputation behaves ethically: the value of building a high type reputation in this situation is so great that no set of client beliefs generates sufficiently large punishments to deter unethical behavior designed to build a personal reputation.

This result generates some insight into the dynamics of reputation building and maintenance in young and unproven banks. In the absence of evidence to the contrary, the clients of such banks assign them a low type reputation $\theta_0$. Because $\theta_0$ is low, no equilibrium client beliefs can induce the bank to behave ethically. In other words, the concern for behavioral reputation that drives the results of Morrison and Wilhelm (2004) is unsustainable in our model because it conflicts with the bank’s desire to build a type reputation.

In light of these remarks, we now investigate equilibria in which the bank engages in a period of type reputation building at the beginning of the game, before behaving ethically in order to sustain its behavioral reputation. Such equilibria comprise three behavior states, which we label “phases”:

**Definition 4:** Let $\theta_0$ be the initial reputation of a bank, and let $\theta^* > \theta_0$. Then, if it exists, the phased equilibrium $\mathcal{E}(\theta_0, \theta^*)$ is an equilibrium that comprises the following three phases:

1. The reputation-building phase lasts for the first $T(\theta_0, \theta^*)$ periods. During the reputation-building phase, $\mu_t = \beta_t = 0$;
2. The bank enters the behavioral reputation phase at the end of the reputation-building phase. The bank behaves ethically in the behavioral reputation phase: that is, \( \mu_t = \beta_t = 1 \). The bank remains in the behavioral reputation phase for as long as \( z_t \neq x \). If \( z_t = x \) then a public randomization device determines the game’s phase in the succeeding period. With probability \( \pi_{t+1} \) the bank enters the punishment phase; with complementary probability it remains in the behavioral reputation phase;

3. During the punishment phase \( \mu_t = \beta_t = 0 \). Once the bank enters the punishment phase it never leaves it.

We write \( v_B(\theta_0, \theta^*) \) for the expected time 0 value of S-type banks in the equilibrium \( \mathcal{E}_0(\theta_0, \theta^*) \).

Reputation considerations follow a well-defined life cycle in a phased equilibrium. Young banks concentrate upon building type reputation. Their behavior is anticipated and tolerated by their clients, and it is priced accordingly. After the reputation-building phase has played out, banks maintain a reputation for ethical behavior for as long as possible. Their clients anticipate this, and they pay a correspondingly higher fee for services. When clients have a poor experience from a bank that they expect to behave ethically the bank loses its reputation with some probability \( \pi_{t+1} \). The existence of a phased equilibrium therefore depends upon the existence of a punishment probability \( \pi_{t+1} \) for which the incentive compatibility constraint (12) is satisfied. We know from Proposition 2 that such a probability cannot exist for any \( \theta^* \) for which there is a \( \bar{\theta} \) satisfying equation (17). Assume instead that \( \theta^* \) is sufficiently high to ensure that no \( \bar{\theta} \) satisfies condition (18):

\[
\bar{\theta} x + (1 - \bar{\theta}) \nu_D > \phi(\theta^*). 
\]  

(18)

With this \( \theta^* \) a punishment probability \( \pi_{t+1} \) that satisfies the incentive compatibility constraint (12) exists. To see this, note that the bank’s standardized value in the punishment phase is less than \( x \); its value if it realizes \( z = 1 \) is unchanged in the behavioral reputation phase, because no updating of type reputation occurs (see equation (13)), and its value increases if it remains in the behavioral reputation phase after \( z = x \), because it then receives a boost to its type reputation. A standard continuity argument then implies that there exists a \( \pi_{t+1} \) that ensures that the IC constraint binds at time \( t \).

We have therefore proved the following result:

**Proposition 3:** Let \( \theta^* \) be sufficiently high to ensure that there is no \( \bar{\theta} \) satisfying condition (18). Then there exists a phased equilibrium \( \mathcal{E}(\theta_0, \theta^*) \).

Proposition 3 demonstrates the existence of phased equilibria, in which ethical behavior forms part of a value-maximizing equilibrium strategy, for high enough \( \theta^* \).\(^{19}\) The intuition

\(^{19}\) In the more complex game outlined in footnote 16 the optimal equilibria are again phased equilibria. However, in contrast to the analysis of the main text, equilibria in the more complex game always approximate the first best when \( \lambda \) and \( \delta \) are high enough.
for this result is that, when reputation is close to 1, the marginal value of a higher reputation is low, so that the scale, and hence the efficiency costs, of the punishment required to sustain ethical behavior is limited. The costs of sustaining ethical behavior are much higher for banks that have more to gain from a higher type reputation and, hence, by Proposition 2, such banks will not behave ethically in equilibrium. Note finally that in the phased equilibrium of Proposition 3, the S-bank’s reputation never drops below $\theta^*$ in the behavioral reputation phase: it either remains constant (when $z = 1$) or it increases (when $z = x$ and it does not enter the punishment phase). Furthermore, the likelihood of entering the punishment phase decreases as $\theta$ increases and the costs of doing so increase.

Proposition 3 demonstrates that ethical choices can form part of a value-maximizing equilibrium for S-type banks that have a sufficiently high reputation $\theta > \theta^*$. We have not yet proved the stronger claim that the highest possible S-type value is achieved by such an equilibrium: that is, that there is an optimal phased equilibrium. Proposition 4 establishes that, for sufficiently patient banks, this is the case:

**Proposition 4:** Let $\hat{\theta}^*$ maximize $v_B(\theta_0, \theta^*)$ across the class of phased equilibria $E_0(\theta_0, \theta^*)$ and suppose that $x < v_B(\theta_0, \hat{\theta}^*)$. Then

1. The phased equilibrium $E_0(\theta_0, \hat{\theta}^*)$ is optimal;
2. There is no optimal equilibrium for which $\mu_t > 0$ when $t < T(\theta_0, \hat{\theta}^*)$.

The highest possible per-period income that an S-type without a behavioral reputation can earn is $x$. The assumption in Proposition 4 that $x$ is less than $v_B(\theta_0, \theta^*)$ therefore guarantees that S-types can be punished by the loss of their behavioral reputation.

Finally, we characterize the length of the reputation-building phase of an optimal phased equilibrium. We write

$$T^*(\theta_0, \delta) \equiv T(\theta_0, \hat{\theta}^*)$$

(19)

for the length of the reputation-building phase of the optimal phased equilibrium $E(\theta_0, \hat{\theta}^*)$ when the discount factor is $\delta$.

**Proposition 5:** The length of the reputation-building phase is increasing in the patience of the S-type firm: $T^*(\theta_0, \delta)$ is increasing in $\delta$.

VI. Institutional Design

The problems of Section V arise because the bank has to choose between the benefits of being perceived to be smart, and of being perceived as ethical, and there is conflict between the actions required to create the associated type and behavioral reputations. In this Section, we examine a possible institutional solution to this conflict.
Suppose that the bank is split into an advisory and an execution division. The advisory division has perfect information as to the state of the world. The execution division does not know the state of the world, and it is capable of taking an action $A \in \{1, 2\}$. As in Section V the execution division can be smart, in which case it can take action 1 or 2, or dumb, in which case it can only take action 1. The execution division knows its type, but neither the advisory unit nor its clients do. The advisory unit observes the state of the world $\omega$ and tells the client what services he requires. The client then requests an action of the execution division. The client cannot observe the execution unit’s type or the action that it takes, but the client’s payoff $\mathit{z}$ is common knowledge. Finally, we make the simplifying assumption that, if the advisory division announces state 2 to the client, the execution division is only able to take action 2. Our qualitative results would not be affected by a relaxation of this assumption, but they would be much more complex.

As in Section III, there are many potential clients, each of whom bids for the services of each division. We start by considering the (out-of-equilibrium) case in which the client wins only the execution division’s services. As the execution division does not know $\omega$, by assumption (1), it is optimal for the client if the execution division always takes action 2; as this action also builds the execution division’s type reputation there is no conflict between the client and the bank. The fee for the execution division in this case is

$$\hat{\phi} (\theta_t) \equiv \theta_t x + (1 - \theta_t) \nu_D.$$

We will verify that, when the client retains both the advisory division and the execution division together, it is able to use the advisory division’s information to ensure that the execution division takes the action that maximizes the client’s income. The two units together therefore generate $\phi (\theta_t)$ of value, where $\phi (\cdot)$ is given by equation (16).

We assume that a proportion $\lambda \in (0, 1)$ of the additional surplus generated when the client deals with both divisions of the bank is captured by the advisory division, and that the rest goes to the execution unit. The respective divisions therefore earn fees $\phi_A (\theta_t)$ and $\phi_E (\theta_t)$, where

$$\begin{align*}
\phi_A (\theta_t) &= \lambda (\phi (\theta_t) - \hat{\phi} (\theta_t)) = \lambda \theta_t p (1 - x); \\
\phi_E (\theta_t) &= \theta_t ((1 - \lambda) (1 - x) p + x) + (1 - \theta_t) \nu_D. 
\end{align*}$$

The execution division captures its information rent and a fraction $(1 - \lambda) > 0$ of the advisory surplus.

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20We think of the advisory division as encompassing activities that are less susceptible to formal contracting such as the traditional investment banking function of advising clients on capital raising transactions and mergers and acquisitions. We envision the execution division encompassing functions that are more susceptible to formal contract such as capital market operations, institutional and retail brokerage, and proprietary trading and market making.
We now exhibit an equilibrium of the new game in which the advisory division tells the truth about type and a type S execution division takes action $\omega$ in state $\omega$. The equilibrium is sustained by client action beliefs $\beta = 0$ whenever $z = x$ after the advisory division announces that $\omega = 1$. Precisely as in the proof of Proposition 3, these beliefs ensure that $\mu = 0$, and so generate a credible punishment phase.

With the above beliefs, the advisory bank will tell the truth. If it were to claim that $\omega = 2$ when the true state was 1 then it would benefit from an enhanced S-type execution division reputation (see equation (20)). But it does not know the execution division’s type, and so runs the risk of an immediate loss of revenue if it is D-type. This must be weighed against the inevitability of long-run reputation building (see Lemma 3). A sufficiently patient advisory division will prefer waiting to risking a current lie.

When state 1 is announced, the S-type execution division will receive a standardized payoff close to $(1 - \lambda)(1 - x)p+x$ from truth telling, because its type is almost fully revealed in the long run by Lemma 3. If it takes action 2 then its standardized payoff will be $x$. Because $\lambda < 1$ the former payoff is dominant for a sufficiently patient execution division.

This argument yields Proposition 6:

**Proposition 6:** If $\delta$ is sufficiently close to 1 then there is an equilibrium in which the advising unit always tells the truth and the execution division selects action $\omega$ in state $\omega$.

Proposition 6 demonstrates that the presence of an independent advisory division can resolve the incentive problems that led to inefficient reputation building and punishment phases in the equilibrium of Proposition 3. The advisory unit’s desire to maintain a behavioral reputation for always telling the truth about the state of the world keeps it on the straight and narrow, and its information is sufficient to negate the execution wing’s incentive to build a type reputation at the expense of its clients. Thus Proposition 6 provides an economic rationale for “Chinese Walls” separating functional units in full-service banks. Of course, for this argument to work it is essential that the advisory division be genuinely independent. Its truth telling incentives derive from its inability to observe the execution division’s type. If the execution division could bribe the advisory division to misreport state 1 then the advisory division could infer its type, and the efficient equilibrium would break down.

The efficient equilibrium of Proposition 6 is not the only one for the divisional bank.

**Proposition 7:** When $\theta_0$ is low enough there is an inefficient equilibrium in which the S-type execution division chooses action 2 when state 1 is announced. This equilibrium generates a higher surplus for the execution division than that of Proposition 6.

If the advisory division tells the truth the execution division can reveal itself to be S-type with certainty by taking action 2 once after state 1 is announced. After that there is no need
for further reputation building, so client beliefs $\beta = 1$ are sustainable thereafter. These beliefs generate very high fees in exchange for short-term lower fees from the anticipated signaling. For sufficiently low $\theta_0$ the former outweigh the latter, so that the execution division prefers the suboptimal equilibrium of Proposition 7.

Since low $\theta_0$ implies weak type reputation, Proposition 7 suggests that the clients of reputable advisory operations could suffer from conflicts of interest if their bank’s execution division is less well established or has lost key people with whom the firm’s type reputation is associated. As we noted in the introduction, both conditions were met by the late 1990s as a number of investment banks were acquired by commercial banks and both individuals and teams of bankers became more mobile.

VII. Technological Shocks

So far we have assumed that the bank’s type is fixed. We now relax this assumption by introducing technological shocks to the model that render the bank’s knowledge obsolete. In any period, the probability that a shock occurs is $\kappa > 0$; after a shock the bank’s type is $S$ with probability $\theta$, irrespective of its type prior to the shock. We assume that shocks are publicly observable.

We allow the bank to make an announcement about its type in each period. Such an announcement would have no effect in the fixed type model of earlier sections, since D-banks would have no incentive to tell the truth. But, in the repeated game setting, the ability to make announcements allows first best to be achieved when $\delta$ is close to 1.

Proposition 8: For any $\kappa < 1$ there exists a $\tilde{\delta} < 1$ such that for all $\delta > \tilde{\delta}$ the following constitutes an equilibrium of the game

1. The bank truthfully reveals its type after every shock;
2. For every $t$ the bank takes the state-appropriate action: $\beta_t = \mu_t = 1$;
3. If a bank announces high type after a shock and subsequently a payoff prior to the next shock $z_T$ is 0, $\beta_{\tau} = \mu_{\tau}$ for all $\tau > T$.

This equilibrium achieves the first best. $S$-banks are paid $\nu_S(1)$ in each period, and $D$-banks are paid $\nu_D$.

Proof: $S$-banks clearly have no incentive to mis-represent their type. It is therefore sufficient to prove that D-banks do not wish to lie. The equilibrium has a recursive structure, and we can write the respective equilibrium payoffs of D- and S-banks as follows:

$$V_D = \nu_D + \delta \left[ \kappa (\theta_0 V_S + (1 - \theta_0) V_D) + (1 - \kappa) V_S \right]$$

$$V_S = \nu_S + \delta \left[ \kappa (\theta_0 V_S + (1 - \theta_0) V_D) + (1 - \kappa) V_S \right].$$
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It follows immediately that

\[ V_S = \frac{\nu_D \delta (1 - \theta_0) \kappa + \nu_S (1 - \delta + \delta \theta_0 \kappa)}{(1 - \delta) (1 - \delta (1 - \kappa))} \]

\[ V_D = \frac{\nu_D \delta (1 - \theta_0) \kappa + \nu_D (1 - \delta + \delta \theta_0 \kappa)}{(1 - \delta) (1 - \delta (1 - \kappa))} \]

If a bank is caught in a lie in period \( t \) then its per-period payoff is \( D \) until the next technology shock, after which it is

\[ \nu_P = \nu_D (1 - \theta_0) + \theta_0 \nu_S (0), \]

so that the value \( V_L \) of a bank caught lying satisfies

\[ V_L = \nu_D + \delta \left[ \frac{\kappa}{1 - \delta} \frac{v_P}{1 - \delta} + (1 - \kappa) V_L \right]. \]

This yields the following expression for \( V_L \):

\[ V_L = \frac{1}{1 - \delta (1 - \kappa) (\nu_D + \kappa \delta v_P)}. \]

If a D-bank deviates by announcing itself as an S-bank after a shock, it will continue lying until the next shock, because truth-telling will result in an immediate punishment for sure. The payoff from deviating \( V_{dev} \) is

\[ V_{dev} = \nu_S + \delta \left\{ (1 - p_0) \left[ \kappa (\theta_0 V_S + (1 - \theta_0) V_D) + (1 - \kappa) V_d \right] + p_0 \left[ \kappa V_P + (1 - \kappa) V_L \right] \right\}, \]

where \( p_0 = \mathbb{P}[z = 0] = (1 - p)(1 - q) \). A low type has no incentive to deviate precisely when \( V_d \leq V_D \). As \( \delta \rightarrow 0 \) this expression tends to the following requirement:

\[ \frac{p_0 (v_P - \nu_D (1 - \theta_0) - \nu_S \theta_0)}{p_0 + \kappa (1 - p_0)}, \]

which is less than zero because \( v_P - \nu_D (1 - \theta_0) - \nu_S \theta_0 = \theta_0 (\nu_S (0) - \nu_S (1)) < 0 \). The result is then immediate. Q.E.D.

The intuitive reason that D-banks do not lie in equilibrium is that a technology lasts \( 1/\kappa \) periods in expectation. If the bank is caught lying in those periods then \( \beta \) is zero for all subsequent periods, and it loses its behavioral reputation forever. As \( \delta \rightarrow 1 \) the value of this loss tends to infinity; the gain from lying is a higher fee income, whose expected value is finite, because it has a finite life. The D-bank’s tradeoff therefore favors truth telling.

First best is achieved in the equilibrium of Proposition 8 because the bank’s repeated type announcements allow it to create a behavioral reputation for telling the truth about its type. Moreover, the fact that type is truthfully announced in equilibrium obviates the bank’s need to engage in damaging type-reputation building.

We believe that Proposition 8 would be robust to a set-up in which technological shocks were observed only by the bank. Even in this set-up a bank would be caught lying with
significant probability, and would then lose all of the benefits of its behavioral reputation. In fact, even if the low type’s lying is never fully revealed, as it is in this model with a 0 payoff, we are able to show that close to first best outcomes can be achieved for high enough $\delta$, because clients are able to make statistical inferences from their payoffs, and so establish when the bank has lied a great deal.

A. Shocks that Affect the Bank’s Probability of Survival

Proposition 8 demonstrates that, when the bank has a high degree of patience, first best can be achieved. In this section we ask what the consequences are of lower patience, when technological shocks could force a bank out of business. Casual empiricism suggests that this is a reasonable assumption; it might reflect obsolescence of an existing technology, or a wave of fresh competitors. We do not attempt to model the reasons for failure, but we assume that both S- and D-banks have a failure probability $h$ whenever a technological shock occurs. Making $h$ higher for D-banks would have no qualitative effect upon our results, because D-banks are not strategic.

We focus upon the case with small $\kappa$, high $h$, and $\delta$ close to 1. A large $h$ reduces the value of a reputation for truth telling; Proposition 9 demonstrates that, as a result, it can reverse the efficiency result of Proposition 8.

**PROPOSITION 9:** When $\kappa$ is small, $h$ is large, and $\delta$ is close to 1, there is an equilibrium in which each shock results in a reputation-building phase followed by a behavioral reputation phase, as in Proposition 3. There is no equilibrium in which D-banks announce their type truthfully.

*Proof:* Similar to parts of the proof of Propositions 3 and 8, and hence omitted. Q.E.D.

VIII. Discussion

Our model predicts that the investment-banking industry is subject to reputational life-cycle effects that reflect both a tension between type- and behavioral-reputation concerns and technological shocks that reshape this tension. In this section we use the model to shed light on the events that reshaped the industry during the last half century and, specifically, why it appears that reputation concerns have diminished and therefore have been less effective in governing behavior. Carrying out this exercise requires an observable proxy for the state of a bank’s reputation concerns. We propose that a measure of the strength of bank/client relationships serves this purpose; banks that are perceived as (or have reputations for) having consistently acted in their clients’ best interest in the past and having the skills necessary for a transaction at hand will have stronger client relationships in the sense that they attract
more repeat business. In turn, to the extent that a strong relationship poses a barrier to entry, the attendant rent stream will strengthen reputation concerns.

Figure 2 charts a measure of bank/client relationship strength from 1944 to 2008. The relationship strength variable is developed for each of the top 30 banks by dollar value of securities underwritten during a given decade beginning with 1940-1949. We collected the details of securities offerings between 1933 and 1969 from two sources. Counsel for several defendants in United States v. Henry S. Morgan, et al assembled details of all underwritten issues of $1,000,000 or more from July 26, 1933 to December 31, 1949.\(^{21}\) Data for 1950s and 1960s deals were collected from the Investment Dealers’ Digest.\(^{22}\) Data for issues between 1970 and 2008 were taken from the Thomson Reuters SDC database.

In a given year, for each bank we identify every issuer whose transaction the bank managed. We calculate the total dollar proceeds raised by each such issuer in the preceding 10 years. The strength of the bank’s relationship with the issuer is defined to be the fraction of proceeds underwritten by the bank. For the individual banks reported in Figure 2 the annual relationship strength measure is the average relationship strength among clients for which the bank underwrote a transaction in that year. The annual top 30 measure averages across the annual relationship strength measures for each bank in that year.

Figure 2 reveals that client relationships among the banks that were most active in securities underwriting strengthened during the post-1933 (Glass Steagall) Banking Act period through the 1950s. Relationship strength leveled out during the 1960s when, on average, investment banks had underwritten about 85% by value of their clients’ securities issuance during the preceding 10 years. From the 1970s forward, investment-banking relationships suffered a steady and substantial decline. We report the path of this relationship strength variable for 3 banks (Goldman Sachs, Merrill Lynch, and Morgan Stanley) that were present in the top 30 throughout the sample period to illustrate that while the strength of individual banks’ relationships took different paths to their pinnacle, they followed similar, and ultimately, convergent declines.\(^{23}\)

Proposition 3 of our model predicts that reputable investment banks in a stable tech-


\(^{23}\)See Morrison and Wilhelm (2007, Ch.7) for discussion of the early paths to industry leadership taken by these 3 banks.
Figure 2. Investment bank relationship strength 1944-2008. The strength of a bank’s relationship with a given issuer is defined to be the fraction of the proceeds raised by that issuer in the preceding 10 years that were underwritten by the bank. We show the average strength of the relationships that the illustrated banks have with those clients that issued in any given year. The annual top 30 measure averages across the annual relationship strength measures for each bank in that year.

nological environment work hard to maintain a strong behavioral reputation. We contend that this was the state of affairs through to the late 1950s. Wall Street firms began to computerize their back offices in the 1960s, a process which culminated in the 1970 decision to permit New York Stock Exchange member firms to go public.24 Thereafter the industry was subject to increasingly frequent technological, legal and regulatory shocks.25 Alongside these environmental shocks, the industry replaced a generation of retiring partners for whom few successors were developed during the post-World War II period and witnessed increasing banker mobility and therefore decreasing tenure with any given firm.26 As individual bankers stand at the foundation of the relationships described in Figure 2, it is not likely a coincidence that increasing mobility corresponded with weakening client relationships.

Proposition 8 predicts that investment banks with a sufficiently long-term perspective

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24See Morrison and Wilhelm (2008) for further detail on events immediately preceding and following the shift from private to public ownership among investment banks.
25See Morrison and Wilhelm (2007, Ch.8) for details.
26See Morrison and Wilhelm (2007, Ch.9) for details.
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should respond to such shocks by maintaining their existing behavior patterns and, hence, their reputations; but Proposition 9 predicts those banks that feared for their survival should have lost their reputations and responded by shifting into a reputation-building phase. Both types of behavior occurred in the industry. Banks that anticipated long-term benefits from reputation maintenance behaved in the way that we characterize above as “ethical.” For other players, new technologies and relaxed regulatory strictures served to devalue their existing behavioral reputations, either because their existing markets were obsolete in the changed environment, or, alternatively, were commoditized; the natural response to such a devaluation was to build a new type reputation, through innovation and entry into new markets. Innovation and new markets attracted new entrants who also needed to build type reputations.

We present two examples of this pattern. The first concerns the impact of financial innovation upon the previously relationship-intensive merger and acquisition advisory business. The second is the development of new financial instruments for managing risk.

A. Hostile Takeover Activity in the 1980s

Nickel Co.’s successful 1974 bid for ESB is identified as the turning point when hostile takeovers went from being a rarity to a relatively common event.27 Morgan Stanley’s representation of Nickel Co. would have concerned its broader clientele: hostile takeovers were viewed from the outset as an affront to client relationships as they threatened the existence of the firm.28 By contrast, Goldman Sachs’ defense of ESB provided the foundation for a business strategy that enabled the firm’s rise to the top ranks of the M&A advisory business.29 Drexel Burnham Lambert, a second tier bank with little reputational stake in advisory businesses, was responsible for the innovation that fueled the burst of hostile takeover activity in the 1980s. The junk bond market, developed and dominated by Michael Milken, provided the rapid access to liquid financing that often was central to a successful hostile takeover. Table 1 summarizes the roles played by these three banks as well as the activity of other major M&A advisory banks in the hostile takeover market between 1978 and 1989 (Source: Thomson Reuters SDC database).

Drexel Burnham Lambert and Goldman Sachs adopted opposing positions in the hostile takeover market: the former concentrated on the bidder’s side of the market, while the latter worked almost exclusively for target firms. Both strategies are consistent with our model.

28See Armour and Skeel (2007, pp. 1752-1753) and citations therein. Armour and Skeel note that leading law firms also viewed takeovers as “unsavory” thereby creating an opportunity for Joseph Flom of Skadden to become “the leading takeover lawyer by taking cases the white shoe firms refused to touch.”
29Ironically, Goldman’s role in the transaction stemmed from Bob Hurst’s arrival from Merrill Lynch where Hurst counted ESB among his clients. See Ellis (2009, p. 274)
Traders vs. Relationship Managers

<table>
<thead>
<tr>
<th>Goldman Sachs</th>
<th>Lehman</th>
<th>Morgan Stanley</th>
<th>First Boston</th>
<th>Drexel Burnham Lambert</th>
<th>Salomon Brothers</th>
<th>Lazard Frères</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target (deals)</td>
<td>67</td>
<td>25</td>
<td>38</td>
<td>51</td>
<td>10</td>
<td>24</td>
</tr>
<tr>
<td>($bn)</td>
<td>129</td>
<td>47</td>
<td>89</td>
<td>77</td>
<td>15</td>
<td>59</td>
</tr>
<tr>
<td>Bidder (deals)</td>
<td>4</td>
<td>40</td>
<td>30</td>
<td>22</td>
<td>58</td>
<td>12</td>
</tr>
<tr>
<td>($bn)</td>
<td>9</td>
<td>62</td>
<td>101</td>
<td>43</td>
<td>97</td>
<td>13</td>
</tr>
</tbody>
</table>


Drexel had a negligible presence in advisory businesses prior to its creation of the junk bond market. Its strong interest was therefore in building a type reputation for expertise in the technology of the hostile takeover. Hostile takeovers relied upon rapid financing and execution, and reputations were readily built on the hostile side of the market.

Relative to Drexel, Goldman was a well-established advisory house in 1978. As Figure 2 indicates the firm maintained strong client relationships longer than its peers and and thus could have anticipated long-term reputational rents from its advisory clients. Unlike most of its peers, Goldman remained a private partnership through the 1980s going public only in 1999. Aside from limiting the mobility of key bankers the firm’s relatively modest balance sheet limited opportunities, at least temporarily, for innovation in capital-intensive risk-taking activities. Thus our model predicts that the firm would have found its best interest in maintaining a behavioral reputation by not engaging in business that clients would view as threatening. In fact John Whitehead justified the firm’s policy of not working for hostile bidders “partly as a matter of business ethics, but primarily as a matter of business judgment.”

Morgan Stanley maintained a relatively balanced stance in the hostile takeover market. The firm was among the first to treat M&A advisory as a revenue producing business when Robert Greenhill founded of its M&A department in 1972. And like Goldman, Morgan Stanley maintained strong client relationships through the 1960s. But in addition to its early decline in relationship strength relative to Goldman shown in Figure 2, the firm maintained less than half the number of client relationships maintained by Goldman during the 1960s. Moreover, the firm’s partnership ranks roughly doubled (from 66 to 125 partners) during the 6 years (1980-1985) preceding the firm’s 1986 public offering and it had been on a steeper

---

30Although the firm’s capitalization was among the top 5 investment banks through most of the 1980s, it was small relative to the most heavily capitalized banks. For example, in 1986 the top 4 banks by capitalization were Merrill Lynch ($7.6B), Salomon Brothers ($4.7B), Shearson Lehman ($3.9B), and Goldman ($2B). See Finance Magazine and Securities Industry Databank and Factbook (Securities Industry Association).

31Ellis (2009, p. 271)

32See Table 8.1 in Morrison and Wilhelm (2007).
growth path since the mid 1970s. Over the same time period, Goldman Sachs grew from 61 to 89 partners.\footnote{Partnership figures are the New York Stock Exchange’s Directories of Member Firms. See Figure 4 in Morrison and Wilhelm (2008) for Morgan’s partnership growth path from 1952 to 1986. The Goldman partnership was relatively stable through the 1970s before increasing from 52 partners in 1978 to 63 in 1979.} The combination of rapid growth followed by the firm’s IPO gives rise to conditions under which Morrison and Wilhelm (2004) predict that it would be increasingly difficult for the firm to maintain a reputation for developing and preserving the human capital on which a type reputation might rest. Although Morgan Stanley’s status was more nearly that of Goldman than that of Drexel, from the perspective of our model the firm might rationally have been more inclined than Goldman to risk its client relationships.

B. Risk Management and Trading

The advent of financial engineering and modern portfolio management techniques caused a severe dislocation in financial markets. They created opportunities for innovation and new entry into the investment banking sector. One of the most significant entrants was Bankers Trust, formerly a mid-sized New York-based commercial bank with relatively modest corporate-client relationships, struggling to achieve efficient scale.\footnote{Morrison and Wilhelm (2007, pp. 244-249) provide a brief account of the precipitous and subsequent decline of Bankers Trust. See Guill (2009) for a detailed account from the perspective of an insider.} By the end of the 1980s Bankers Trust was recognised as the leading innovator in risk management and the new over-the-counter derivatives markets.

While Bankers aggressively pursued opportunities in the derivatives markets, it was not the only player. More established institutions also entered the market. In particular, J.P. Morgan was another early entrant into the derivatives markets. But the two institutions were very different, and adopted very different approaches to the new markets. Banker’s Trust appears to have placed a much lower value on client relationships that J.P. Morgan. For example, Bankers was a pioneer of the originate-to-distribute lending model: the firm had a policy of lending only when it was confident that it could immediately selling the exposure to a third party.\footnote{See Carol J. Loomis, “A Whole New Way to Run a Bank,” \textit{Fortune}, September 7, 1992, p.80.} But an important element of a banking relationship is the management and renegotiation of impaired loans. Bankers’ policy of selling its assets eliminated this key element of a banking relationship, and suggests a lack of concern for the client-centric behavior that is the foundation of our behavioral reputation.\footnote{In 1991 George Vojta, Bankers’ Vice Chairman, stated that the firm had “de-emphasized the old-style relationship management that’s so closely linked with lending” and claimed to have built relationships “on the basis of innovation rather than lending” where “very often the solutions [to a client’s problem] involves a transaction.” [emphasis added] See Ida Picker, “Bankers Trust’s Amazing Risk Machine,” \textit{Institutional Investor}, August 1, 1992, p.29.}

In 1993 three of Bankers Trust’s clients, Procter & Gamble, Gibson Greetings, and Mead, suffered substantial losses on derivatives trades that were widely perceived as serving Bankers
Trust’s interests rather than its clients’. By 1995, Bankers faced narrowing margins as the derivatives business was being commoditized. The bank sought to build new client relationships by “spending a significant amount of time in educating our clients” as a means of having them embrace the bank’s continued focus on innovation.37

From the perspective of our model, Bankers’ persistent innovation strategy may have been driven by a combination of rapid adoption of innovative ideas that, increasingly, narrowed margins as competitors grew more sophisticated and the perception among its current and prospective clients that Bankers’ was willing to exploit their relative lack of expertise. The former effectively acted as a technology shock by undermining the value of Bankers’ type reputation, while the latter made it difficult to use its expertise to cement client relationships and thereby weakened incentives to build and protect behavioral reputation.

In contrast to Bankers Trust, a high point of J.P. Morgan’s activities in the derivatives market was its creation of the pioneering Bistro synthetic loan securitization structure, which enabled the firm to manage its risk exposure to a client without upsetting the existing client relationship. J.P. Morgan had a long history of client relationships with Fortune 500 companies from which it could reasonably expect to earn a continuing stream of rents; our model implies that Morgan therefore had a far stronger interest in protecting its behavioral reputation.38 Just as noteworthy as the innovation was Morgan’s willingness to pull back from the market it created as competitors pushed the structure to levels that it could not support. In doing so, Morgan sacrificed market share to its competitors and by 2004 was lagging behind competitors that continued to innovate in the structured finance business.39

The ABACUS 2007-AC1 transaction, sponsored by Goldman Sachs, is a noteworthy and considerably more complex extension of the synthetic structure pioneered by J.P. Morgan.40 The transaction was the subject of a civil complaint against Goldman Sachs by the Securities and Exchange Commission, under which the SEC alleged that Goldman had violated the anti-fraud provisions of federal securities regulations when marketing the transaction.41 In Congressional testimony, Lloyd Blankfein, Chairman and CEO of Goldman, observed that in transactions like ABACUS “clients are buying […] an exposure. […] They are not

38Bistro enabled Morgan to retain a $10B loan portfolio on its balance sheet while insuring against default loss with credit protection purchased from an off-balance sheet special purpose vehicle (the Bistro Trust). Because Morgan retained ownership of the loans, this “synthetic collateralized loan obligation (CLO) had no impact on client relationships. See Farman and Froot (1999), ‘Collateralized Loan Obligations and the Bistro Trust,’ Harvard Business School Press (Case no. 9-299-016).
39See Tett (2009, Ch. 6).
40Griffin, Lowery and Saretto (2012) suggest that increasing complexity in structured products prior to the financial crisis reflected a deliberate attempt by securitization professionals to evade monitoring by banks’ senior management. Biais and Landier (2012) study an OLG model in which such product complexity arises endogenously as agents seek to raise their principal’s monitoring costs and thereby increase rents extracted from the principal. Limited competition in the labor market (for agents) sustains the inefficient equilibrium.
41For a discussion of the deal and its consequences, see Davidoff, Morrison and Wilhelm (2012).
coming to us to represent what our views are."\(^{42}\)

Blankfein’s statement is a clear expression of an arms-length arrangement. In contrast to the traditional advisory functions of investment banks, the trading and execution functions that support transactions like ABACUS are more susceptible to formal contracting and thus less dependent on behavioral reputation as a governance mechanism. It is worth noting that Goldman Sachs in 2007 was very different from the firm as it was described in the preceding subsection. In the aftermath of the firm’s 1999 IPO, revenues produced by the firm’s trading and principal investments businesses rapidly outpaced investment-banking revenues and by a wide margin.\(^{43}\) As such, it is natural for the firm to be more concerned with its (type) reputation for competence in creating a particular exposure than with its (behavioral) reputation for client care. If Goldman believes that the technological environment is changing so rapidly that there is little to be earned from a long-term behavioral reputation, then Proposition 9 of our model suggests that it will respond to each new shock by working anew to build a strong type reputation.

Proposition 6 suggests that a high-reputation firm can resolve the problems of the previous paragraph by separating its advisory businesses from its execution and risk-bearing businesses, and designing appropriate incentive structures. In practice, it seems extremely difficult to erect a genuinely impermeable Chinese wall within a full-service investment bank. The difficulty of doing so provides a market incentive for bankers whose greatest asset is a behavioral reputation to establish standalone advisory firms that aim to avoid even the appearance of conflict. It is not likely a coincidence that many of the highest profile M&A bankers since the 1980s have followed this path.\(^{44}\) In fact, these firms typically identify seeking freedom from conflicts in full-service banks as the motivation for their founding and as a distinguishing feature of the services they provide. Clients have responded positively as evidenced by market share that is out proportion to their size as many appear in the top 20 of the league tables for M&A advisory and fairness opinions.\(^{45}\)

\(^{42}\)See Davidoff et al. (2012, pp. 543-544).

\(^{43}\)In 1998 investment banking produced $3.2B in revenue while trading and principal investments produced $2.4B. In 2007, trading and principal investments produced almost $25.6B while investment banking produced $5.6B.

\(^{44}\)Bruce Wasserstein and Joseph R. Perella might be credited with pioneering this movement with their founding of Wasserstein Perella in 1988 after having run First Boston’s M&A business. Other prominent examples and (their founding dates) include Eric Gleacher (Gleacher Partners, 1990) who started Lehman Brothers’ M&A business; Geoff Boisi (Beacon Group, 1993) who ran Goldman Sachs’s M&A business through the 1980s; Robert Greenhill (Greenhill & Co., 1996); Roger Altman (Evercore Partners, 1996) after heading Blackstone’s M&A; Joseph Perella, Terry Meguid and Peter Weinberg (Perella Weinberg, 2006) after leading investment banking operations at Morgan Stanley and Goldman Sachs (Weinberg), and Ken Moelis (Moelis & Co., 2007) who headed the investment-banking business at UBS. Note also that Lazard Frères maintained a boutique business model even after their 2005 IPO engineered by Bruce Wasserstein.

\(^{45}\)See “Mergers and Acquisitions Review” for year end 2011 published by Thomson Reuters.
IX. Conclusion

We have presented a model in which a bank can build a type reputation for its competence, and a behavioral reputation for ethical behavior. Type reputation formation is in conflict with behavioral reputation maintenance and, in our basic set-up, there is no equilibrium in which a bank with low type reputation takes the ethical actions needed to maintain a behavioral reputation. Our analysis reveals a reputational life-cycle effect: young banks with poor type reputations engage in a reputation building phase, during which they take actions to reveal their abilities even when those actions are not in the interests of their clients; only when their type reputation is established do banks settle into an ethical behavior pattern, acting in their clients’ best interests even when the short-term effects upon the bank of doing so are unappealing.

We investigate the extent to which institutional design choices can resolve the efficiency problem highlighted above. Our analysis indicates that the first best is achievable when the execution division to which type reputation adheres is supplemented by an independent advisory division that does not know the type of the execution division. In other words, apparently ethical behavior is an equilibrium phenomenon in markets that have independent advisory banks. This is one way to understand the increasing importance of such firms in the investment banking business.

Finally, we demonstrate that banks can establish a reputation for telling the truth about their types when those types change frequently. Provided the banks are long-lived, their truth-telling reputations are sufficient to generate first best outcomes. If they are not then the outcome reduces to a series of back-to-back equilibria of the type we study in our base model.

Our analysis is motivated by an interest in understanding why investment banks, that for so long sought to gain their clients’ trust, increasingly appear content to maintain arms-length relationships. Our analysis suggest that the combination of technological changes and commercial bank entry that have given rise to large, complex, full-service banks aggravated a tension between concerns for type reputation and reputation for ethical behavior. Advisory functions remain less susceptible to formal contract and therefore more dependent on gaining and preserving their clients’ trust. But technological advances have increased the capacity for monitoring and measuring performance in the execution of brokerage and risk bearing functions (including the placement of securities with investors) and thereby have made them more susceptible to (arms-length) formal contracting.

We do not contend that being perceived as behaving ethically is of no concern for these functions but rather that it is of less concern in both absolute terms and relative to advisory functions. Our model demonstrates that when a bank’s execution functions have well-established reputations for competence the bank is more likely to enjoy a stable reputa-
tion for ethical behavior. If it were feasible to maintain strict independence between advisory and execution functions, a bank could even avert the incentive distortion that might otherwise arise with, for example, the introduction of a new operational unit. Moreover, what we might think of as modest but steady technological change can reinforce a delicate balance between type- and behavior- reputation concerns.

Conversely, our analysis also illustrates how profound technological shocks that expose banks to a high risk of failure upset this balance. In this setting, our model predicts instability characterized by destruction of both type- and behavioral- reputation followed by periods of rebuilding type-reputation during which clients expect affected banks to place their interests before those of the client if by doing so they enhance their type-reputation. We suggest that this perspective can shed light on the timing of, for example, the first wave of hostile takeover activity during the 1980s and recent events that have followed commercial bank entry into investment banking.

Finally, to the extent that behavioral-reputation concerns complement or substitute for formal regulation, the current regulatory policy debate might benefit from a clear understanding of their role in governing the behavior of investment banks. Our model sheds light on why reputation concerns vary across banking functions, how they interact with one another, and how they might be expected to evolve in the future.

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Traders vs. Relationship Managers


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Proofs

Proof of Lemma 3

Lemma 3 is a result with general implications. It is easiest to prove a more general version of the result, and then to demonstrate that Lemma 3 is a special case of that result.

**Lemma 5:** Let \((\Omega, g)\) be a Borel measurable space. For all \(\epsilon, \psi > 0\) and \(\theta^+ \in (0, 1]\), there exists a positive integer \(K\) such that for all \(\theta_0 \in [\theta^+, 1]\), for any measures \(P, \hat{P}, \tilde{P}\) on \((\Omega, g)\) with \(P = \theta_0 \hat{P} + (1 - \theta_0) \tilde{P}\), and for every filtration \(\{g^t\}_{t \geq 0}, g^t \subset g\), we have

\[
\hat{P}[\# \{t > 0 : \theta_t < 1 - \psi\} \geq K] < \epsilon,
\]

where \(\#\{\cdot\}\) denotes the number of elements in the set \(\{\cdot\}\), whenever
\[
\sum_{\varpi_{t+1} \in g^{t+1}} |\hat{P}^t(\varpi_{t+1}) - \tilde{P}^t(\varpi_{t+1})| \geq \kappa > 0, \forall t. \tag{21}
\]

\(\Omega\) is the space of the outcomes of the game including all the private and public information. Condition (21) says that the probability measures \(\hat{P}\) and \(\tilde{P}\) on the end-of-period outcomes \(\varpi_{t+1} \in g^{t+1}\) are sufficiently different. \(\varpi_{t+1}\) is one realization of all the public information up to time \(t + 1\) in our game. \(^{46}\) \(\hat{P}\) is the probability conditional on a type S, \(\tilde{P}\) is the probability conditional on a type D in an equilibrium. \(\hat{P}^t(\tilde{P}^t)\) is the probability induced by a type S (type D) in period \(t\). \(P\) is the probability belief of the clients, which is a mixture between \(\hat{P}\) and \(\tilde{P}\) with prior \(\theta_0\).

Lemma 3 follows from Lemma 5 if condition (21) is satisfied. To see that this is the case, note that

\[
\sum_{\varpi_{t+1} \in g^{t+1}} |\hat{P}^t(\varpi_{t+1}) - \tilde{P}^t(\varpi_{t+1})| \geq |\hat{P}^t(z_t = x) - \tilde{P}^t(z_t = x)| + |\hat{P}^t(z_t = 0) - \tilde{P}^t(z_t = 0)| = 2(1 - p)(1 - q).
\]

Condition (21) is satisfied with \(\kappa = 2(1 - p)(1 - q)\).

Note that Lemma 5 requires that \(K\) be independent of the particular measures and probability spaces; it follows that Lemma 5 holds uniformly across all equilibria.

The proof is similar to the proof of Lemma 2 in Sorin (1999). We proceed by proving a series of lemmas. First, notice that from the clients perspective, i.e., under measure \(P\), the reputation of a bank is a martingale. That is, \(\theta_t = E_t[\theta_{t+1}]\), which follows from the law of reiterated expectation. The following lemma bounds the one step ahead expected martingale difference.

**Lemma 6:** Suppose \(X^t\) is an uniformly bounded martingale under \(P\) with \(0 < X^t < 1 \forall t\). For all \(\eta > 0\) and \(K \geq 1\),

\[
P(\#\{t \geq 0 : E_t|X^{t+1} - X^t| \geq \eta\} \geq K) \leq \frac{1}{K \eta^2}. \tag{22}
\]

**Proof:** Fix \(m > 0\). Under \(P\)

\[
E\left[\sum_{t=0}^{m} (X^{t+1} - X^t)\right]^2 = E\sum_{t=1}^{m} (X^{t+1} - X^t)^2
\]

\(^{46}\)Notice that \(\varpi_{t+1}\) need not to be discrete. If \(\varpi_{t+1}\) is continuous, (21) is \(\int_{G^{t+1}} |\hat{P}^t(\varpi_{t+1}) - \tilde{P}^t(\varpi_{t+1})|d\varpi_{t+1}\), here \(\hat{P}^t(\varpi_{t+1})\) and \(\tilde{P}^t(\varpi_{t+1})\) is the probability density functions. Technically, the left hand side of (21) is the total variation of \(\hat{P}^t - \tilde{P}^t\). Our proof goes through for continuous \(\varpi_{t+1}\) by changing summation to the corresponding integration.
Lemma 7: Under $\eta$ which is (22). The last inequality follows because $E_{t+1}(X^{t+s} - X^{t+s-1}) = 0$ for $s > 1$. But

$$1 \geq E[X^{m+1} - X^0]^2 = E \left[ \sum_{t=0}^{m} (X^{t+1} - X^t) \right]^2 = E \sum_{t=0}^{m} (X^{t+1} - X^t)^2$$

As $m \to \infty$. Then we have

$$1 \geq E \sum_{t=0}^{\infty} (X^{t+1} - X^t)^2$$

$$= E \sum_{t=0}^{\infty} E_t(X^{t+1} - X^t)^2$$

$$\geq P(\#\{t \geq 0 : E_t(X^{t+1} - X^t)^2 \geq \eta^2 \} \geq K)\eta^2K$$

$$\geq P(\#\{t \geq 0 : E_t|X^{t+1} - X^t| \geq \eta \} \geq K)\eta^2K$$

which is (22). The last inequality follows because $E_t|X^{t+1} - X^t| \geq \eta$ implies $E_t(X^{t+1} - X^t)^2 \geq \eta^2$ by Jensen’s inequality and therefore

$$P(\#\{t \geq 0 : E_t|X^{t+1} - X^t| \geq \eta \} \geq K) \leq P(\#\{t \geq 0 : E_t(X^{t+1} - X^t)^2 \geq \eta \} \geq K).$$

Q.E.D.

The next lemma bounds $(1 - \theta_t)$ using condition (21).

Lemma 7: Under $P$, $(1 - \theta_t) \leq \frac{E_t[\theta_{t+1} - \theta_t]}{\theta_t \kappa}$.

Proof: For any outcome $\omega_{t+1} \in g^{t+1}$

$$E_t[\theta_{t+1} - \theta_t] = E_t \left[ \frac{\theta_t \hat{P}^t(\omega_{t+1}) - \theta_t}{P^t(\omega_{t+1})} - \theta_t \right]$$

$$= \theta_t E_t \left[ \frac{|\hat{P}^t(\omega_{t+1}) - P^t(\omega_{t+1})|}{P^t(\omega_{t+1})} \right]$$

$$= \theta_t E_t \left[ \frac{\hat{P}^t(\omega_{t+1}) - \theta_t \hat{P}^t(\omega_{t+1}) - (1 - \theta_t) \hat{P}^t(\omega_{t+1})}{P^t(\omega_{t+1})} \right]$$

$$= (1 - \theta_t) \theta_t E_t \left[ \frac{P^t(\omega_{t+1}) - \hat{P}^t(\omega_{t+1})}{P^t(\omega_{t+1})} \right]$$

$$= (1 - \theta_t) \theta_t \sum_{\omega_{t+1} \in g^{t+1}} \frac{|\hat{P}^t(\omega_{t+1}) - \bar{P}^t(\omega_{t+1})|}{P^t(\omega_{t+1})} P^t(\omega_{t+1})$$

$$\geq (1 - \theta_t) \theta_t \kappa$$
Q.E.D.

These two lemmas show that $(1 - \theta_t)$ cannot be too large for too many periods if $\theta_t$ is not too small. The next lemma shows that the case where $\theta_t$ is small cannot have a large measure under $\hat{P}$.

**Lemma 8**: $\hat{P}[\theta_t \leq l\theta_0, \forall t] \leq lP[\theta_t \leq \gamma\theta_0, \forall t]$ for $l > 0$.

**Proof**: By Bayes rule, we have

$$\theta_t(\varpi_t) = \frac{\theta_0\hat{P}(\varpi_t)}{P(\varpi_t)}$$

If $\varpi_t$ is such that $\theta_t(\varpi_t) \leq l\theta_0$,

$$\frac{\theta_0\hat{P}(\varpi_t)}{P(\varpi_t)} \leq l\theta_0, \text{ or}$$

$$\hat{P}(\varpi_t) \leq lP(\varpi_t).$$

Integrating over $L = \{\varpi_t|\theta_t(\varpi_t) \leq \gamma\theta_0, \exists t\}$ yields the desired result. Q.E.D.

Notice that Lemma 8 implies that $\hat{P}[\theta_t \leq l\theta_0, \forall t] \leq l$.

Finally, we can prove Lemma 5. By Lemma 7, we have

$$\hat{P}(\#\{t \geq 0 : \theta_t < 1 - \psi\} \geq K) \leq \hat{P}(\{\#\{t \geq 0 : \frac{1}{\kappa \theta_t} E_t[|\theta_{t+1} - \theta_t| > \psi]\} \geq K\})$$

$$\leq \hat{P}(\{\#\{t \geq 0 : \frac{1}{\kappa \theta_t} E_t[|\theta_{t+1} - \theta_t| > \psi]\} \geq K\} \cap \{\theta_t < \gamma\theta_0, \forall t\})$$

$$+ \hat{P}(\{\#\{t \geq 0 : \frac{1}{\kappa l\theta_0} E_t[|\theta_{t+1} - \theta_t| > \psi]\} \geq K\} \cap \{\theta_t < \gamma\theta_0, \forall t\})$$

$$\leq \hat{P}(\{\#\{t \geq 0 : \frac{1}{\kappa l\theta_0} E_t[|\theta_{t+1} - \theta_t| > \psi]\} \geq K\}) + \hat{P}[\theta_t \leq l\theta_0, \forall t]$$

Because $P \geq \theta_0\hat{P}$,

$$\hat{P}(\{\#\{t \geq 0 : \frac{1}{\kappa l\theta_0} E_t[|\theta_{t+1} - \theta_t| > \psi]\} \geq K\}) \leq \frac{1}{\theta_0} P(\{\#\{t \geq 0 : \frac{1}{\kappa l\theta_0} E_t[|\theta_{t+1} - \theta_t| > \psi]\} \geq K\})$$

$$\leq \frac{1}{\theta_0} \frac{1}{K(l\theta_0\psi)^2}.$$  

The second inequality follows from Lemma 6. Therefore,

$$\hat{P}(\{\#\{t \geq 0 : \theta_t < 1 - \psi\} \geq K\}) \leq \frac{1}{\theta_0} \frac{1}{K(l\theta_0\psi)^2} + l, \quad (23)$$

notice $\hat{P}[\theta_t \leq l\theta_0, \forall t] \leq l$ by Lemma 8.

Choose $\gamma = \frac{1}{2} \epsilon$. The RHS of (23) is $\frac{1}{K\psi^2\theta_0^2(\frac{1}{2} \epsilon)^2\kappa^2} + \frac{1}{2} \epsilon$. Pick $K$ large enough so that

$$\frac{1}{K\psi^2\theta_0^2(\frac{1}{2} \epsilon)^2\kappa^2} \leq \frac{1}{2} \epsilon,$$

that is

$$K > K^* = \frac{1}{\psi^2\theta_0^3(\frac{1}{2} \epsilon)^3\kappa^2},$$

we have $\hat{P}(\{\#\{t \geq 0 : \theta_t < 1 - \psi\} \geq K\}) \leq \epsilon$.

Remark: $K^*$ only depends on $\psi, \epsilon, \kappa$, and $\theta_0$. $K^*$ thus applies for any game or filtrations satisfying (21) and $\theta_0 \in [\theta^+, 1]$. 

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Proof of Lemma 4.

Let $\epsilon = \gamma/2$ and $\psi = 1 - \bar{\theta}$. By Lemma 3 there exists an integer $M$ such that $\mathbb{P}[\text{There are more than } K \text{ periods in which } \theta_t < \bar{\theta}] < \epsilon$. Then

$$v_t \geq \epsilon \mathbb{E}[v|\text{More than } K \text{ periods with reputation } < \bar{\theta}]$$

$$+ (1 - \epsilon) \mathbb{E}[v|\text{Fewer than } K \text{ periods with reputation } > \bar{\theta}]$$

$$\geq (1 - \epsilon) \mathbb{E}[v|\text{Fewer than } K \text{ periods with reputation } > \bar{\theta}].$$

Now note that

$$A = \mathbb{E}[v|\text{Every period for which } \theta < \bar{\theta} \text{ is replaced } \bar{\theta}]$$

$$- \text{Avg. additional equivalent income from replacing } \theta \text{ by } \bar{\theta} \text{ in those periods}$$

$$\geq v[B] - (1 - \delta) \left( \sum \text{DFs in periods where replacement occurred} \right)$$

$$\geq v[B] - (1 - \delta^K),$$

where the final line is the sum of the discount factors if the replacement occurs in the first $K$ periods.

$v[B]$ is greater than or equal to the value that would be obtained if $\theta$ in each of the replacement periods was $\bar{\theta}$ and $\beta$ was equal to 0. That value is equal to $(1 - \bar{\theta}) \nu_D + \bar{\theta}x$. Hence,

$$v_t \geq (1 - \epsilon) \left[ (1 - \bar{\theta}) D - (1 - \delta^K) \right]$$

$$= (1 - \bar{\theta}) \nu_D + \bar{\theta}x - \epsilon \left[ (1 - \bar{\theta}) \nu_D + \bar{\theta}x \right] - (1 - \epsilon) (1 - \delta^K)$$

$$\geq (1 - \bar{\theta}) \nu_D + \bar{\theta}x - \epsilon - (1 - \epsilon) (1 - \delta^K)$$

$$\geq (1 - \bar{\theta}) \nu_D + \bar{\theta}x - \gamma/2 - (1 - \delta^K)$$

The result followed immediately for any $\delta$ sufficiently large to ensure that $1 - \kappa^K < \gamma/2$.

Proof of Proposition 4.

**Definition 5:** Consider any equilibrium $\mathcal{E} = (\beta, \mu, \theta)$ of the game. The maximal equilibrium path $h^1 \equiv \{z^1_0, z^1_1, z^1_2, \ldots, z^1_t, \ldots\}$ of $\mathcal{E}$ is defined by setting $z^1_0 = \emptyset$ and for $t \geq 0$ defining $z^1_{t+1}$ inductively as follows:

$$z^1_{t+1} = \begin{cases} 
  x, & \text{if } \mu(z^1_0, z^1_1, \ldots, z^1_t) = 0; \\
  1, & \text{if } \mu(z^1_0, z^1_1, \ldots, z^1_t) > 0.
\end{cases}$$
The maximal equilibrium path \((z_0^1, z_1^1, \ldots, z_t^1)\) to time \(t\) is denoted \(h_t^1\). The maximal strategy \(\mu^1 = \{\mu_0^1, \mu_1^1, \ldots\}\) associated with \(E\) is given for all \(t \geq 0\) by \(\mu_t^1 = \mu (h_t^1)\), and the maximal reputation \(\theta^1 = \{\theta_0^1, \theta_1^1, \ldots\}\) associated with \(E\) is given by \(\theta_0^1 = \theta_0\) and, for all \(t \geq 1\), by \(\theta_t^1 = \theta (h_t^1)\).

For \(t \geq 1\), \(z_t^1\) is the highest possible payoff achievable within \(E\) given history \(\{z_0^1, z_1^1, \ldots, z_{t-1}^1\}\). \(\mu_t^1\) is the strategy that an S-firm follows on the maximal equilibrium path, and \(\theta_t^1\) is the time \(t\) reputation of a firm on the maximal equilibrium path.

We write \(v_t^1\) for value as at time \(t\) of the maximal equilibrium path:

\[
v_t^1 \equiv (1 - \delta) \sum_{i=t}^{\infty} \delta^{i-t} R (\theta_i^1, \mu_i^1).
\]

and \(v^* (\theta_0)\) for the time 0 value of an S-firm under the optimal equilibrium \(E^*\). Since, by the incentive compatibility constraint (12), for any \(t\) the firm has higher value when \(z_t = 1\) realizes than when \(z_t = x\) realizes, we must have

\[
v^* (\theta_0) \leq v_0^1.
\]

Note that, by definition, we must have \(v^* (\theta_0) \geq v_B (\theta_0, \hat{\theta}^* )\).

**Lemma 9:** Under the conditions of Proposition 4, every element \(\mu_t^1\) of the maximal strategy of any optimal equilibrium \(E\) is 0 or 1.

**Proof:** Let \(\tau\) be the largest subset of \(N\) such that \(0 < \mu_t^1 < 1\) for every \(t \in \tau\) and suppose for a contradiction that \(\tau\) is non-empty. We construct an equilibrium \(\hat{\mu} = (\hat{\beta}, \hat{\mu}, \hat{\theta})\) that yields a strictly higher firm value than \(v^* (\theta_0)\). The strategy \(\hat{\mu}\) under \(\hat{E}\) is defined along the maximal equilibrium path \(h^1\) of \(E\) as follows:

\[
\forall t \in N, \hat{\mu} (h_t) = \begin{cases} 1, & t \in \tau; \\ \mu (h_t), & \text{otherwise}. \end{cases}
\]

Hence, by construction, the maximal equilibrium path \(h^1\) under \(\hat{E}\) is the same as the corresponding maximal equilibrium path \(h^1\) under \(E\), and the maximal strategy \(\hat{\mu}^1\) is equal to the maximal strategy \(\mu^1\) except when \(0 < \mu^1 (h_t^1) < 1\), in which case \(\mu (h_t^1) = 1\).

\(\hat{\mu}\) results in weakly higher fee revenue than \(\mu\) along \(h^1\), so that \(R (\theta_t^1, \hat{\mu}_t^1) \geq R (\theta_t^1, \mu_t^1)\). But changing the strategy from \(\mu\) to \(\hat{\mu}\) also changes the reputation from \(\theta\) to a new function \(\hat{\theta}\), which is obtained from \(\hat{\mu}\) via Bayes’ Law. Given a payoff \(x\), reputation updating is performed as follows:

\[
\hat{\theta}_{t+1} (\hat{\mu}, \langle h_t, 1 \rangle) = \frac{\hat{\theta}_t \hat{\mu}}{\hat{\theta}_t \hat{\mu} + (1 - \hat{\theta}_t)}.
\]
\( \hat{\theta}^1 \) therefore agrees with \( \theta^1 \) on any path that ends before the lowest time in \( \tau \). On maximal paths that extend beyond \( \tau \), because \( \hat{\theta}_{t+1} \) is increasing in \( \mu, \hat{\theta}^1 > \theta^1 \). Hence, reputations and revenues along the maximal path \( \hat{h}^1 \) are the same as those along the corresponding path \( h^1 \) before the lowest element of \( \tau \) and are higher thereafter. It follows that \( \hat{v}^1 > v^1 \).

The equilibrium \( \hat{E} \) comprises two states: \textit{punishment} and \textit{non-punishment}. The time 0 state is non-punishment. For as long as the game history \( h \) is an initial segment of the maximal equilibrium path \( \hat{h}^1 \) the game remains in the non-punishment phase. If the history first deviates from the maximal equilibrium path at time \( t \) then there are two cases. First, if \( \hat{\mu}^1_t = 0 \) (so that deviation involves a time \( t \) payoff of 1) then the game enters the punishment phase immediately. Second, if \( \hat{\mu}^1_t = 1 \) (so that deviation involves a time \( t \) payoff of \( x \)) then a public randomisation device is used to determine whether or not the game enters the punishment phase; we write \( \pi_t \) for the probability that the game enters the punishment phase. Once the game has entered the punishment phase it remains there. Hence, the maximal equilibrium path \( \hat{h}^1 \) is the non-punishment path of the equilibrium \( \hat{E} \).

The equilibrium belief \( \hat{\beta} \) is 0 in the punishment phase. In the non-punishment phase with history \( h = \langle h_{t-1}, z_t \rangle \) there are two cases. First, if \( h \) is an initial segment of \( \hat{h}^1 \) then \( \hat{\beta}(h) = \hat{\mu}(h) \). Second, if \( h \) is not an initial segment of \( \hat{h}^1 \) then \( z_t = x \) is a deviation from the maximal equilibrium path and the public randomisation device did not result in punishment. In this case, \( \hat{\beta}(h) = \hat{\mu}^1((h_{t-1}, 1)) \), so that beliefs are formed as if the game had proceeded along the maximal equilibrium path although, of course, the firm’s reputation \( \theta_t \), is enhanced.

In the punishment phase the firm cannot build a behavioral reputation and hence sets \( \mu = 0 \), as in Proposition 3. In the non-punishment phase it sets \( \mu = \beta \) provided the incentive compatibility constraint (26) holds:

\[
v_{t+1} \left( \theta_{t+1}(h_t, 1), \hat{\beta}, \hat{\mu}, \hat{\theta} \right) \geq v_{t+1} \left( \theta_{t+1}(h_t, x), \hat{\beta}, \hat{\mu}, \hat{\theta} \right). 
\] 

Suppose now that Condition (26) binds. Then the beliefs \( \hat{\beta} \) are correct in equilibrium along the path \( \hat{h}^1 \). Suppressing dependence upon \( \hat{\beta}, \hat{\mu} \) and \( \hat{\theta} \) to simplify notation, the standardised value of the firm in the non-punishment phase at any time \( t \) is then

\[
v_t(\theta_t) = \begin{cases} 
(1 - \delta) R(\theta_t, 1) + \delta v_{t+1}(\theta_{t+1}(h_t, 1)), & \hat{\mu}^1_t = 1; \\
(1 - \delta) R(\theta_t, 0) + \delta v_{t+1}(\theta_{t+1}(h_t, x)), & \hat{\mu}^1_t = 0,
\end{cases}
\]

where the first equality is a consequence of the assumption that Condition (26) binds. Using these equalities inductively along the maximal equilibrium path gives us

\[
\hat{v}_t(\theta_t) = (1 - \delta) \sum_{i=t}^{\infty} \delta^{i-1} R(\hat{\theta}^1_i, \hat{\mu}^1_i) = \hat{v}^1(\theta_t).
\]
That is, when Condition (26) binds, $\hat{E}$ constitutes an equilibrium for which the firm value is $\hat{v}^1$, which exceeds the value of the optimal equilibrium by Equation (25), and, hence, gives us the desired contradiction. If we can exhibit a punishment probability $\pi_t$ for which Condition (26) binds then the proof is complete. The existence of this probability follows from a standard continuity argument. First, if $\pi_t$ were zero we would have

$$\hat{v}_{t+1} \left( \langle h_t, x \rangle, \hat{\beta}, \hat{\mu}, \hat{\theta} \right) \geq \hat{v}_{t+1} \left( \langle h_{t}, 1 \rangle, \hat{\beta}, \hat{\mu}, \hat{\theta} \right),$$

because $\theta_t \langle h_t, x \rangle > \theta_t \langle h_t, 1 \rangle$.

On the other hand, if $\pi_t$ were 1 then the continuation value would be less than $x$, which is less than $v_B (\theta_0, \hat{\theta}^*)$ by assumption in Proposition 4, which, in turn is less than $\hat{v}^1$. This concludes the proof of the Lemma. Q.E.D.

We now prove Proposition 4. Consider the time 0 optimal equilibrium $E^*$: that is, the equilibrium that achieves the maximum value $V^* (\theta_0)$ when the reputation is $\theta_0$. We claim that the equilibrium achieves the maximal value $V^*$ at every point on the maximal equilibrium path $h^1$.

Let $T \geq 0$ be the first period at which the firm sets $\mu > 0$ along the maximal equilibrium path. Because $\mu = 0$ for $t < T$, the incentive compatibility constraint need not be satisfied when $t < T$. Hence, we can fix $\beta$ and $\mu$ as we please when $t < T$ along the maximal equilibrium path. Suppose that at some $\tau < T$ the maximal equilibrium value $V^*$ was not realised at $h_{\tau}^1$. Then $\beta$ and $\mu$ could be altered to increase the firm’s time $\tau$ value; by Equation (10) this would raise the time 0 value of the firm and, hence, would contradict the statement that the time 0 equilibrium was optimal. The time 0 equilibrium therefore remains optimal along $h^1$ for $t < T$.

For $t = T$ we have $\mu = 1$, by Lemma 9. The firm remains on the maximal equilibrium path only if $z_{T+1} = 1$. If this happens then, by Lemma 2, the equilibrium remains optimal. Moreover, by equation (15), we have

$$v^* (\theta (h_T^1)) \leq R (\theta (h_T^1), 1, 1). \tag{27}$$

The optimal standardized value is attained with history $h_T^1$, and, by equation (27), is no greater than $R (\theta (h_T^1), 1, 1)$. This value is achieved in the behavioral reputation phase in a phased equilibrium. Hence the optimal value can be achieved along a path that has $\mu = 0$ for $T - 1$ periods before entering a behavioral phase. Part 2 of the Proposition follows from Lemma 9 and the fact that the reputation-building phase is on the maximal equilibrium path.
Proof of Proposition 5.

The firm value at \( t = 0 \) for a given \( T \) periods of reputation-building phase is

\[
v(\theta_0, T, \delta) = (1 - \delta) \left[ \sum_{t=0}^{T-1} \delta^t R(\theta_t, 0) + \frac{\delta^T}{1 - \delta} v(\theta_T) \right]
\]

\[
= (1 - \delta) \sum_{t=0}^{T-1} \delta^t R(\theta_t, 0) + \delta^T R(\theta_T, 1)
\]

The second equality follows because \( v(\theta_T) = R(\theta_T, 1) \). In the reputation-building phase,

\[
\theta_{t+1} = \theta_{t+1}(x) = \frac{\theta_t}{\theta_t + (1 - \theta_t)(1 - p)q} \quad \text{for } t \leq T.
\]

Therefore we can find the optimal \( T^* \) by solving

\[
\max_T v(\theta_0, T)
\]

(29)

Let \( \delta_1 > \delta_2 \). We need to show that \( T_1 \equiv T^*(\theta_0, \delta_1) \geq T_2 \equiv T^*(\theta_0, \delta_2) \).

For ease of notation, we drop * for the rest of the proof. Suppose not. We have \( T_2 > T_1 \). \( v(\theta_0, T_2, \delta_1) - v(\theta_0, T_1, \delta_1) \leq 0 \) implies

\[
v(\theta_0, T_2, \delta_1) - v(\theta_0, T_1, \delta_1)
\]

\[
= (1 - \delta_1) \sum_{t=T_1}^{T_2-1} \delta_t^i [R(\theta_t, 0) - R(\theta_{T_1}, 1)] + \delta_{T_2}^i [R(\theta_{T_2}, 1) - R(\theta_{T_1}, 1)]
\]

\[
\leq 0.
\]

(30)

The equality follows because

\[
v(\theta_0, T_1, \delta_1) = (1 - \delta_1) \sum_{t=0}^{T_1-1} \delta_t^i R(\theta_t, 0) + \delta_{T_1}^i R(\theta_{T_1}, 1)
\]

\[
= (1 - \delta_1) \left[ \sum_{t=0}^{T_1-1} \delta_t^i R(\theta_t, 0) + \sum_{t=T_1}^{T_2-1} \delta_{T_1}^i R(\theta_{T_1}, 1) \right] + \delta_{T_2}^i R(\theta_{T_1}, 1).
\]

(30) is equivalent to

\[
(1 - \delta_1) \sum_{i=0}^{T_2-T_1-1} \delta_t^i [R(\theta_{T_1+i}, 0) - R(\theta_{T_1}, 1)] + \delta_{T_2-T_1}^i [R(\theta_{T_2}, 1) - R(\theta_{T_1}, 1)] \leq 0.
\]

(31)

Notice that \( (1 - \delta_1) \sum_{i=0}^{T_2-T_1-1} \delta_t^i + \delta_{T_2-T_1}^i = 1 \). Therefore, \( \{(1 - \delta_1) \delta_t^0, (1 - \delta_1) \delta_t^1, \ldots, \delta_{T_2-T_1}^i\} \) is a probability measure, with density \( f_1(i) \), on state \( i \in \{0, T_2 - T_1\} \). Thus, (31) is

\[
E_1[\Delta R_i] \leq 0,
\]

(32)

\[47\] T_1^* \in T^*(\theta_0, \delta_1) \geq T_2^* \in T^*(\theta_0, \delta_2) \text{ if } T^*(\theta_0, \delta_1) \text{ is not unique.}
where
\[ \Delta R_i = \begin{cases} R(\theta_{T_1+i}, 0) - R(\theta_{T_1}, 1), i \in \{0, T_2 - T_1 - 1\} \\
R(\theta_{T_2}, 1) - R(\theta_{T_1}, 1), i = T_2 - T_1 \end{cases} \]

\[ E_1[.] \] the the expectation under measure \( f_1(i) \).

Similarly, \( v(\theta_0, T_2, \delta_2) - v(\theta_0, T_1, \delta_2) \geq 0 \) implies
\[ E_2[\Delta R_i] \geq 0. \tag{33} \]

Notice that because \( \delta_1 > \delta_2 \), \( f_1(i) \) first-order stochastically dominates \( f_2(i) \). This is because CDF of \( f_1 \) is
\[ F_1(i) = \begin{cases} 1 - \delta_1^{i+1}, i \in \{0, T_2 - T_1 - 1\} \\
1, \ i = T_2 - T_1 \end{cases} \]

\( F_2(i) \) is similarly derived and therefore we can conclude that \( F_1(i) \leq F_2(i) \). Notice also that \( \Delta R_i \) is strictly increasing in \( i \) because \( \theta_{T_1+i} \) is strictly increasing in \( i \). First-order stochastic dominance implies
\[ E_1[\Delta R_i] > E_2[\Delta R_i] \geq 0 \]

which contradicts (32). Q.E.D.